ANALYTICAL METHOD OF 2.5D EXTERIOR SOUND FIELD SYNTHESIS BY USING MULTIPOLE LOUDSPEAKER ARRAY

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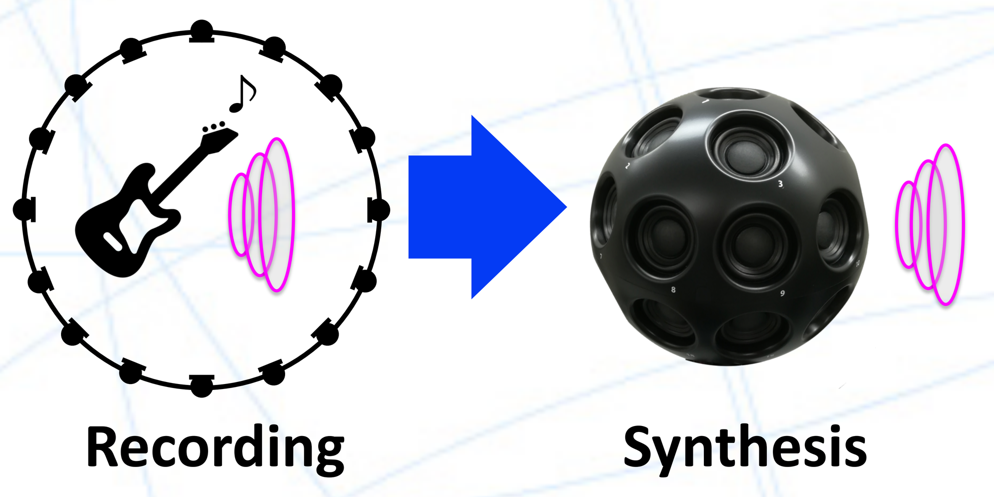
I. Introduction

◆Generating arbitrary exterior sound field



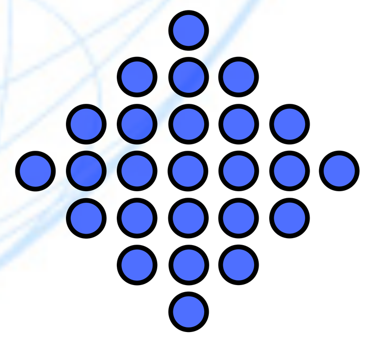
Conventional methods of sound field synthesis use circular or spherical loudspeaker array

◀ : Microphone

Contribution

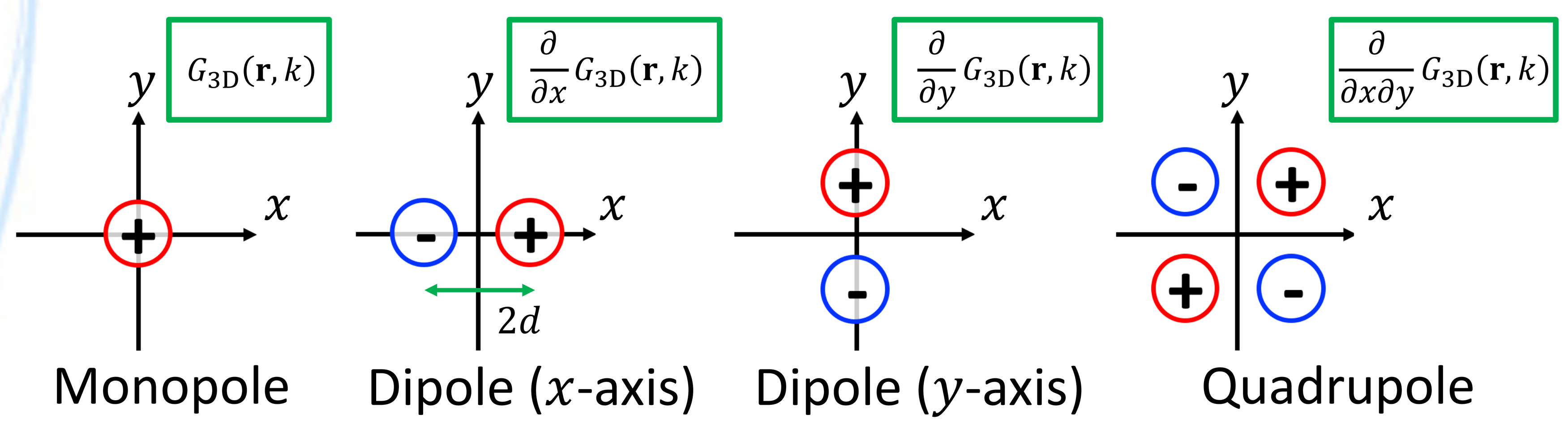
We proposed a 2.5D exterior sound field synthesis that uses a 2D multipole loudspeaker array.

We derived an analytical method that converts the expansion coefficients of spherical harmonics into weighting coefficients of multipole sound source superposition.



II. Multipole Sound Source

◆Expressed as the partial derivative of Green's function



$$S_\mu^\nu(\mathbf{r},k) = (d)^{\mu+\nu}\frac{\partial^{\mu+\nu}}{\partial x^\mu \partial y^\nu}G_{3D}(\mathbf{r},k) = G_{3D}(\mathbf{r},k)(-jdk)^{\mu+\nu}\cos^\mu\phi\sin^\nu\phi$$

$S_\mu^\nu(\mathbf{r},k)$: Sound field at an arbitrary point $\mathbf{r}$ by (μ,ν)-th multipole sound source

$G_{3D}(\mathbf{r},k) = -\frac{jk}{4\pi}h_0^{(2)}(kr)$: 3D Green's function

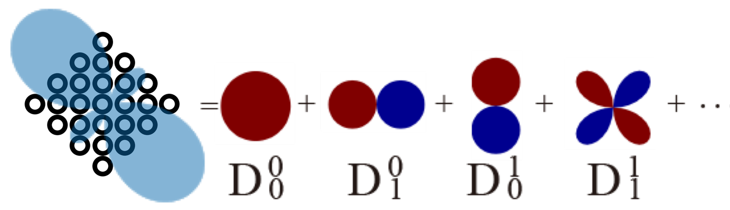
$h_n^{(2)}(\cdot)$: n-th order spherical Hankel function of the second kind

μ,ν : Number of partial derivatives with respect to x and y

III. Proposed Method

◆Deriving the weighting of multipole sources

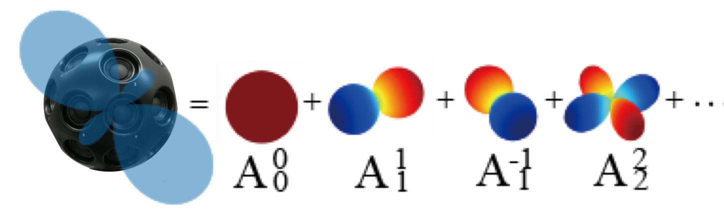
Sound field by multipole superposition



$$S(\mathbf{r},k) = \sum_{\mu=0}^{\infty}\sum_{\nu=0}^{\infty} D_\mu^\nu S_\mu^\nu(\mathbf{r},k)$$

Analytical conversion

Sound field by spherical harmonics



$$S(\mathbf{r},k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} A_n^m h_n^{(2)}(kr)Y_n^m(\theta,\phi)$$

$$Y_n^m(\theta,\phi) = \underbrace{\sqrt{\frac{4\pi}{2n+1}\cdot\frac{(n-m)!}{(n+m)!}}P_n(\cos\theta)}_{F_n^m(\theta)}e^{jm\phi}$$

$P_n(\cdot)$: Legendre function

IV. Analytical Conversion Method

◆Conversion of spherical harmonics for multipole superposition

Sound field by spherical harmonics

$$S(\mathbf{r},k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} A_n^m h_n^{(2)}(kr)Y_n^m(\theta,\phi)$$

$Y_n^{-m}(\theta,\phi) = (-1)^m Y_n^m(\theta,\phi)^*$

$$= h_n^{(2)}(kr)\left[\sum_{n=0}^{\infty}\{A_n^0F_n^0(\pi/2)\} + \sum_{n=1}^{\infty}\sum_{m=1}^{n}F_n^m(\pi/2)\{A_n^m e^{jm\phi} + (-1)^m A_n^{-m}e^{-jm\phi}\}\right]$$

When (kr) is sufficiently large, the relation of $h_n^{(2)}(kr) = j^n h_0^{(2)}(kr)$ can be used

Applying Euler's equation $e^{jm\phi} = (\cos\phi + j\sin\phi)^m$ and using binomial expansion

$$S(\mathbf{r},k) = h_0^{(2)}(kr)\left[\sum_{n=0}^{\infty}\{j^nA_n^0F_n^0(\pi/2)\} + \sum_{n=1}^{\infty}\sum_{m=1}^{n}\sum_{\nu=0}^{m} j^{n+\nu}F_n^m(\pi/2)\binom{m}{\nu}\{A_n^m + (-1)^{m+\nu}A_n^{-m}\}\cos^{m-\nu}\phi\sin^\nu\phi\right]$$

Sound field by multipole superposition

$$S(\mathbf{r},k) = \sum_{\mu=0}^{\infty}\sum_{\nu=0}^{\infty} D_\mu^\nu S_\mu^\nu(\mathbf{r},k) = h_0^{(2)}(kr)\sum_{\mu=0}^{\infty}\sum_{\nu=0}^{\infty} D_\mu^\nu\left(-\frac{jk}{4\pi}\right)(-jdk)^{\mu+\nu}\cos^\mu\phi\sin^\nu\phi$$

Replace the μ by $m-\nu$ $(0 \le m \le n, 0 \le \mu, 0 \le \nu)$

$$S(\mathbf{r},k) = h_0^{(2)}(kr)\sum_{m=0}^{\infty}\sum_{\nu=0}^{m} D_{m-\nu}^\nu\left(-\frac{jk}{4\pi}\right)(-jdk)^m\cos^{m-\nu}\phi\sin^\nu\phi$$

Comparison of the coefficients of $\cos^{m-\nu}\phi\sin^\nu\phi$

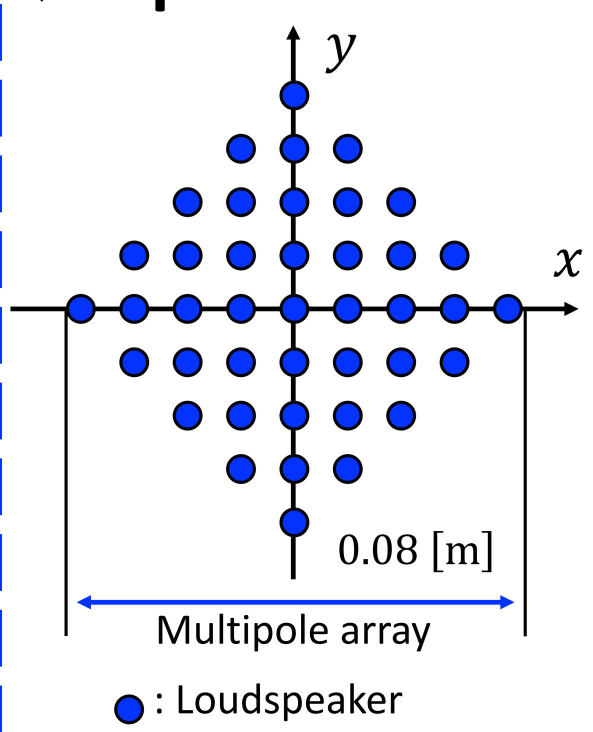
$$D_{m-\nu}^\nu\left(-\frac{jk}{4\pi}\right)(-jdk)^m = \begin{cases}\sum_{n=0}^{\infty}A_n^0F_n^0(\pi/2) & (m=0)\\ \sum_{n=m}^{\infty}j^{n+\nu}F_n^m(\pi/2)\binom{m}{\nu}\{A_n^m + (-1)^{m+\nu}A_n^{-m}\} & (m>0)\end{cases}$$

Replace m by $\mu+\nu$

$$D_\mu^\nu = -\frac{4\pi}{jk}\cdot\begin{cases}\sum_{n=0}^{\infty}j^nA_n^0F_n^0(\pi/2) & (m=0)\\ \frac{1}{(-jdk)^\mu}\sum_{n=\mu+\nu}^{\infty}j^{n+\nu}F_n^{\mu+\nu}(\pi/2)\binom{\mu+\nu}{\nu}\{A_n^{\mu+\nu} + (-1)^{\mu+2\nu}A_n^{-\mu-\nu}\} & (m>0)\end{cases}$$

V. Experiments

◆Experimental conditions



Number of loudspeakers	41
Interval of loudspeaker [d]	0.01 [m]
Array length	0.08 [m]
Maximum order of spherical harmonics	4

Target sound field : modeled using randomly spherical harmonics

Evaluation with the synthesis error

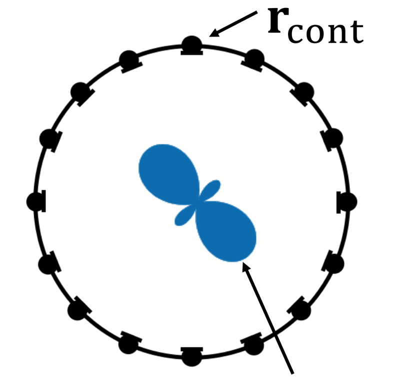
$$E(\mathbf{r}) = 10\log_{10}\frac{|S_{des}(\mathbf{r}) - S_{syn}(\mathbf{r})|^2}{|S_{des}(\mathbf{r})|^2}$$

Pressure-matching method

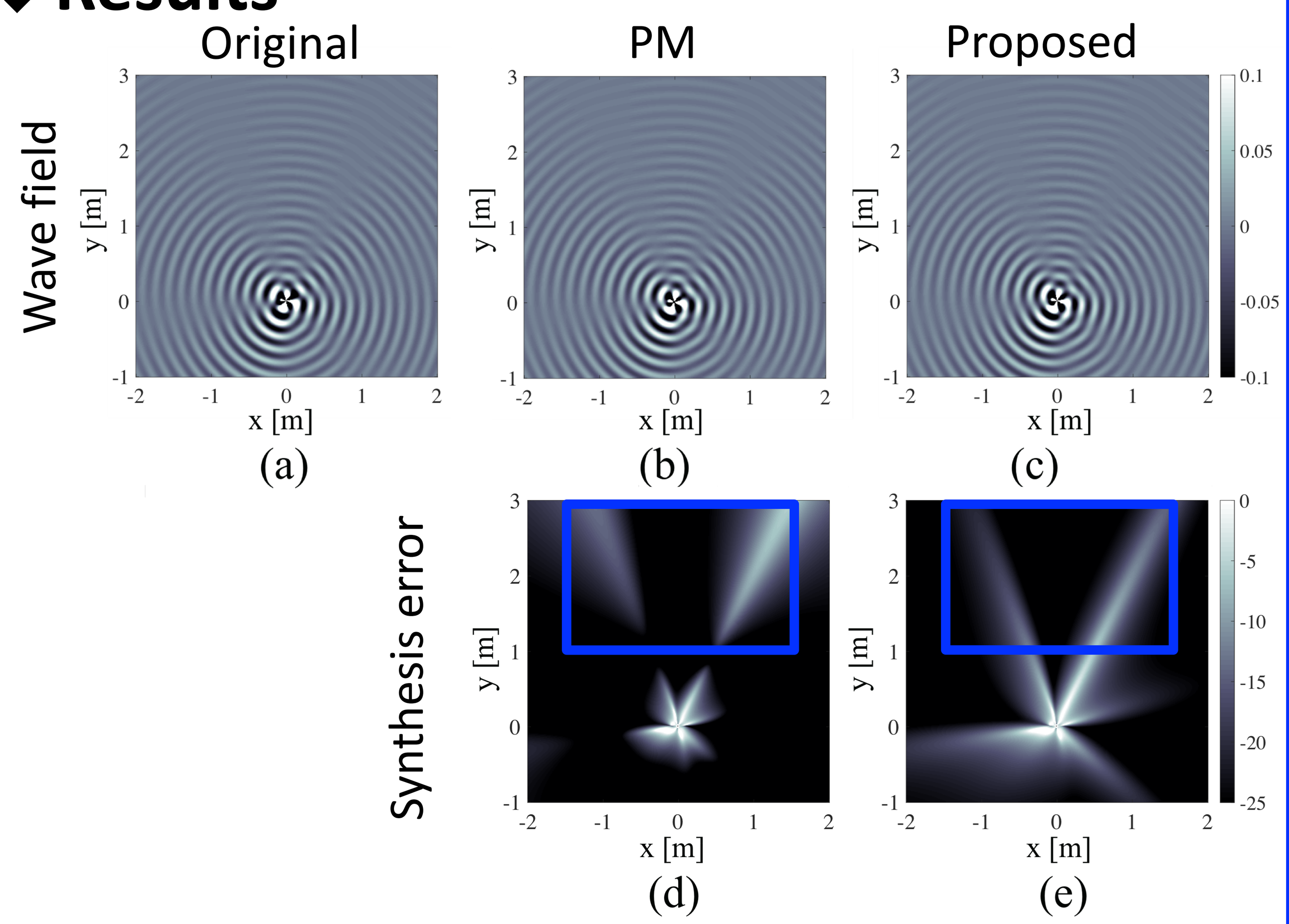
$$\mathbf{d}_\mu^\nu = (\mathbf{G}^H\mathbf{G} + \lambda\mathbf{I})^{-1}\mathbf{G}^H$$

$$\mathbf{d}_\mu^\nu = [D_0^0, D_1^0, \cdots, D_\mu^\nu]$$

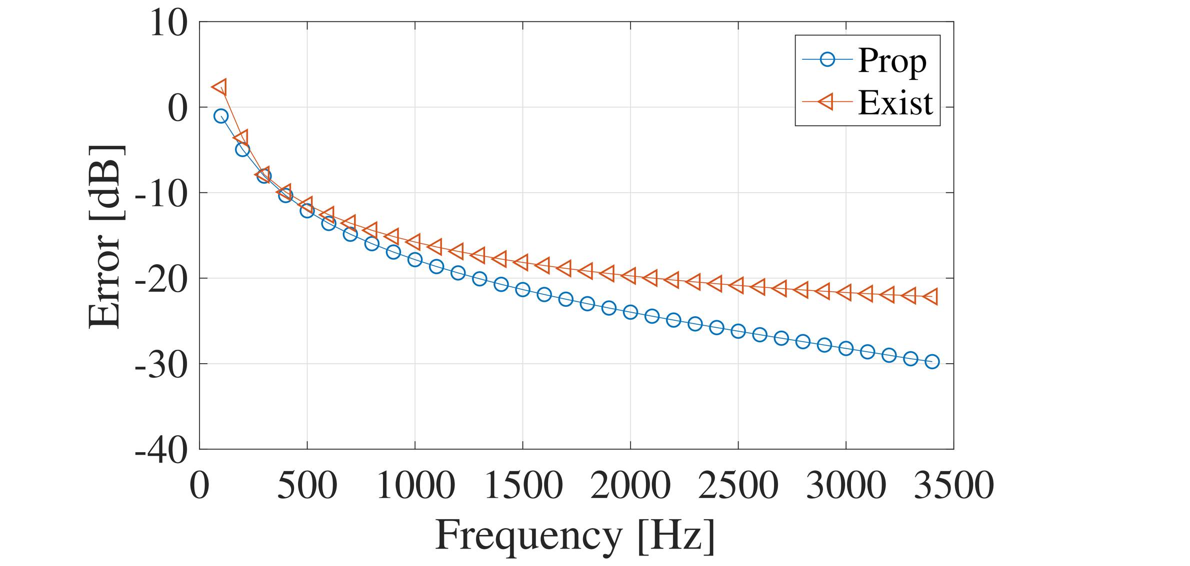
$$\mathbf{G} = [S_0^0(\mathbf{r}_{cont}), S_1^0(\mathbf{r}_{cont}), \cdots, S_\mu^\nu(\mathbf{r}_{cont})]$$



◆Results



Reproduced sound field : sine wave 2 [kHz]



Averaged synthesis error$(-1.5 < x < 1.5, 1.0 < y < 3.0)$