

# NON-LOCAL OPERATIONAL ANISOTROPIC DIFFUSION FILTER

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# Medical Image Filtering

- Remove high-frequency noise;
  - Preserve signal;
  - Do not insert artifacts;
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- Anisotropic Diffusion Filters

# Filtering Generations

- 1) Local and isotropic
  - Gaussian, median
- 2) Local and anisotropic
  - Bilateral and diffusion
- 3) Non-local and anisotropic
  - NLM, BM3D

# Anisotropic Diffusion Filter

- Iterative process simulating thermal energy flow.

$$I_s^{t+1} \approx I_s^t + \frac{\lambda}{|\eta_s|} \sum_{p \in \eta_s} g(\nabla I_{s,p}^t, \gamma) \nabla I_{s,p}^t$$

- $I_s^t$  – intensity of pixel  $s$  in instant  $t$
- $\lambda$  – diffusion rate scalar
- $\gamma$  – smoothing strength
- $\eta_s$  – pixels adjacent to  $s$
- $\nabla I_{s,p}^t$  or  $x$  – magnitude of intensity directional gradient from  $s$  to  $p$
- $g(\ )$  – edge stopping function

# Edge Stopping Functions

$$I_s^{t+1} \approx I_s^t + \frac{\lambda}{|\eta_s|} \sum_{p \in \eta_s} g(\nabla I_{s,p}^t, \gamma) \nabla I_{s,p}^t$$

Let  $\gamma = \lambda / |\eta_s|$

$$g(x, \gamma) = \exp(-x^2 / 2\gamma^2)$$

$$g(x, \gamma) = \left[1 + (x/\gamma)^2\right]^{-1}$$

$$g(x, \gamma) = \begin{cases} \left[1 - (x^2 / 5\gamma^2)\right]^2, & |x| \leq \gamma\sqrt{5} \\ 0, & \text{otherwise} \end{cases}$$

(Tukey's Function - Black et al. 1998)

# Third Generation Diffusion Filters

- Trivial extensions with no parametric evaluation:
  - M Yang et al., *Non-local means theory based Perona-Malik model for image denoising* - Neurocomputing, vol. 120, pp. 262–267, 2013.
    - Original edge-stopping function.
  - J Yuan, *Improved anisotropic diffusion equation based on new non-local information scheme for image denoising*- IET Computer Vision, vol. 9, no. 6, pp. 864–870, 2015.
    - Tukey's stopping function.
    - Computes adjacent patches on each iteration.

# Optimal Non-Local Anisotropic Diffusion Filter

$$I_s^{t+1} \approx I_s^t + \frac{1}{|H_s|} \sum_{p \in H_s} \frac{g(\nabla I_{s,p}^t, \gamma)}{d_{s,t}} \nabla I_{s,p}^t$$

- $H_s$  – Local and Non-local adjacency.
- $d_{s,t}$  – distance between pixels  $s$  and  $t$ .
  - Non-local adjacency is treated as a new dimension.
  - Local and non-local dimensions may be weighted differently.
  - Distance weight eliminates  $\lambda$  parameter.

# Optimal Non-Local Anisotropic Diffusion Filter

$$I_s^{t+1} \approx I_s^t + \frac{1}{|H_s|} \sum_{p \in H_s} \frac{g(\nabla I_{s,p}^t, \gamma)}{d_{s,t}} \nabla I_{s,p}^t$$

- Parameter estimation:

- Initial  $\gamma$

- 5% high threshold of the sum of directional gradients

- $G_s = \sum_{p \in \eta_s} I_s - I_p$

- Stopping criteria

- $\gamma^t \leq \varepsilon I^M$

- $I^M$  is the maximum image intensity.

- $\varepsilon=0.01$  in our experiments.



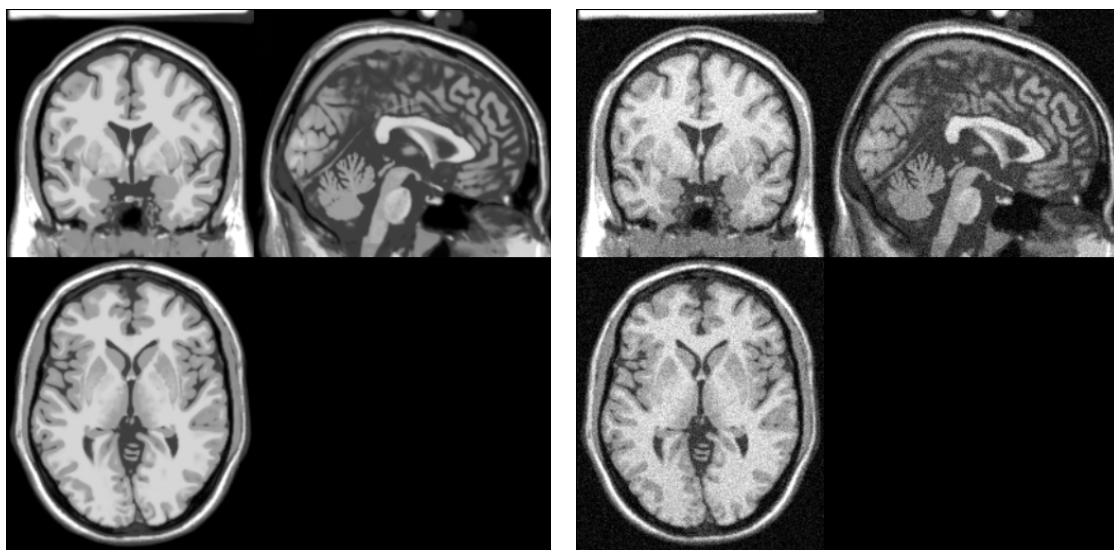
# Optimal Non-Local Anisotropic Diffusion Filter

$$I_s^{t+1} \approx I_s^t + \frac{1}{|H_s|} \sum_{p \in H_s} \frac{g(\nabla I_{s,p}^t, \gamma)}{d_{s,t}} \nabla I_{s,p}^t$$

- New parameters:
  - $\gamma$ -reduction rate or conservativeness (R $\gamma$ )
  - Non-local similar patch search radius (SR)
  - Non-local patch radius (PR)
  - Non-local dimension distance unit (PD)
  - Number of non-local patches (#P)

# Parameter Estimation

- BrainWeb Phantom
  - Weighted Anisotropic Diffusion Filter – WADF (local)



$\gamma R$	IQI	MSE	PSNR
$0.16\gamma^{t-1}$	0.990	3.74	27.6
$0.32\gamma^{t-1}$	0.995	2.99	28.8
$0.48\gamma^{t-1}$	0.996	2.84	29.1
$0.64\gamma^{t-1}$	<b>0.997</b>	2.79	29.2
$0.80\gamma^{t-1}$	0.995	<b>2.78</b>	<b>29.3</b>
$0.96\gamma^{t-1}$	0.994	2.80	29.2

# Parameter Estimation

- BrainWeb Phantom
  - Non-Local Weighted Anisotropic Diffusion Filter – NL-WADF

SR	PR	PD	#R	$\gamma_R$	IQI	MSE	PSNR
2.0	1.1	0.5	2	$0.32\gamma^{t-1}$	<b>0.997</b>	2.72	<b>29.5</b>
4.0	1.1	0.5	2	$0.32\gamma^{t-1}$	<b>0.997</b>	<b>2.71</b>	<b>29.5</b>
2.0	1.9	2.0	1	$0.64\gamma^{t-1}$	<b>0.997</b>	2.79	29.2
3.0	1.9	0.5	1	$0.64\gamma^{t-1}$	<b>0.997</b>	2.78	29.2
4.0	1.9	1.0	2	$0.64\gamma^{t-1}$	<b>0.997</b>	2.79	29.2

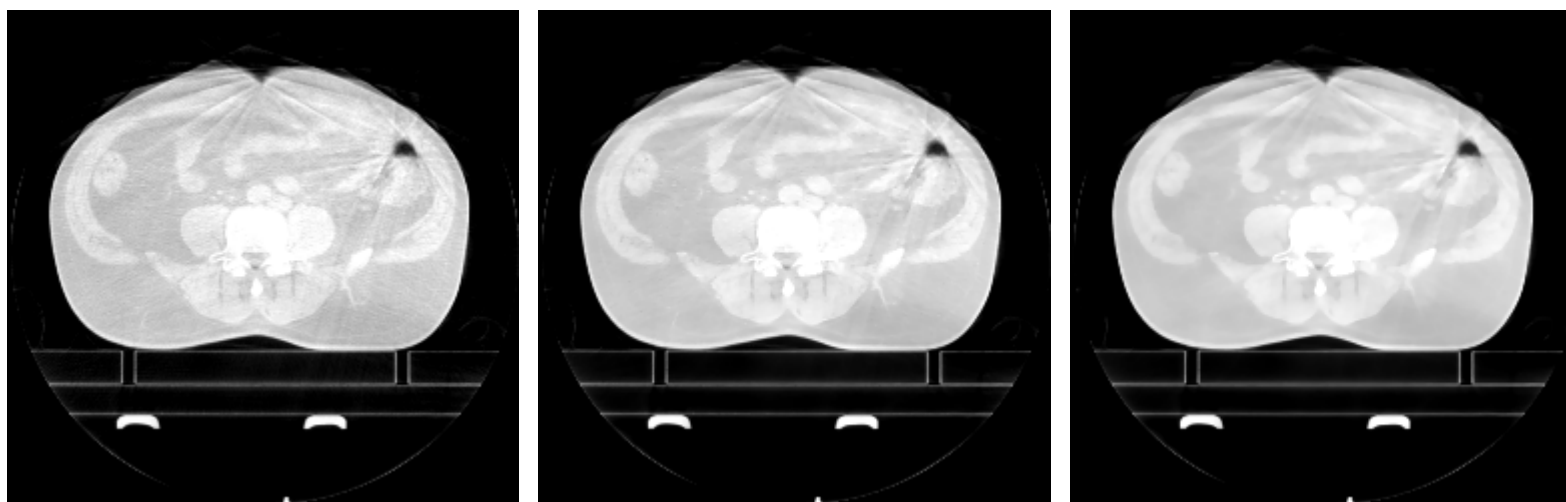
# Quantitative Evaluation

- BrainWeb Phantom
  - Comparing NL-WADF with other non-local filters:

Noise level	IQI			MSE			PSNR		
	5%	7%	9%	5%	7%	9%	5%	7%	9%
NL-WADF	0.974	0.993	0.971	23.5K	47,0K	110K	28.5	25.2	21.8
NLM-3D	<b>1.002</b>	0.970	0.963	<b>21.0K</b>	<b>38.5K</b>	<b>101K</b>	<b>29.0</b>	<b>26.4</b>	<b>22.2</b>
BM4D	0.973	<b>1.000</b>	<b>0.975</b>	21.7K	44.0K	104K	28.9	25.8	22.1

# Qualitative Evaluation

- CBCT/kV radiotherapy images.



$\gamma^0$	$0.2\sigma_{Gs}$	$0.4\sigma_{Gs}$	$0.6\sigma_{Gs}$	$0.8\sigma_{Gs}$	$\sigma_{Gs}$
Score	4.9	5.0	4.9	4.9	4.8

Thank you!