Tensor-based Blind fMRI Source Separation Without the Gaussian Noise Assumption – A β-Divergence Approach

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November 13, 2019

Magnetic Resonance Imaging

- Exploits magnetic properties of tissues.
- Anatomy of the body - structural.

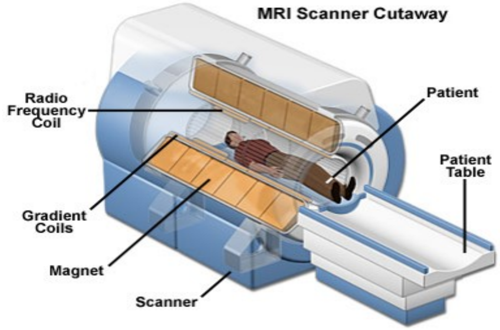
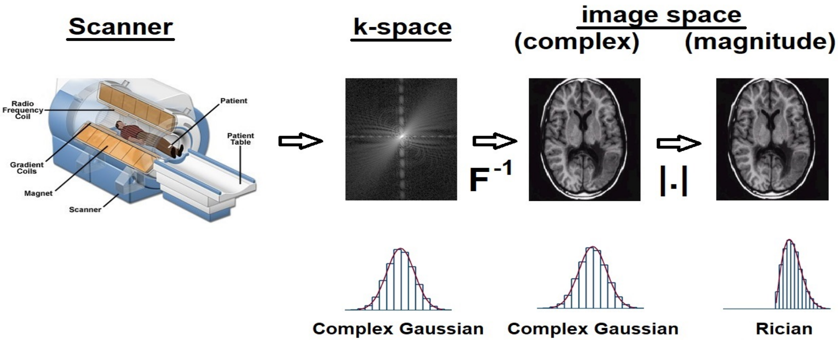


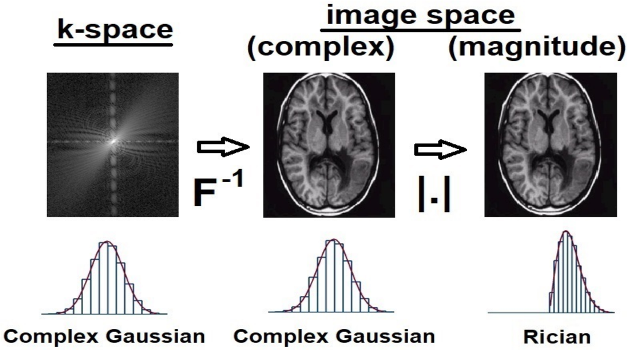
Figure: MRI scanner representation [1]

[1]'MRI a guided tour", Available online: https://nationalmaglab.org/education/magnet-academy

Signal Acquisition (Single Coil)



Noise Characteristics (Single Coil)

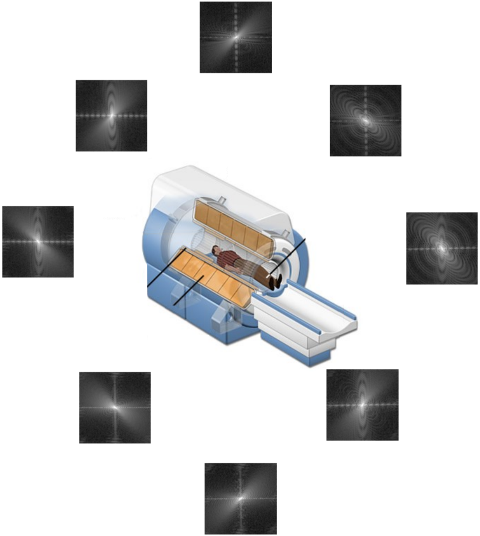


Magnitude Image $M(p) = \sqrt{(A_r(p) + N_r(p))^2 + (A_i(p) + N_i(p))^2}$

Rician Noise $p(M) = \frac{M}{\sigma^2} e^{-\frac{M^2+A^2}{2\sigma^2}} I_0\left(\frac{AM}{\sigma^2}\right)$

With p being a random voxel location, $A = \sqrt{A_r^2 + A_i^2}$ and the standard deviation of real and imaginary part being the same and equal to σ.

Signal Acquisition (Multiple Coil)



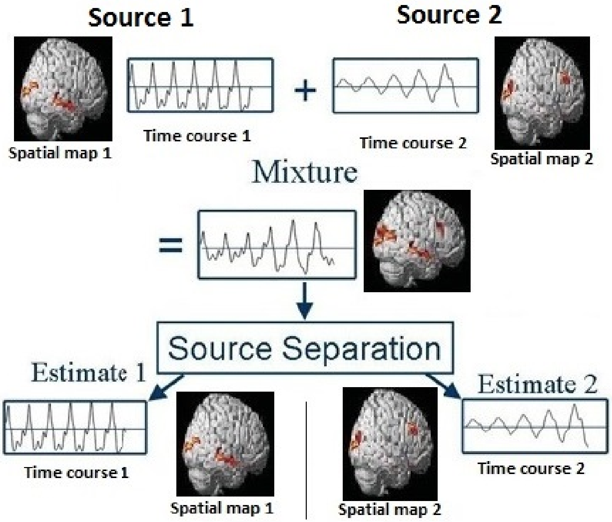
- Used in parallel imaging to increase the acquisition rate.
- Reconstruction process is needed, for combining the signals from each individual coil.
- Reconstruction algorithms do not use linear mappings.
- The assumption of a single value of σ to characterize the whole data set is no longer valid.

Functional Magnetic Resonance Imaging

- Measures Blood Oxygen Level Dependent (BOLD) signal.
- Activation of the neurons - functional.
- BOLD fluctuation is modelled by the haemodynamic response function (HRF).

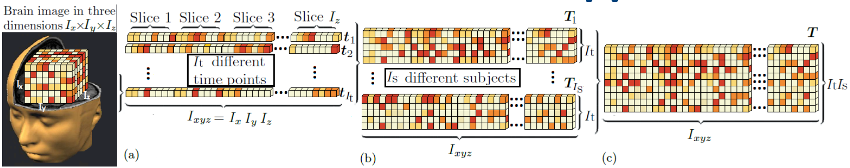
-*Determination of brain connectivity*
-*Localization of activated sources*

Blind Source Separation (BSS) for fMRI

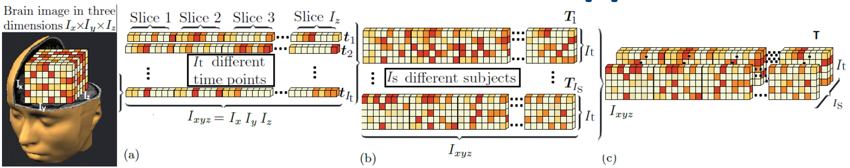


BSS for fMRI

Matrix-based approach



Tensor-based approach



[2]C. Chatzichristos et al,"Blind fMRI Source Unmixing via Higher-Order Tensor Decompositions", J. Neuroscience Methods, Vol. 315, pp 17-47, Mar. 2019

Tensor BSS for fMRI

The multi-way nature of the data is preserved in multi-linear (tensor) models, which, in general:

- Produce unique (modulo scaling and permutation ambiguities) representations under mild conditions.
- Can improve the ability of extracting spatiotemporal modes of interest.
- Facilitate neurophysiologically meaningful interpretations

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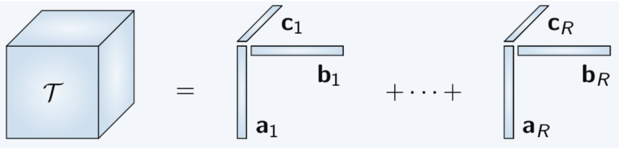

Least-squares (LS) optimization problem:

$$\min_{\mathbf{A},\mathbf{B},\mathbf{C}} \frac{1}{2}\|\mathscr{T} - [\![\mathbf{A},\mathbf{B},\mathbf{C}]\!]\|_F^2$$

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Other cost functions

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- Euclidean distance implicitly assumes additive Gaussian noise
- Other cost functions might be better depending on data/noise distribution
- Poisson process: Kullback–Leibler (KL) divergence
- Multiplicative Gamma noise: Itakura–Saito (IS) divergence

β-divergences

- β-divergences interpolate between LS distance, KL divergence and IS divergence [2]

$$d_\beta(x,y) = \begin{cases} \dfrac{x^\beta + (\beta-1)y^\beta - \beta x y^{\beta-1}}{\beta(\beta-1)} & \beta \in \mathbb{R} \setminus \{0,1\} \\ x \log \dfrac{x}{y} - x + y & \beta = 1 \text{ (KL)} \\ \dfrac{x}{y} - \log \dfrac{x}{y} - 1 & \beta = 0 \text{ (IS)} \end{cases}$$

[3]M. Vandecapelle et al,"Rank-one Tensor Approximation with β-divergence Cost Functions", EUSIPCO 2019

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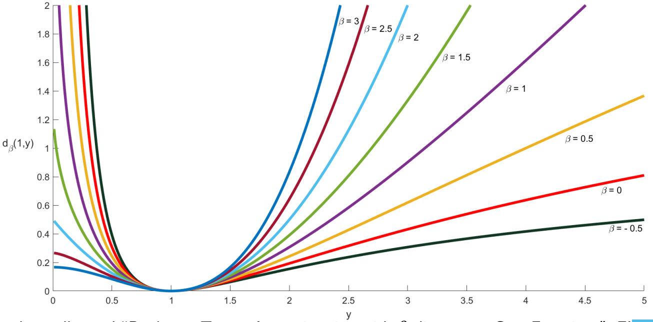
β-divergences

- β-divergences interpolate between LS distance, KL divergence and IS divergence [2]
- If $\beta = 2$, we obtain the Euclidean distance
- For $\beta > 2$, errors on larger values are penalized more heavily than for the LS criterion; for $\beta < 2$, the converse is true



[3]M. Vandecapelle et al,"Rank-one Tensor Approximation with β-divergence Cost Functions", EU

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Simulations

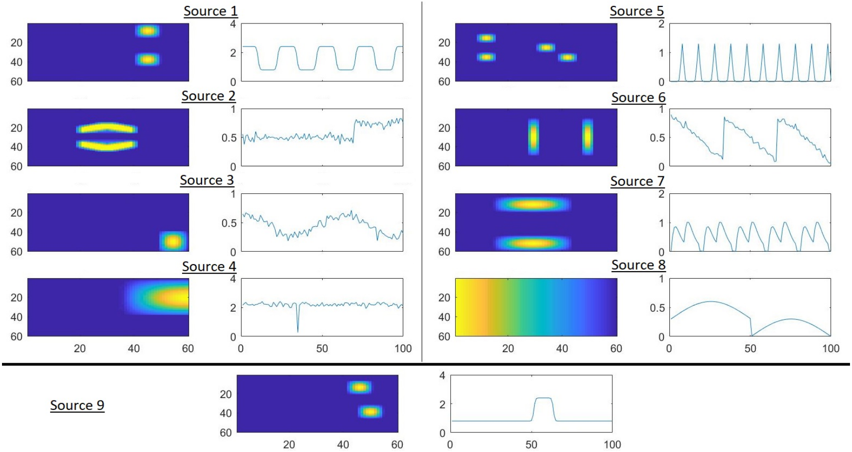

Figure: Sources used in the simulations [3]

[3] V. Calhoun et al, "Independent component analysis of fMRI data in the complex domain", Magn. Reson. Med, vol. 48, no. 1, pp. 180–192, Jul. 2002

First simulation with same noise variance

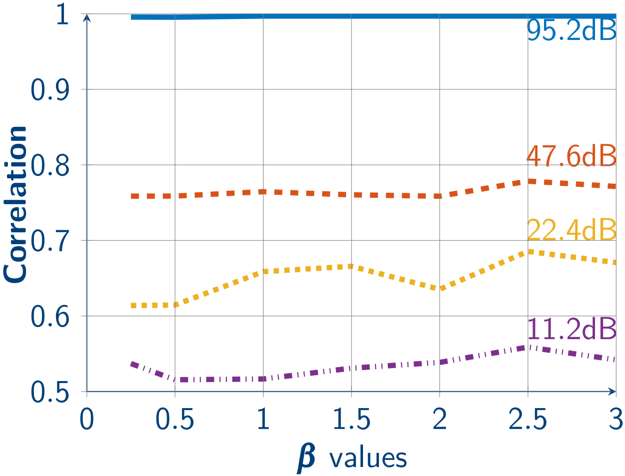

Figure: First simulation with the 8 sources used in [3].

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First simulation with same noise variance

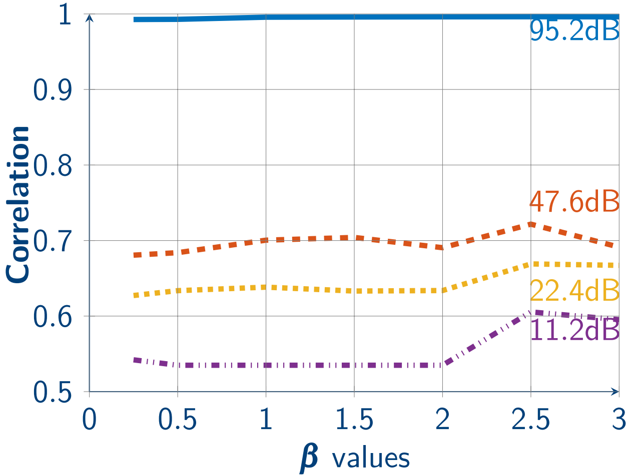

Figure: Source 9 with high overlap is included in the initial sources

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Second simulation with different noise variance

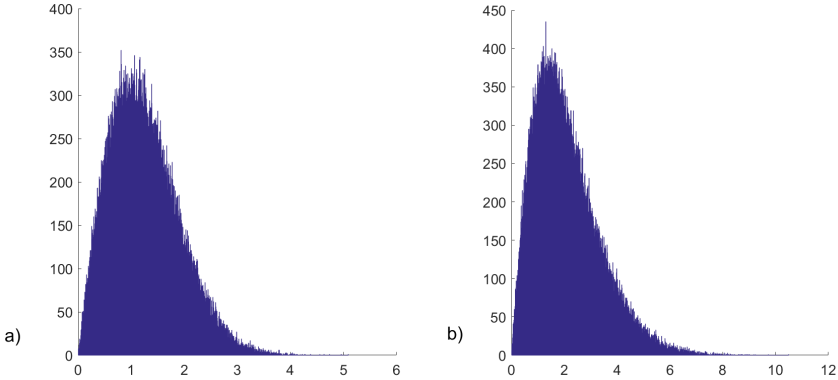

Figure: Histograms of the observed values at one spatial point. a) Real and imaginary noise variances are equal; b) the noise variance of the imaginary part is five times that of the real part.

Second simulation with different variance

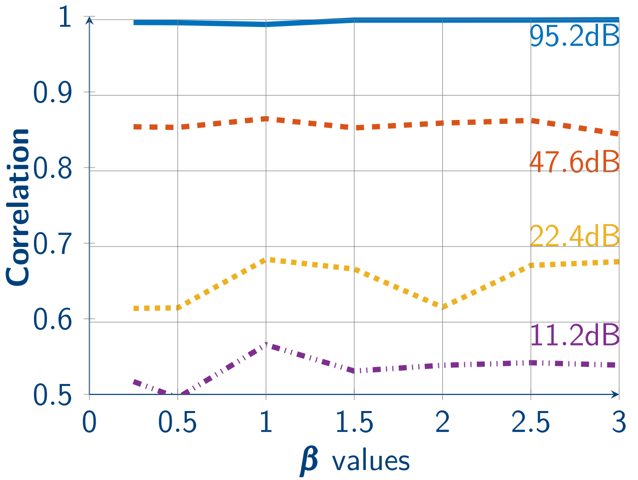

Figure: Second simulation with the 8 sources [3] and different noise variance in the real and imaginary parts of the sources.

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Conclusions

- First time that the Gaussian noise assumption and its influence on the fMRI BSS performance are tested in a tensorial framework.
- $\beta = 1$ (KL divergence) performs best in cases where different noise variances affect the real and imaginary data.
- $\beta = 2.5$ gives the best separation results in all other cases.

Future work

- Application in real data
- Use of regularizers that force independence or sparsity in the spatial maps will be investigated