Fast Adaptive Bilateral Filtering of Color Images

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IEEE International Conference on Image Processing, Taipei (2019)

Nonlinear edge-preserving smoothing[1]:

$$\boldsymbol{g}(\boldsymbol{i}) = \eta(\boldsymbol{i})^{-1} \sum_{\boldsymbol{j}\in\Omega} \omega(\boldsymbol{j})\phi\big(\boldsymbol{f}(\boldsymbol{i}-\boldsymbol{j}) - \boldsymbol{f}(\boldsymbol{i})\big)\boldsymbol{f}(\boldsymbol{i}-\boldsymbol{j}),$$

$$\eta(\boldsymbol{i}) = \sum_{\boldsymbol{j}\in\Omega} \omega(\boldsymbol{j})\phi\big(\boldsymbol{f}(\boldsymbol{i}-\boldsymbol{j}) - \boldsymbol{f}(\boldsymbol{i})\big),$$

where

- $\boldsymbol{f}$ and $\boldsymbol{g}$ are the input and output RGB images.
- $\boldsymbol{f}(\boldsymbol{i})$ and $\boldsymbol{g}(\boldsymbol{i})$ are vectors.
- ω and ϕ = Gaussian kernels with variance ρ^2 and σ^2.
- Ω = Neighborhood for averaging.

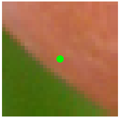
[1] Tomasi and Manduchi, 1998

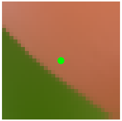
Role of σ

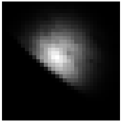

Input.

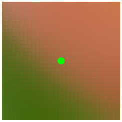

Output, $\sigma = 30$.

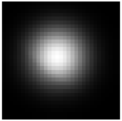

Output, $\sigma = 200$.






Weights




Weights

Adaptation of σ

- σ (width of range kernel) controls the extent of blurring.
- A fixed σ either over or under smooths.
- Useful for controlling the blur in different regions, e.g., more blur to remove coarse textures in images.
- σ is allowed to change at each pixel (a rule is required).
- Proposed for a couple of applications (for grayscale images):
  - Image sharpening[2].
  - JPEG deblocking[3].

[2] Zhang and Allebach, 2008.
[3] Zhang and Gunturk, 2009.

- Make the width of the range kernel a function of $\boldsymbol{i}$.
- Moreover, allow center[4] to be different from $\boldsymbol{f}(\boldsymbol{i})$.

$$\boldsymbol{g}(\boldsymbol{i}) = \eta(\boldsymbol{i})^{-1} \sum_{\boldsymbol{j}\in\Omega} \omega(\boldsymbol{j})\ \phi_{\boldsymbol{i}}\big(\boldsymbol{f}(\boldsymbol{i}-\boldsymbol{j}) - \boldsymbol{\theta}(\boldsymbol{i})\big)\boldsymbol{f}(\boldsymbol{i}-\boldsymbol{j}),$$

$$\eta(\boldsymbol{i}) = \sum_{\boldsymbol{j}\in\Omega} \omega(\boldsymbol{j})\ \phi_{\boldsymbol{i}}\big(\boldsymbol{f}(\boldsymbol{i}-\boldsymbol{j}) - \boldsymbol{\theta}(\boldsymbol{i})\big)\boldsymbol{f}(\boldsymbol{i}-\boldsymbol{j}).$$

- However, a fixed spatial kernel is used.

[4] Zhang and Allebach, 2008.

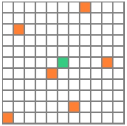
Computation cost

- $O(\rho^2)$ computations per pixel.
- Higher ρ (window size) is used for higher-resolution images.
- e.g. 60 seconds for a 2 megapixel image on a CPU.
- Real-time implementation is challenging.
- Fast approximation: Approximate the original formula and hope to speed it up, without appreciable loss of visual information.

- Several fast algorithms for classical bilateral filtering (gray/color).
- Complexity does not scale with filter width ($O(1)$ implementation).
- Almost all fundamentally require the range kernel to be fixed.
- Filtering reduced to fast convolutions by approximating the range kernel.
- Rules out extension to ABF (range kernel is changing).

- Novel $O(1)$ algorithm for fast ABF of color images.
- Builds on a recently proposed algorithm for gray images[5].
- Trivial channel-by-channel extension to color images (3X cost).
- Filtering in RGB space?
- As explained later, this poses technical challenges.
- Core idea: Express filtering using local (weighted) histograms[6].

[5] Gavaskar and Chaudhury, 2019.
[6] Mozerov and van de Weijer, 2015.



- Local histogram at pixel $\boldsymbol{i}$:

$$h_{\boldsymbol{i}}(\boldsymbol{t}) = \sum_{\boldsymbol{j}\in\Omega} \delta\big(\boldsymbol{f}(\boldsymbol{i}-\boldsymbol{j}) - \boldsymbol{t}\big), \quad \boldsymbol{t} \in \{0,\ldots,255\}^3.$$

- $\boldsymbol{t} = (t_r, t_g, t_b)$ and $\delta(\boldsymbol{t}) = \delta(t_r)\ \delta(t_g)\ \delta(t_b)$.

- Local weighted histogram at pixel $\boldsymbol{i}$:

$$h_{\boldsymbol{i}}(\boldsymbol{t}) = \sum_{\boldsymbol{j}\in\Omega} \omega(\boldsymbol{j})\ \delta\big(\boldsymbol{f}(\boldsymbol{i}-\boldsymbol{j}) - \boldsymbol{t}\big), \quad \boldsymbol{t} \in \{0,\ldots,255\}^3.$$

- Interpretation: Spatially-weighted frequency of RGB value $\boldsymbol{t}$.

ABF in terms of local weighted histograms:

$$\boldsymbol{g}(\boldsymbol{i}) = \eta(\boldsymbol{i})^{-1} \sum_{\boldsymbol{t}} \boldsymbol{t} h_{\boldsymbol{i}}(\boldsymbol{t}) \phi_{\boldsymbol{i}}\big(\boldsymbol{t} - \boldsymbol{\theta}(\boldsymbol{i})\big),$$

and

$$\eta(\boldsymbol{i}) = \sum_{\boldsymbol{t}} h_{\boldsymbol{i}}(\boldsymbol{t}) \phi_{\boldsymbol{i}}\big(\boldsymbol{t} - \boldsymbol{\theta}(\boldsymbol{i})\big),$$

where sum is over RGB values in the neighborhood of $\boldsymbol{i}$.

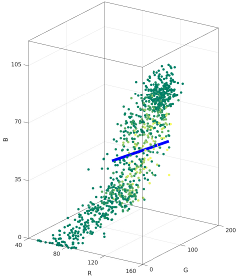
- ABF for grayscale images can be similarly reformulated.
- In grayscale, $h_{\mathbf{i}}(t)$ is a function of a scalar variable.
- For fast algorithm, $h_{\mathbf{i}}(t)$ is approximated using polynomials[7].
- This gave closed-form Gaussian integrals.
- Histogram approximation using fast convolutions (moment matching).
- For color images, $h_{\mathbf{i}}(\boldsymbol{t})$ is a function of a vector variable.
- Polynomial approximation is bad due to sparse data.

[7] Gavaskar and Chaudhury, 2019.

- Motivated by the approach in Mozerov and van de Weijer[8]:
  - $h_{\boldsymbol{i}}(\boldsymbol{t})$ is constant over an interval $[\boldsymbol{a_i}, \boldsymbol{b_i}]$ (in $\mathbb{R}^3$).
  - $h_{\boldsymbol{i}}(\boldsymbol{t})$ is zero elsewhere.
  - Summations are replaced by line integrals:

$$\hat{\boldsymbol{g}}(\boldsymbol{i}) = \hat{\eta}(\boldsymbol{i})^{-1} \int_{[\boldsymbol{a_i},\boldsymbol{b_i}]} \boldsymbol{t}\ \phi_{\boldsymbol{i}}\big(\boldsymbol{t} - \boldsymbol{\theta}(\boldsymbol{i})\big)\, d\boldsymbol{t},$$

$$\hat{\eta}(\boldsymbol{i}) = \int_{[\boldsymbol{a_i},\boldsymbol{b_i}]} \phi_{\boldsymbol{i}}\big(\boldsymbol{t} - \boldsymbol{\theta}(\boldsymbol{i})\big)\, d\boldsymbol{t}.$$



- The integrals, and hence the filter, have a closed-form expression.
- By clever choice of the interval, the computation becomes $O(1)$.

[8] Mozerov and van de Weijer, 2015.

Novelty of our proposal

- In Mozerov and van de Weijer, the interval was chosen to be
  - passing through $\boldsymbol{f}(\boldsymbol{i})$.
  - having direction $\bar{\boldsymbol{f}}(\boldsymbol{i}) - \boldsymbol{f}(\boldsymbol{i})$, where $\bar{\boldsymbol{f}}(\boldsymbol{i}) =$ mean value.
- This makes the algorithm $O(1)$, but is an ad-hoc choice.
- We choose the interval such that it captures linear trend of data.
- To do this, we use the covariance of the local weighted histogram.
- Our proposed algorithm is also $O(1)$.

- Covariance matrix:

$$\mathrm{C}_{\boldsymbol{i}} = \sum_{\boldsymbol{j}\in\Omega} \omega(\boldsymbol{j})\big(\boldsymbol{f}(\boldsymbol{i}-\boldsymbol{j}) - \bar{\boldsymbol{f}}(\boldsymbol{i})\big)\big(\boldsymbol{f}(\boldsymbol{i}-\boldsymbol{j}) - \bar{\boldsymbol{f}}(\boldsymbol{i})\big)^{\top}.$$

- Direction of $[\boldsymbol{a}_{\boldsymbol{i}}, \boldsymbol{b}_{\boldsymbol{i}}]$ = Largest eigenvector of the covariance matrix.

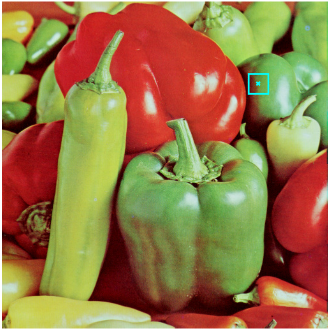
- This should give "best" linear approximation of the set of data points.

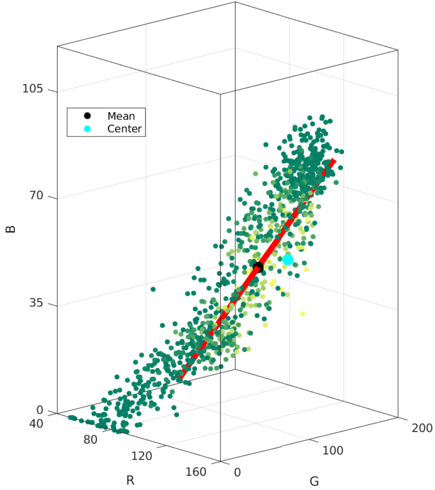
- Proposal:

$$[\boldsymbol{a}_{\boldsymbol{i}}, \boldsymbol{b}_{\boldsymbol{i}}] = \Big[\bar{\boldsymbol{f}}(\boldsymbol{i}) - c\sqrt{\lambda_{\boldsymbol{i}}}\ \boldsymbol{q}_{\boldsymbol{i}}, \bar{\boldsymbol{f}}(\boldsymbol{i}) + c\sqrt{\lambda_{\boldsymbol{i}}}\ \boldsymbol{q}_{\boldsymbol{i}}\Big];$$

$$(\lambda_{\boldsymbol{i}}, \boldsymbol{q}_{\boldsymbol{i}}) = \text{Top eigenpair of } \mathrm{C}_{\boldsymbol{i}},$$

$$c = \text{Positive constant, decides length of the interval.}$$





Mean
Center
B
105
70
35
0
R
40
80
120
160
G
0
100
200

Fast computation of interval endpoints

- Recall: $\boldsymbol{a_i} = \bar{\boldsymbol{f}}(\boldsymbol{i}) - c\sqrt{\lambda_{\boldsymbol{i}}}\ \boldsymbol{q_i}$, $\boldsymbol{b_i} = \bar{\boldsymbol{f}}(\boldsymbol{i}) + c\sqrt{\lambda_{\boldsymbol{i}}}\ \boldsymbol{q_i}$.
- We must find a fast method to compute the end points.
- $O(1)$ Gaussian convolutions come to our rescue.
- $\bar{\boldsymbol{f}}(\boldsymbol{i}) = \omega * \boldsymbol{f}(\boldsymbol{i}) \rightarrow 3$ Gaussian convolutions.
- (p, q)th entry of $\mathrm{C}_{\boldsymbol{i}} = \omega * (f_p f_q)(i) - \big(\omega * f_p(i)\big)\ \big(\omega * f_q(i)\big)$.
- 6 additional Gaussian convolutions to compute $\mathrm{C}_{\boldsymbol{i}}$'s.

Fast computation of interval endpoints

- $(\lambda_{\boldsymbol{i}}, \boldsymbol{q_i})$ computed using power iterations method.
- Power iterations:
  - Initialize $\boldsymbol{q_i}$ as unit vector along $\bar{\boldsymbol{f}}(\boldsymbol{i}) - \boldsymbol{f}(\boldsymbol{i})$.[9]
  - Iterate: $\boldsymbol{q_i} \leftarrow \mathsf{C}_{\boldsymbol{i}} \boldsymbol{q_i} / \|\boldsymbol{q_i}\|$.
- In practice, just one iteration is enough.
- $\lambda_{\boldsymbol{i}} = \boldsymbol{q_i}^\top \mathsf{C}_{\boldsymbol{i}} \boldsymbol{q_i}$.
- Overall, computation of $\boldsymbol{a_i}, \boldsymbol{b_i}$ requires $O(1)$ operations.

[9] Direction used in Mozerov and van de Weijer, 2015.

- Recall:

$$\hat{\boldsymbol{g}}(\boldsymbol{i}) = \hat{\eta}(\boldsymbol{i})^{-1} \int_{[\boldsymbol{a_i}, \boldsymbol{b_i}]} \boldsymbol{t}\, \phi_{\boldsymbol{i}}\big(\boldsymbol{t} - \boldsymbol{\theta}(\boldsymbol{i})\big)\, d\boldsymbol{t},$$

$$\hat{\eta}(\boldsymbol{i}) = \int_{[\boldsymbol{a_i}, \boldsymbol{b_i}]} \phi_{\boldsymbol{i}}\big(\boldsymbol{t} - \boldsymbol{\theta}(\boldsymbol{i})\big)\, d\boldsymbol{t}.$$

- The integrals have closed-form expressions in terms of $\boldsymbol{a_i}, \boldsymbol{b_i}$.
- This was made possible due to the nature of the approximation.
- As computation of $\boldsymbol{a_i}, \boldsymbol{b_i}$ is $O(1)$, computation of $\hat{\boldsymbol{g}}(\boldsymbol{i})$ becomes $O(1)$.

- Closed-form expression (mean + first-order correction):

$$\hat{\boldsymbol{g}}(\boldsymbol{i}) = \bar{\boldsymbol{f}}(\boldsymbol{i}) + \left(2\left(\beta - \alpha e_1 e_2^{-1}\right) - 1\right) c\sqrt{\lambda_{\boldsymbol{i}}}\,\boldsymbol{q}_{\boldsymbol{i}},$$

where

$$\begin{aligned}
\alpha &= \sigma(\boldsymbol{i})/c\sqrt{2\pi\lambda_{\boldsymbol{i}}}, \\
\beta &= \frac{1}{2c\sqrt{\lambda_{\boldsymbol{i}}}}\boldsymbol{q}_{\boldsymbol{i}}^{\top}\left(\boldsymbol{\theta}(\boldsymbol{i}) - \bar{\boldsymbol{f}}(\boldsymbol{i}) + c\sqrt{\lambda_{\boldsymbol{i}}}\mathrm{q}_{\boldsymbol{i}}\right), \\
e_1 &= \exp\left(-\frac{(1-\beta)^2}{\pi\alpha^2}\right) - \exp\left(-\frac{\beta^2}{\pi\alpha^2}\right), \\
e_2 &= \operatorname{erf}\left(\frac{1-\beta}{\sqrt{\pi}\alpha}\right) - \operatorname{erf}\left(-\frac{\beta}{\sqrt{\pi}\alpha}\right).
\end{aligned}$$

- Main point: All computations are $O(1)$.

1. Compute $\omega * (f_p f_q)$, $\omega * f_p$ for $p, q = 1, 2, 3$ using $O(1)$ convolutions.

2. For each pixel $\boldsymbol{i}$,

   2.1 Populate $\mathrm{C}_{\boldsymbol{i}}$ using the above convolved quantities.

   2.2 Estimate dominant eigenpair $(\lambda_{\boldsymbol{i}}, \boldsymbol{q}_{\boldsymbol{i}})$ by power iterations method.

   2.3 Compute α, β, e_1, e_2 in the previous slide.

   2.4 Compute $\hat{\boldsymbol{g}}(\boldsymbol{i})$ using the formula in the previous slide.

Dominant cost = 9 Gaussian convolutions.

Brief overview:

- Objective: Enhance details, but not to the same extent everywhere.
- More enhancement in regions which are more visually salient.
- Can be accomplished using the ABF[10].
- $\sigma(\boldsymbol{i})$ is decided using a saliency map.
- $\boldsymbol{\theta}(\boldsymbol{i}) = \boldsymbol{f}(\boldsymbol{i})$.
- We use our proposed algorithm for color filtering.

[10] Ghosh et al., 2019.

Input (640×960).

Enhanced, $\rho = 5$.

Saliency map.

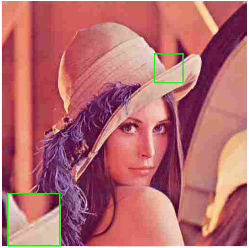
σ map.

Timings: Brute-force $=$ 27 sec., Proposed $=$ 1.4 sec.

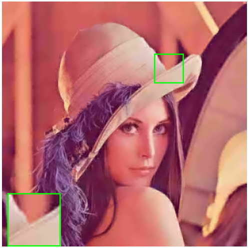
Brief overview:

- Objective: Smooth out blocking artifacts in JPEG-compressed images.
- For grayscale images, can be accomplished using ABF[11][12].
- We extend the same idea to color images.
- $\sigma(\boldsymbol{i})$ is decided using a technique proposed previously [11].
- $\boldsymbol{\theta}(\boldsymbol{i}) = \boldsymbol{f}(\boldsymbol{i})$.
- We use our proposed algorithm for filtering.

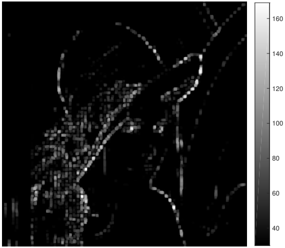
[11] Zhang and Gunturk, 2009.
[12] Gavaskar and Chaudhury, 2019.



Input (512×512).



Deblocked, $\rho = 4$.



σ map.

Original.

Timings: Brute-force $= 8.4$ sec., Proposed $= 0.6$ sec.

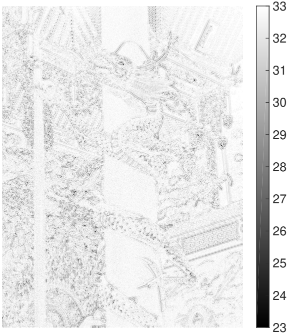
Brief overview:

- Objective: Sharpen a blurred image containing fine noise grains.
- For grayscale images, can be accomplished using ABF[13].
- We extend the idea to color images.
- Both $\sigma(\boldsymbol{i})$ and $\boldsymbol{\theta}(\boldsymbol{i})$ are decided using previously proposed techniques.
- We use our proposed algorithm for filtering.

[13] Zhang and Allebach, 2008.

Input (1600 $\times$ 1200).

Sharpened, $\rho = 4$.



σ map.

Timings: Brute-force = 62 sec., Proposed = 4.4 sec.

Conclusion

- Proposed $O(1)$ algorithm for adaptive bilateral filtering of color images.
- First such algorithm to the best of our knowledge.
- Core idea: Approximate local histogram as uniform along direction of maximum variance.
- Achieves about $15\times$ speedup with reasonable accuracy.
- Useful for detail enhancement, sharpening, and deblocking.
- Better accuracy and extension to non-Gaussian kernels?

Research supported by EMR grant SERB/F/6047/2016-2017 from DST-SERB, Government of India.

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Thanks for listening!