Spatially Regularized Multi-exponential Transverse Relaxation Times Estimation from Magnitude Magnetic Resonance Images Under Rician Noise

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Introduction

Detailed tissue characterization using MRI relaxometry requires (i) estimation of multi-exponential relaxation times T_2 and their associated amplitude A_0 on the image level. However, estimating the model parameters for magnitude data is a large-scale ill-posed inverse problem, in the particular context of Rician noise. This problem has not been solved yet, only local filtering technique was proposed.

We propose

- A parameter estimation method that combines a spatial regularization with a Maximum-Likelihood criterion based on the Rician distribution of the noise.
- A Majorization-Minimization approach alongside an adapted non-linear least-squares Levenberg-Marquardt algorithm.
- A tissue characterization method for exploiting the reconstructed maps by clustering the parameters using a K-means classification algorithm applied to the extracted relaxation time and amplitude maps.

Multi-exponential decay

$$A(\tau,\boldsymbol{\theta_j}) = \sum_{c=1}^{N_c} A_{0_{(c,j)}} e^{-\frac{\tau}{T_{2(c,j)}}}, \quad \text{for } \tau = \tau_1, \dots, \tau_{N_t}$$

Vector of uknown parameters ($\boldsymbol{\theta_j}$); Number of components (N_c); Time sample (τ); Number of time samples (N_t); Signal model (A); Voxel (j).

Rician distribution of the noise

Real + Gaussian Noise, Imaginary + Gaussian Noise → Magnitude Transformation => Rician Noise

$$P_R\left(m_j(\tau) \mid A(\tau,\boldsymbol{\theta_j}), \sigma^2\right) = \frac{m_j(\tau)}{\sigma^2} e^{-\frac{m_j(\tau)^2 + A(\tau,\boldsymbol{\theta_j})^2}{2\sigma^2}} \times I_0\left(\frac{m_j(\tau)A(\tau,\boldsymbol{\theta_j})}{\sigma^2}\right),$$

Measured signal ($m_j(\tau)$); Noise variance (σ^2); Bessel function of first kind and of order zero (I_0).

If the Rician noise distribution is not properly accounted for, this leads to a bias on the estimated parameters.

Proposed parameter estimation algorithm

Regularized maximum-likelihood:

$$F(m,\theta) = J_{MV}(m,\theta) + Reg(\theta)$$

with :

$$J_{MV}(m,\theta) = \sum_{j=1}^{N_v} \sum_{\tau=\tau_1}^{\tau_{N_t}} \left[\frac{A(\tau,\theta_j)^2}{2\sigma^2} - \log\left(I_0\left(D_{j\tau}\right)\right)\right], \quad D_{j\tau} = \frac{m_j(\tau)\, A_j(\tau,\boldsymbol{\theta_j})}{\sigma^2}$$

and :

$$Reg(\theta) = \sum_{p=1}^{N_p} \beta(p) \sum_{i=1}^{N_v} \sum_{i \in V_j} \psi\left(\theta_j(p) - \theta_i(p)\right)$$

Number of voxels (N_v); vector containing the measurement at all the voxels (m); Controls the weight of the spatial regularization ($\beta(p)$); A predefined neighboring region (V_j).

By adopting a Majoration-Minimization technique, the minimization of $F(m,\theta)$ is carried out by a serie of quadratic minimizations of the following criterions:

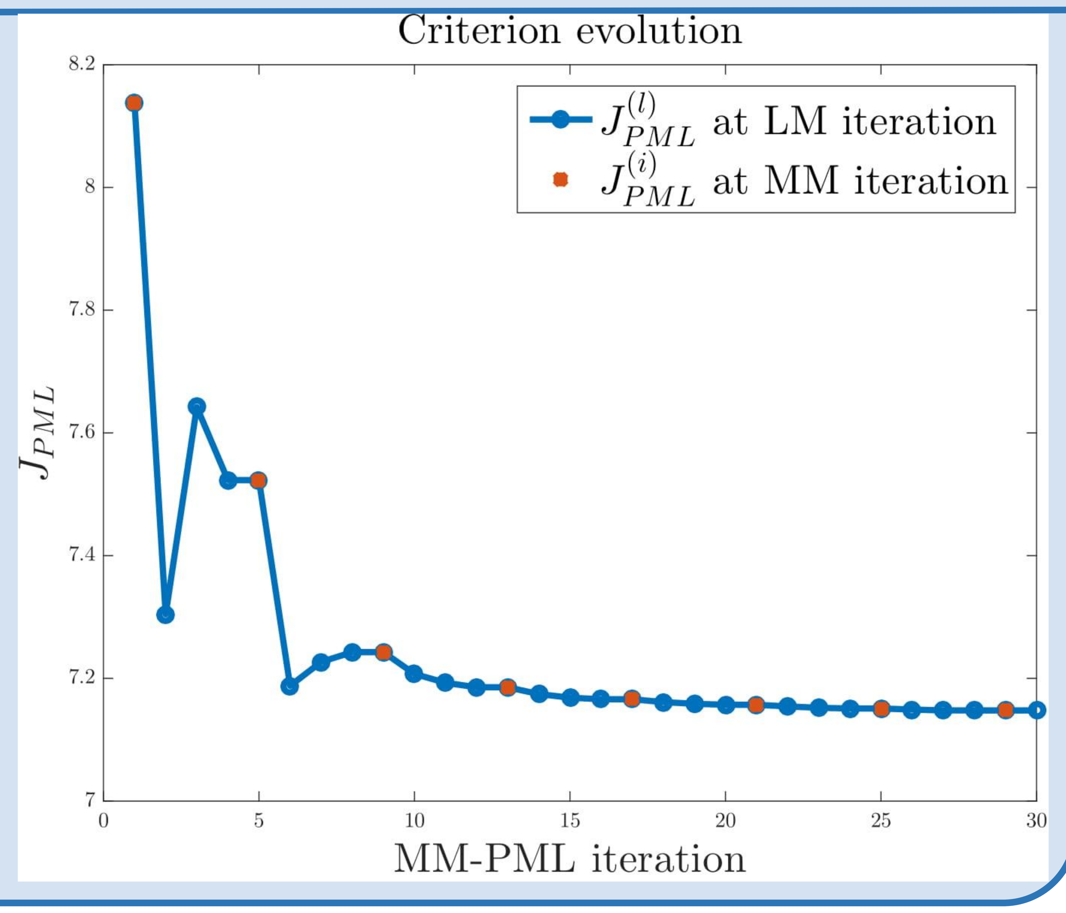
$$Q_{MV}^{(k)}(m,\theta^{(k)}) = \frac{1}{2\sigma^2} \sum_{j=1}^{N_v} \sum_{\tau=\tau_1}^{\tau_{N_t}} \left[m_j(\tau)\, R\left(D_{j\tau}^{(k)}\right) - A(\tau,\theta_j)\right]^2$$

$$Q_{reg}^{(k)}(\theta) = \sum_{p=1}^{N_p} \sum_{j=1}^{N_v} \sum_{i \in V_j} \left[\beta(p)^{\frac{1}{2}} \psi^{\frac{1}{2}} \left(2\theta_j(p) - \left(\theta_j^{(k)}(p) + \theta_i^{(k)}(p)\right)\right)\right]^2$$

- $R(\cdot) = \frac{I_1(\cdot)}{I_0(\cdot)}$, $I_1(\cdot)$ is the first kind modified Bessel function of first order.

At each iteration k, the Levenberg-Marquardt algorithm was used with the following adaptations :

- A maximum step-size that guarantees the parameters positivity.
- A step search approach using a backtracking technique based on the Armijo line search to find a step-size that ensures the convergence of the algorithm.

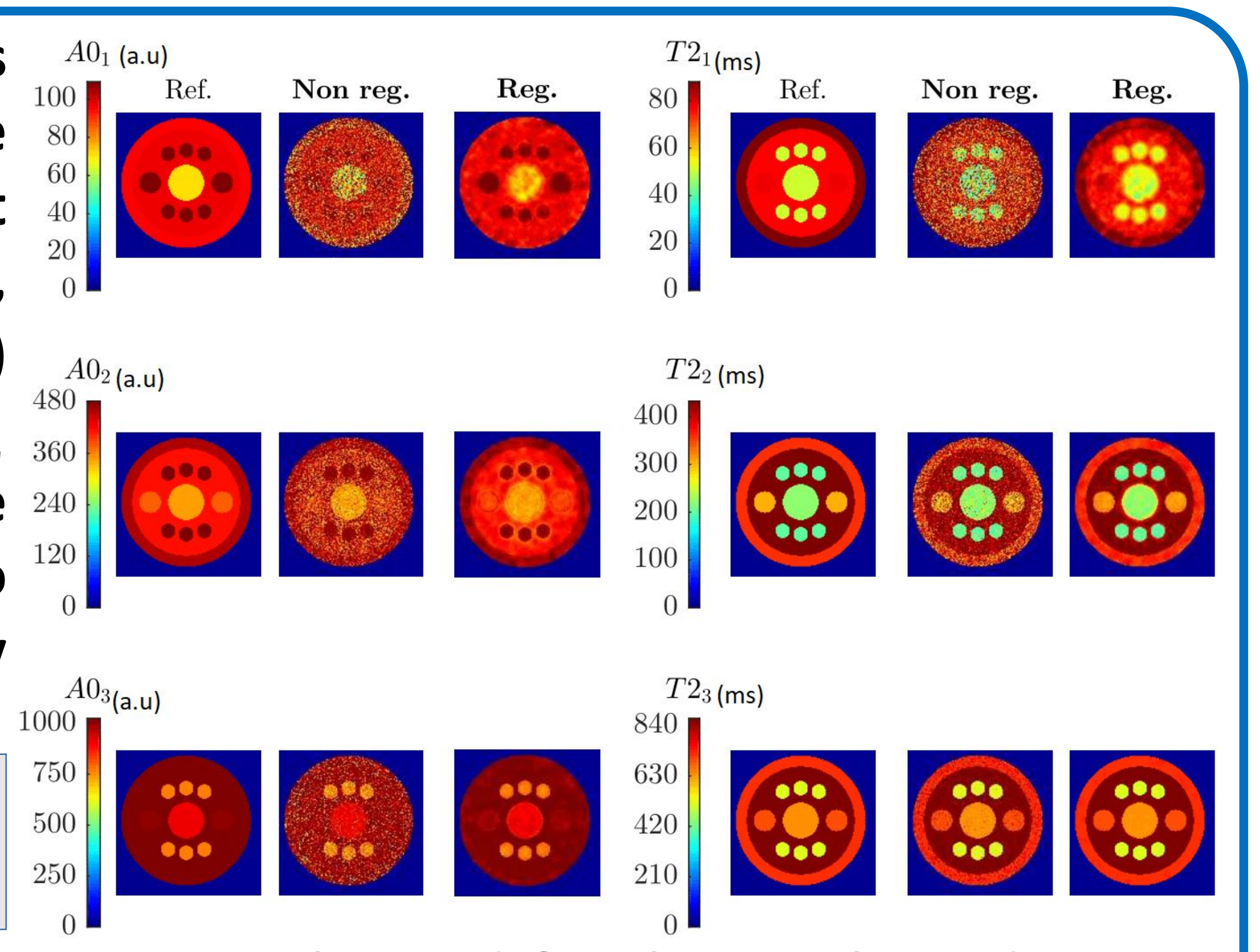


Results

A circular phantom was constructed on an image of 128 × 128 voxels. At each part of the phantom, a tri-exponential ($N_c = 3$) model was generated. T_2 and A_0 values were chosen to be close to parameters typically found in tomato fruits.

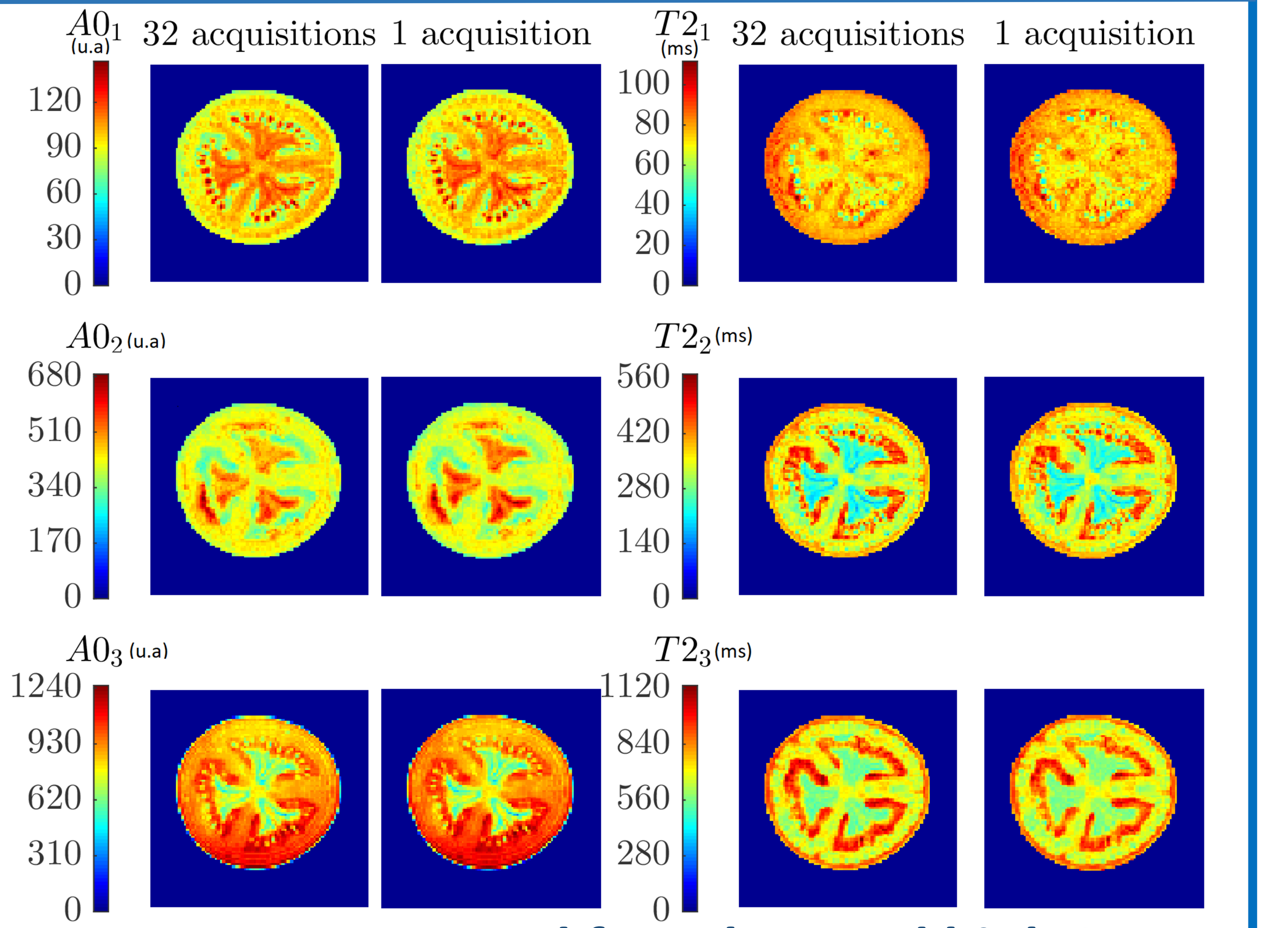
$$NRMSE = 100\sqrt{\frac{1}{N_v N_p}\sum_{j=1}^{N_v}\sum_{p=1}^{N_p}\frac{\left(\theta_j(p) - \theta_j^*(p)\right)^2}{\theta_j^*(p)^2}},$$

$\theta_j^*(p)$ are the reference values.



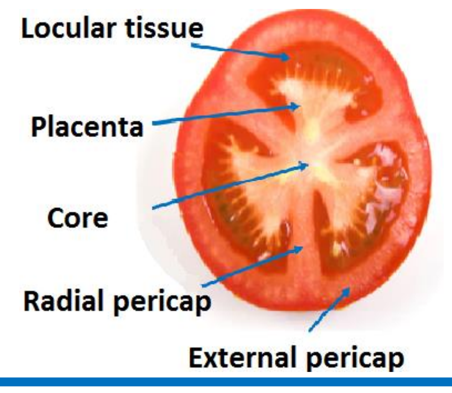
Simulated data : a low NRMSE was obtained for the regularized version 4:47% , for the non regularized version ($\beta = 0$) NRMSE was 22:85%.

For the experimental (Tomato) MRI settings, a Multi-SE sequence was used on a 1.5T MRI scanner with inter-echo spacing of 6.5ms, bandwidth of 260 Hz/pixel, 512 echoes per echo train. The reference maps were acquired with 32 scans in order to obtain higher SNR and are compared to maps acquired with 1 scan (low SNR).



Images reconstructed from low and high SNR data are quite similar, validating the robustness of the method to different noise levels. (NRMSE equal to 5:56%).

Future work



Clustering the image using a classification algorithm (k-means) using the extracted parameters as features.