Towards Multi-rigid Body Localization

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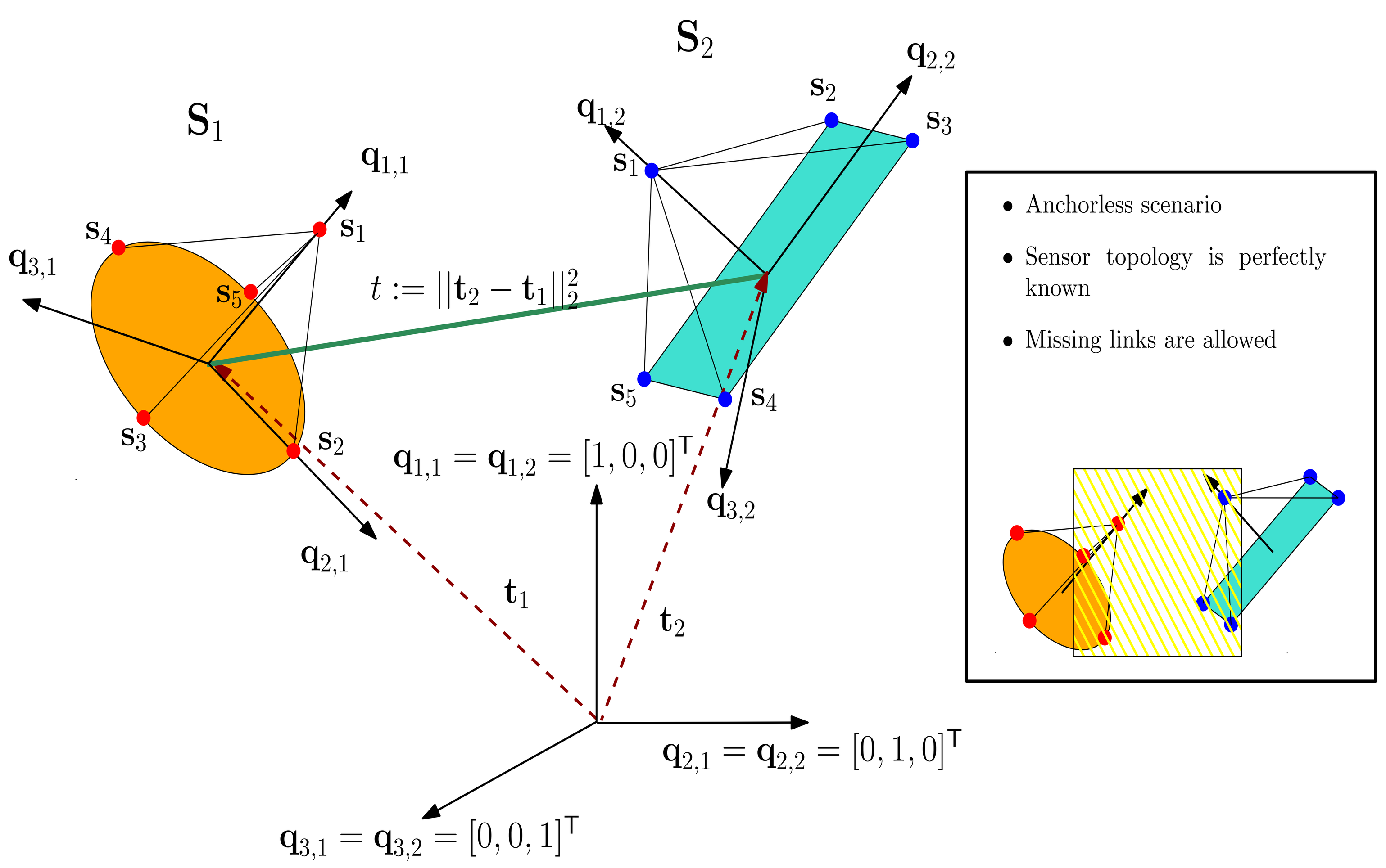
Problem statement

- Jointly estimate the **relative** position and orientation of multiple rigid bodies in $\mathbb{R}^3$.
- Based on range-only measurements between the sensor pairs.
- Without any inertial measurements (e.g., accelerometers) and/or anchors.

Applications

- Control and maneuvering of unmanned aircrafts, drones, underwater vehicles, satellites, and robotics.
- Topology-aware localization.

Problem modeling



- N sensors mounted on each rigid body.
- We know how the sensors are mounted on each rigid body (e.g., during fabrication).
- Absolute position of the sensors, or the rigid bodies itself is not known.

Absolute position of ith body (rigid body transformation [1]):

$$\mathbf{S}_i = \mathbf{Q}_i\mathbf{C}_i + \mathbf{t}_i\mathbf{1}_N^\mathsf{T} \qquad i = 1,2.$$

$\mathbf{S}_i = [\mathbf{s}_{1,i}, \cdots, \mathbf{s}_{N,i}] \in^{3\times N}$

Remarks:

1. $\mathbf{C}_i = [\mathbf{c}_{1,i}, \cdots, \mathbf{c}_{N,i}]$ is the known coordinates of the ith body in the reference frame.
2. $\mathbf{Q}_i$ is the unknown rotation matrix, i.e., $\mathbf{Q}_i^T\mathbf{Q}_i = \mathbf{I}$. It tells how the ith rigid body is rotated in the reference frame.
3. $\mathbf{t}_i \in \mathbb{R}^{3\times 1}$ is the unknown translation of the ith body.

Measurements

Squared range between cross-body sensors (noise free):

$$\mathbf{Y}^{\odot 2} = \boldsymbol{\psi}_1\mathbf{1}_N^\mathsf{T} + \mathbf{1}_N\boldsymbol{\psi}_2^\mathsf{T} - 2\mathbf{S}_1^\mathsf{T}\mathbf{S}_2$$

$\boldsymbol{\psi}_i = [\|\mathbf{s}_{1,i}\|^2, \cdots, \|\mathbf{s}_{N,i}\|^2]^\mathsf{T} \in^{N\times 1}$ and $[\mathbf{Y}]_{m,n} = \|\mathbf{s}_{m,i} - \mathbf{s}_{n,i}\|_2$, $m,n = 1,\cdots,N \quad i = 1,2.$

Proposed estimators

For relative localization (imagine multidimensional scaling):

1. Estimate the relative rotation matrix $\mathbf{Q} = \mathbf{Q}_1^T\mathbf{Q}_2 \in \mathcal{V}_{3,3}$, where
$$\mathcal{V}_{3,3} = \{\mathbf{Q} \in \mathbb{R}^{3\times 3} : \mathbf{Q}^T\mathbf{Q} = \mathbf{I}_3\},$$
2. Estimate the relative translation $t = \|\mathbf{t}_2 - \mathbf{t}_1\|_2^2 \in \mathbf{R}$.

Relative rotation estimator

Step 1: Project out the known vectors $\mathbf{1}_N^\mathsf{T}$ and $\mathbf{1}_N$ from $\mathbf{Y}$

$$\tilde{\mathbf{Y}} = -1/2\boldsymbol{\Gamma}_N\mathbf{Y}^{\odot 2}\boldsymbol{\Gamma}_N = \tilde{\mathbf{S}}_1^\mathsf{T}\tilde{\mathbf{S}}_2 = \mathbf{C}_1^\mathsf{T}\mathbf{Q}\mathbf{C}_2$$

$\tilde{\mathbf{S}}_i = \mathbf{S}_i\boldsymbol{\Gamma}_N$ with projection matrix $\boldsymbol{\Gamma}_N$

Step 2: Orthogonal Procrustes Problem (OPP) [2]:

$$\widehat{\mathbf{Q}} = \underset{\mathbf{Q}\in\mathcal{V}_{3\times 3}}{\text{argmin}}\ \|\check{\mathbf{Y}} - \mathbf{C}_1^\mathsf{T}\mathbf{Q}\|_\mathsf{F}^2 = \mathbf{U}\mathbf{V}^\mathsf{T}$$

$\check{\mathbf{Y}} = \tilde{\mathbf{Y}}\mathbf{C}_2^\dagger$ and $\mathbf{C}_1\check{\mathbf{Y}} =: \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\mathsf{T}$

Relative translation estimator

We can express range in terms of relative translation as:

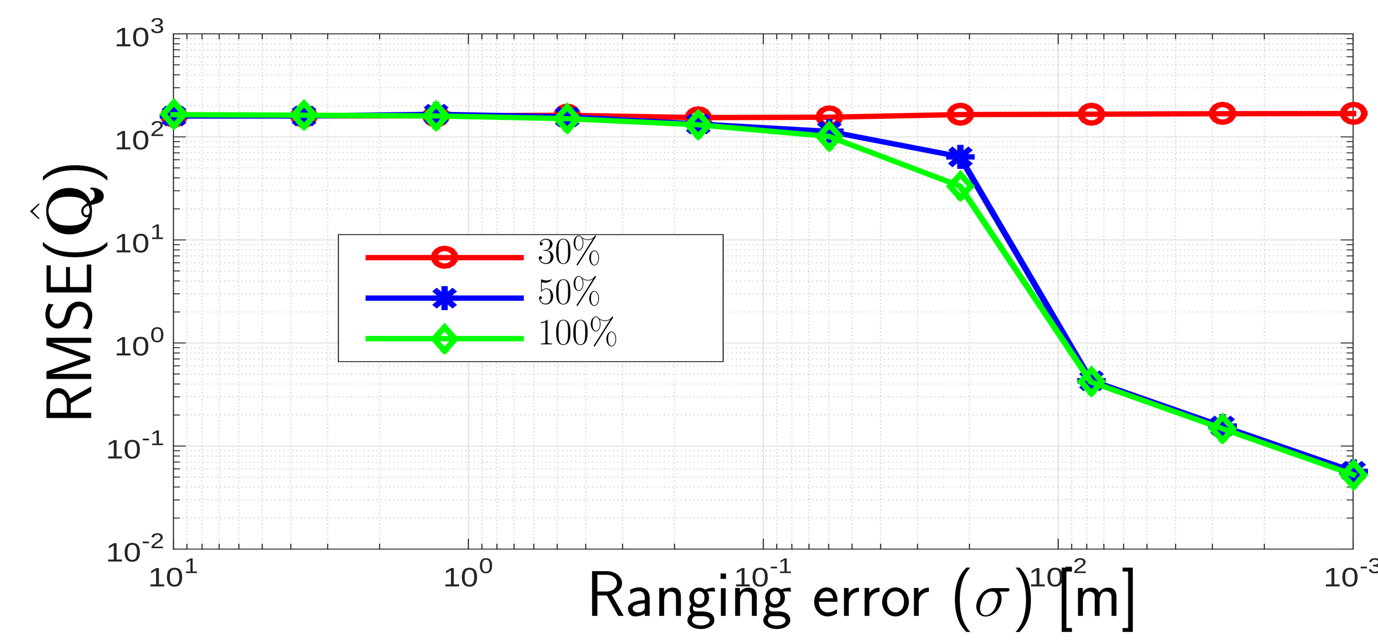
$$\|\mathbf{Y}\|_\mathsf{F}^2 = N\sum_{n=1}^{N}(\|\mathbf{c}_{n,1}\|_2^2 + \|\mathbf{c}_{n,2}\|_2^2) + N^2\underbrace{\|\mathbf{t}_1 - \mathbf{t}_2\|_2^2}_{\text{relative translation}}$$

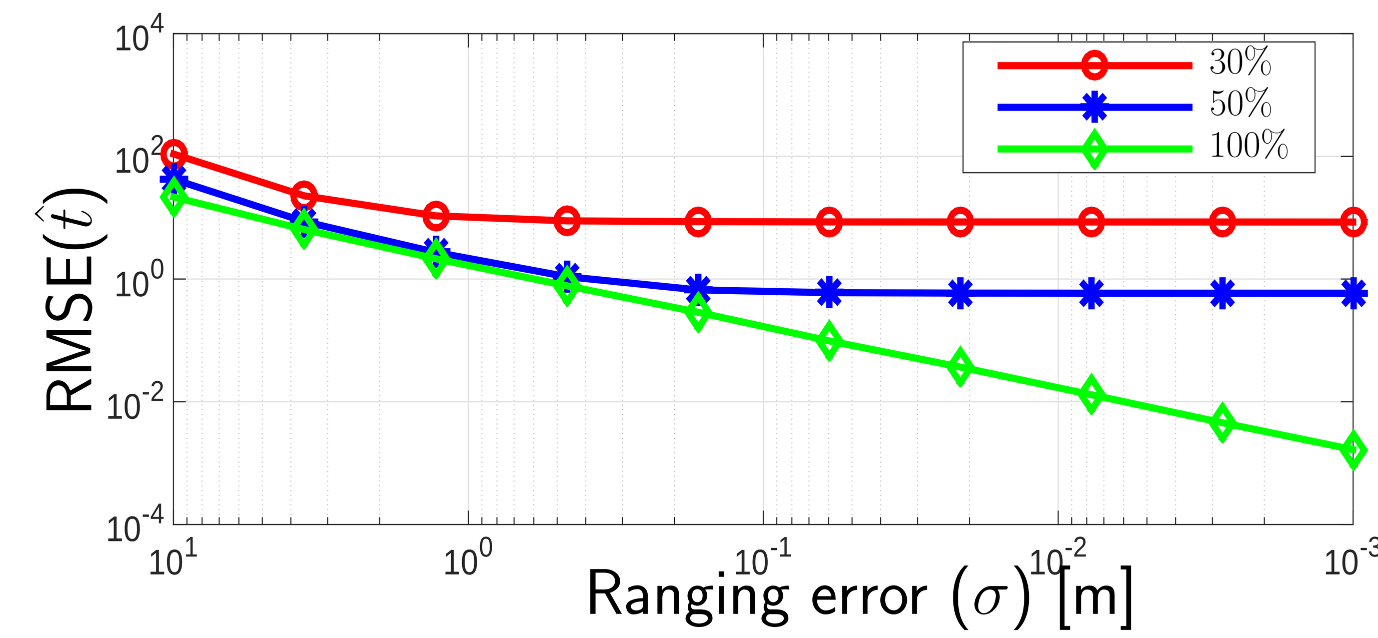
Thus

$$\hat{t} = \frac{1}{N^2}\|\mathbf{Y}\|_\mathsf{F}^2 - \frac{1}{N}\left(\sum_{n=1}^{N}(\|\mathbf{c}_{n,1}\|_2^2 + \|\mathbf{c}_{n,2}\|_2^2)\right)$$

Simulation results

Set-up: $N = 10$, $\{\psi_1,\theta_1,\phi_1\} = \{20°,-25°,30°\}$ and $\{\psi_2,\theta_2,\phi_2\} = \{40°,135°,-75°\}$, $\mathbf{t}_1 = [1,-5,4]^\mathsf{T}$ m, $\mathbf{t}_2 = [-3,1,7]^\mathsf{T}$ m, and 10^4 Monte-Carlo experiments.





[1] S. P. Chepuri, G. Leus,and A.-J van der Veen,"Rigid Body Localization Using Sensor Networks," *IEEE Trans. Sig. Process.*, 2014.

[2] G.H. Golub and C.F. van Loan, Matrix Computations, JHU Press, 1996

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