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Statistical Analysis of Antenna Array Systems with Perturbations in Phase, Gain and Element Positions

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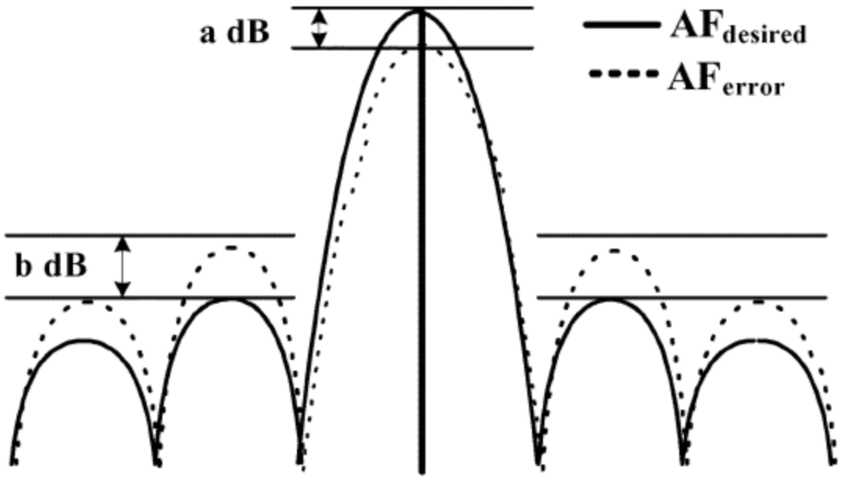
Outline

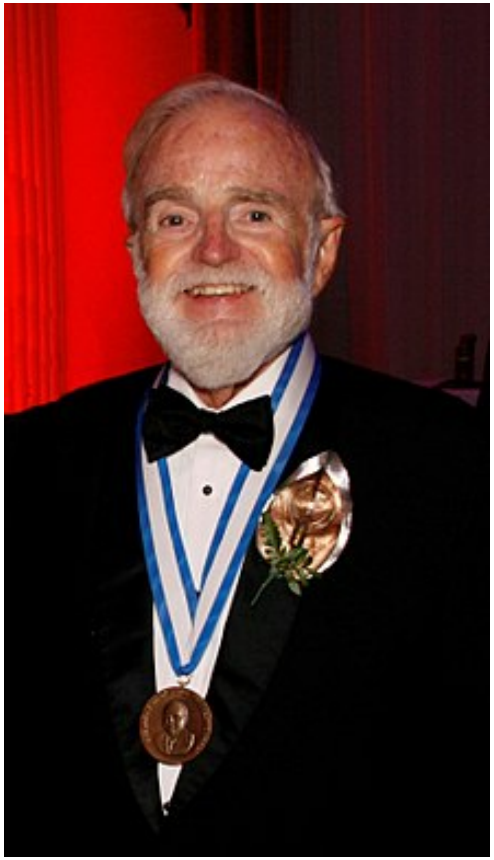
- Motivations behind statistical analysis of antenna array systems
- Perturbation modeling
- Perturbation analysis
- Simulation results
- Conclusion and future works

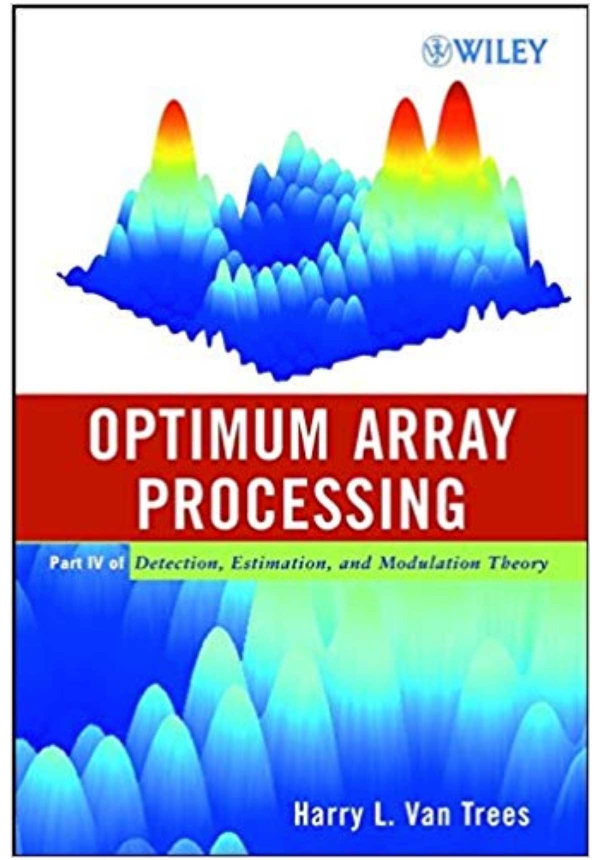
Motivations behind statistical analysis of antenna array systems

- System model for analyzing the effect of variabilitites due to manufacturing processes in beamformer modules
  - variability of phase in the manufacturing process
  - variability of gain in the manufacturing processes
  - variability of element positions in the manufacturing processes
- What will happen to beam pattern, array gain, and sidelobe levels in presence of these variabilities?
- How can we determine maximum allowable variations for a given performance penalty?



Perturbation modeling, a bit of history





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Perturbation modeling

- Phase shifter modeling for analyzing the electromagnetic beam of a linear array with *N* elements

$$B(\theta,\psi) = B(\mathbf{k}) = \mathbf{w}^H \mathbf{v}(\mathbf{k}) = \sum_{i=0}^{N-1} g_i e^{j(\phi_i)} e^{-j\mathbf{k}\mathbf{p}_i}$$

- Variability modeling for phase, gain and element positions*

$$B(\mathbf{k}) = \sum_{i=0}^{N-1} g_i (1+\Delta g_i) e^{(j(\phi_i+\Delta\phi_i)-j\mathbf{k}\mathbf{p}_i)}$$

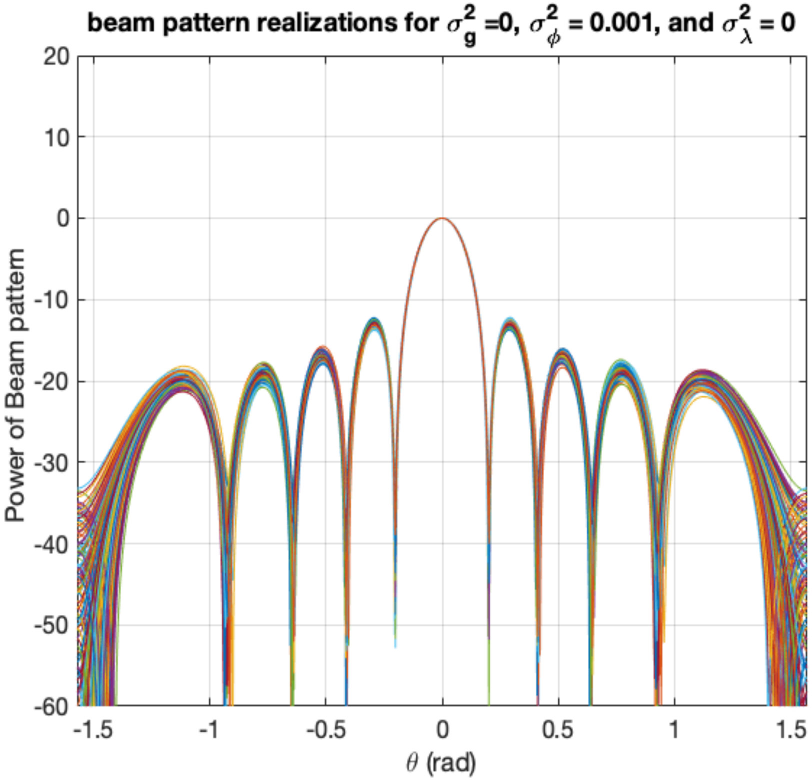
$$\mathbf{k} = \frac{2\pi}{\lambda}\begin{bmatrix} sin(\theta)cos(\psi) & sin(\theta)sin(\psi) & cos(\theta) \end{bmatrix}$$

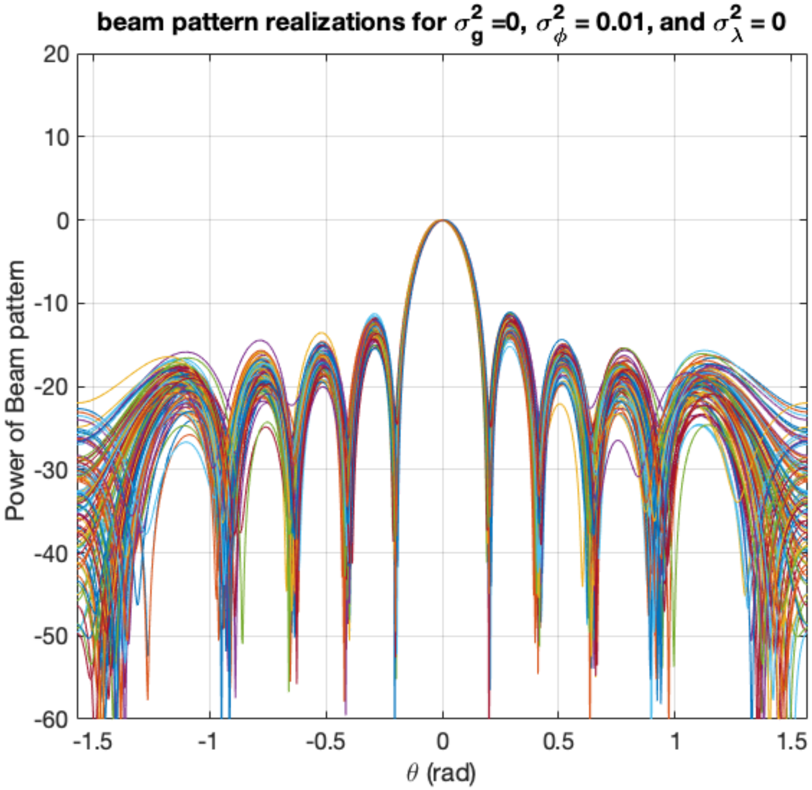
- Where all perturbations are considered as uncorrelated zero-mean Gaussian random variables

* H. L. Van Trees, Optimum array processing: Part IV of detection, estimation, and modulation theory, John Wiley & Sons, 2004.

Perturbation analysis

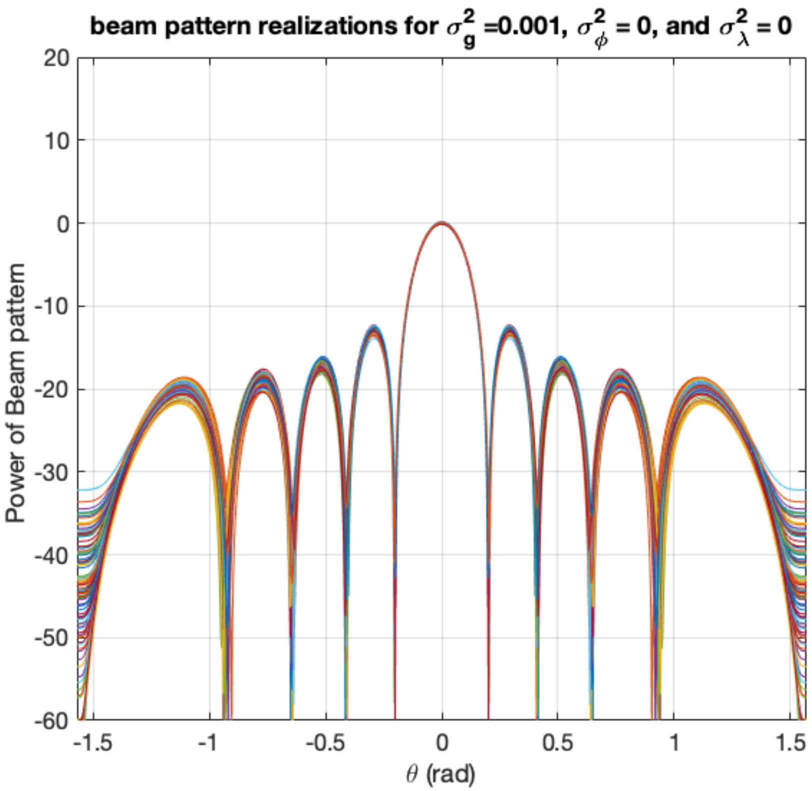
- Variability in manufacturing processes of phase $e^{j(\phi_i+\Delta\phi_i)}$

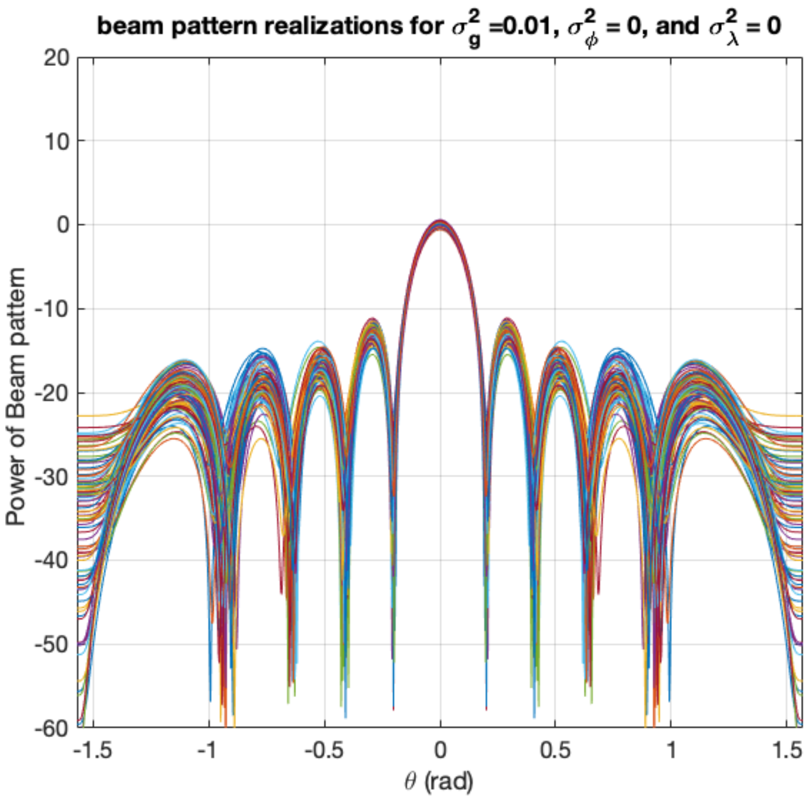




Perturbation analysis

- Variability in manufacturing processes of gain $g_i(1 + \Delta g_i)$

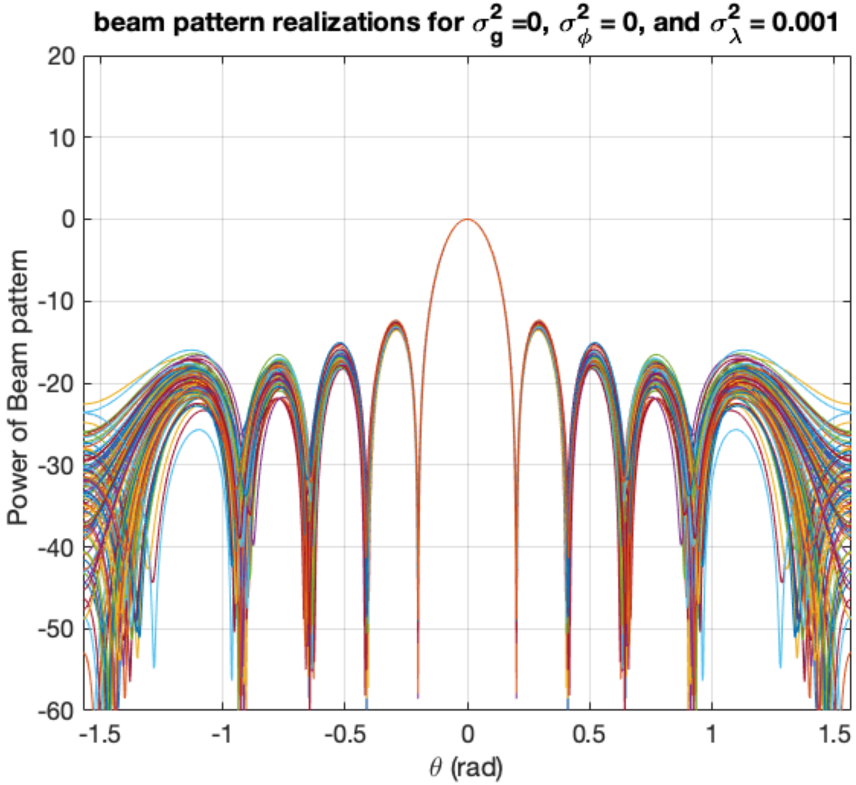


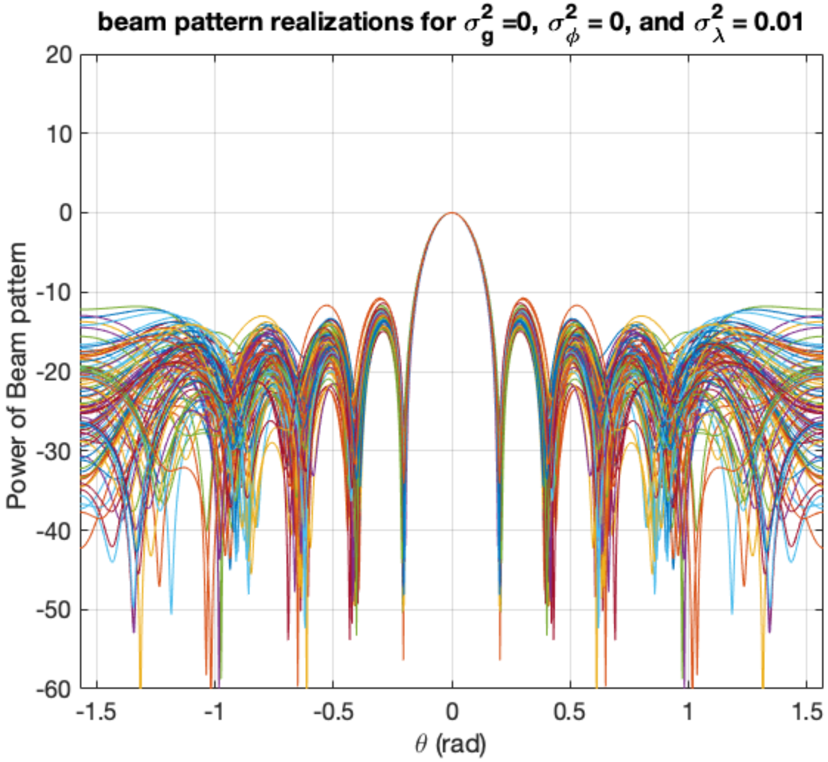


Perturbation analysis

- Variability in manufacturing processes of position of elements

$$\mathbf{p}_i = \mathbf{p}_i^c + [0 \quad 0 \quad \Delta p_i]^T$$





Perturbation analysis

- Variations of beam power

$$\Lambda = \mathbb{E}[|B(\mathbf{k})|^4] - (\mathbb{E}[|B(\mathbf{k})|^2])^2$$

- Mean of beam power

$$\mathbb{E}\big[|B(\mathbf{k})|^2\big]$$
$$= |B^c(\mathbf{k})|^2 e^{-(\sigma_\phi^2+\sigma_\lambda^2)} + \Big((1+\sigma_g^2) - e^{-(\sigma_\phi^2+\sigma_\lambda^2)}\Big) \sum_{i=0}^{N-1} g_i^2$$

- And then we need to calculate

$$\mathbb{E}[|B(\mathbf{k})|^4] = \mathbb{E}[B(\mathbf{k})^H B(\mathbf{k}) B(\mathbf{k})^H B(\mathbf{k})]$$

Perturbation analysis

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$$\Lambda = \mathbb{E}[|B(\mathbf{k})|^4] - (\mathbb{E}[|B(\mathbf{k})|^2])^2$$

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- And then we need to calculate

$$E[|B(\mathbf{k})|^4] = E[B(\mathbf{k})^H B(\mathbf{k}) B(\mathbf{k})^H B(\mathbf{k})]$$
$$= \sum_{i=0}^{N-1}\sum_{l=0}^{N-1}\sum_{m=0}^{N-1}\sum_{q=0}^{N-1} E\Big[g_i(1+\Delta g_i) g_l(1+\Delta g_l) g_m(1+\Delta g_m) g_q(1+\Delta g_q)$$
$$e^{j(\phi_i+\Delta\phi_i-\phi_l-\Delta\phi_l+\phi_m+\Delta\phi_m-\phi_q-\Delta\phi_q)} e^{-j\mathbf{k}(\mathbf{p}_i-\mathbf{p}_l+\mathbf{p}_m-\mathbf{p}_q}\Big].$$

Perturbation analysis

- Lemma 1*: expected value of multiplication of four jointly Gaussian random variables

$$E[\Delta g_i \Delta g_l \Delta g_m \Delta g_q] = E[\Delta g_i \Delta g_l]E[\Delta g_m \Delta g_q] + E[\Delta g_i \Delta g_m]E[\Delta g_l \Delta g_q]$$

$$+E[\Delta g_i \Delta g_q]E[\Delta g_l \Delta g_m],$$

- Lemma 2*: expected value of product of normal exponential random variables

$$E\left[\prod_{i=1}^{K} e^{a_i z_i}\right] = E\left[e^{\mathbf{a}^T \mathbf{z}}\right] = e^{\mathbf{a}^T \mathbf{m} + 0.5\mathbf{a}^T \Sigma \mathbf{a}}$$

- Where ***m*** is the mean vector and Σ is the covariance matrix

* A. Papoulis and S. U. Pillai, Probability, random variables, and stochastic processes, p. 258, Tata McGraw-Hill Education, 2002.

Perturbation analysis

- Using Lemma 1 & 2

$$\Lambda = \sum_{i=0}^{N-1}\sum_{l=0}^{N-1}\sum_{m=0}^{N-1}\sum_{q=0}^{N-1}\Big[\Big(g_i g_l g_m g_q e^{j(\phi_i-\phi_l+\phi_m-\phi_q)} e^{-j\mathbf{k}(\mathbf{p}_i^c-\mathbf{p}_l^c+\mathbf{p}_m^c-\mathbf{p}_q^c)}\Big)$$

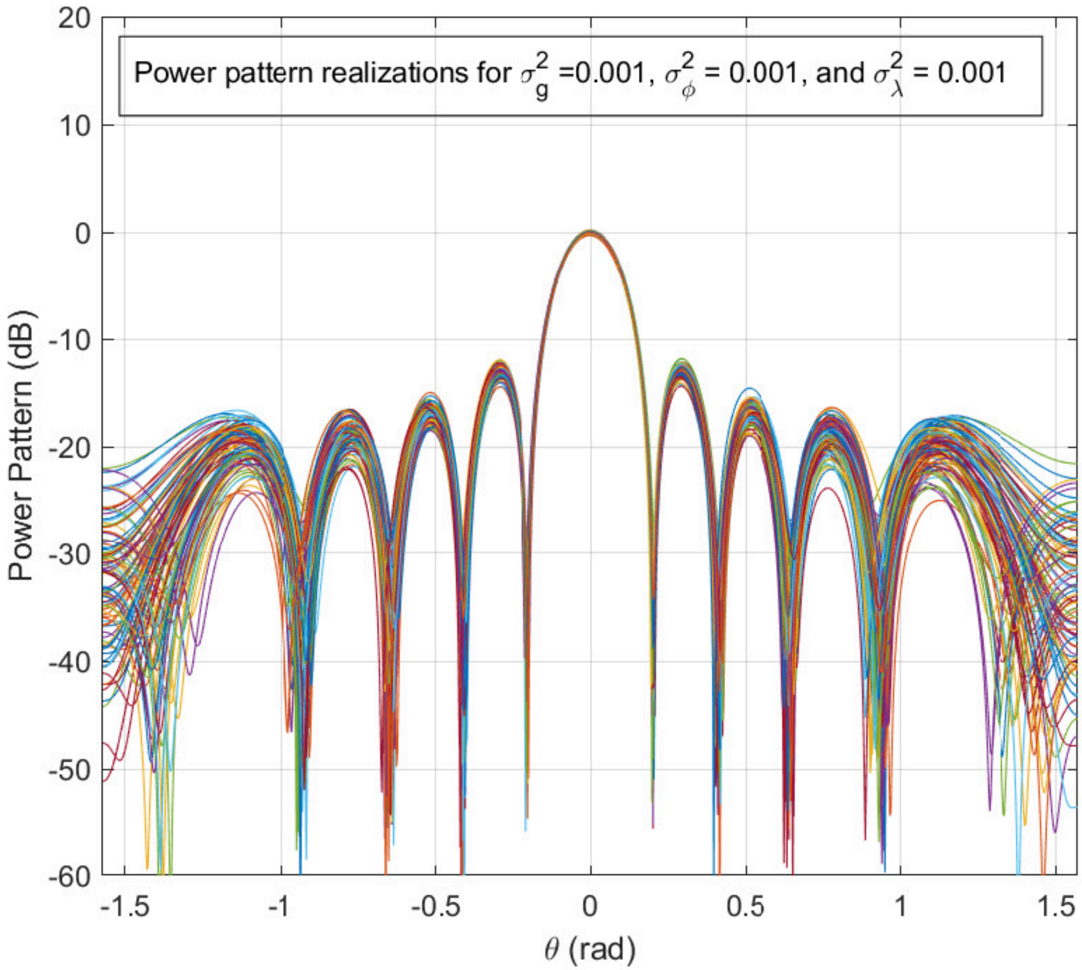
$$\Big(1+(\delta_{mq}+\delta_{im}+\delta_{iq}+\delta_{lm}+\delta_{lq}+\delta_{il})\sigma_g^2+(\delta_{il}\delta_{mq}+\delta_{im}\delta_{lq}+\delta_{iq}\delta_{lm})\sigma_g^4\Big)$$

$$\Big(e^{\sigma_\phi^2(-2+(\delta_{mq}-\delta_{im}+\delta_{iq}+\delta_{lm}-\delta_{lq}+\delta_{il}))}\Big)\Big(e^{\sigma_\lambda^2(-2+(\delta_{mq}-\delta_{im}+\delta_{iq}+\delta_{lm}-\delta_{lq}+\delta_{il}))}\Big)\Big]$$

$$-\Big(|B^c(\mathbf{k})|^2 e^{-(\sigma_\phi^2+\sigma_\lambda^2)}+[(1+\sigma_g^2)-e^{-(\sigma_\phi^2+\sigma_\lambda^2)}]\sum_{i=0}^{N-1} g_i^2\Big)^2$$

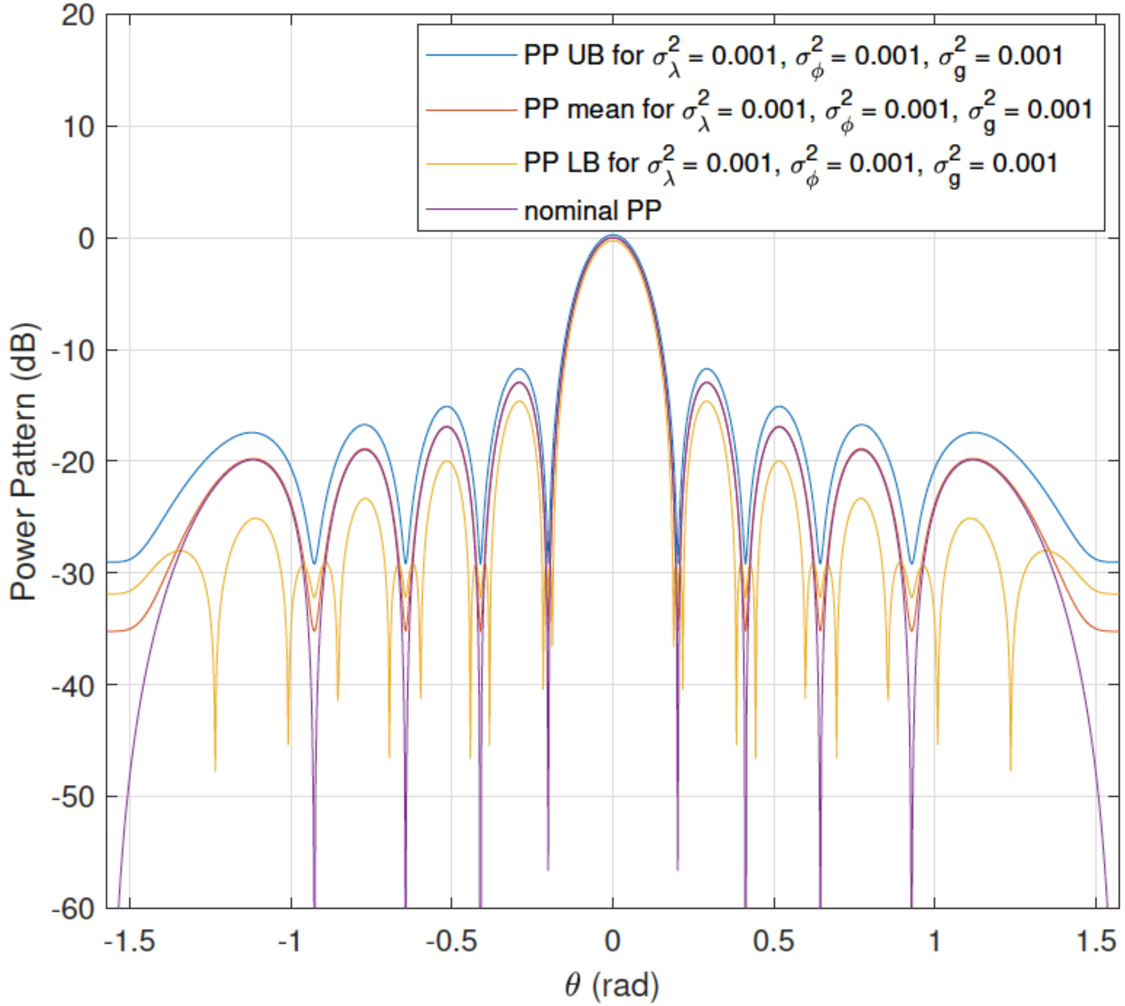
Simulation results

- Monte-Carlo simulations for 100 realizations
- One standard deviation = 0.0188



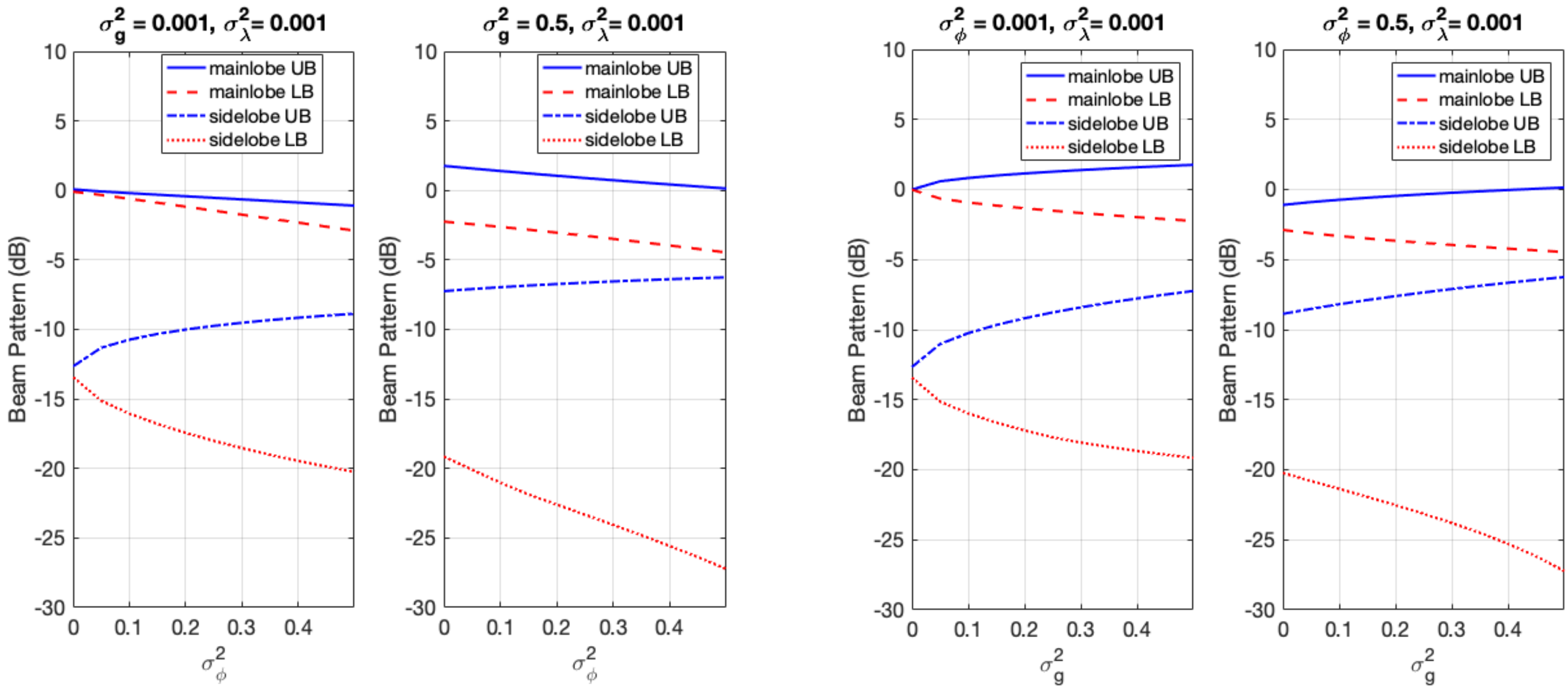
Simulation results

- Statistical bounds
- One standard deviation = 0.0200
- Maximum three standard deviations is considered (0.06)



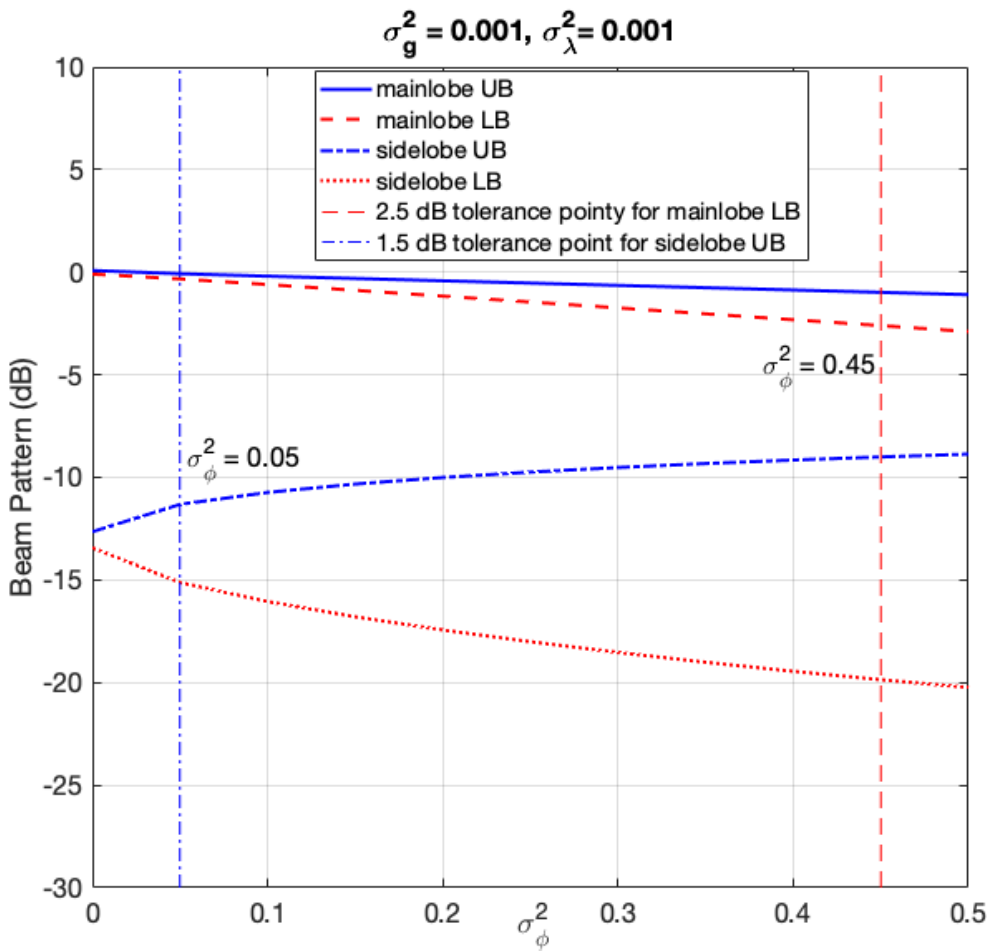
Simulation results

- Side-lobe, main-lobe variation analysis



Simulation results

- Tolerance analysis
- For a certain amount of variation in manufacturing process, we can determine maximum allowable variations for a given performance penalty



Conclusion & Future works

- We model the phase shifters with three parameters for manufacturing process variability analysis, and can predict the maximum variations in the beam pattern.
- We can make tolerance study based on statistical derivations we made. We can indicate that for a certain amount of loss in the main lobe or a certain amount of increase in the side lobes, how much each parameter is free to variate in the manufacturing process.
- Future works:
  - We can make similar study for frequerncy selective variations, such as beam squint
  - We can make similar statistical modeling and analysis for other components in the beamformer module and take into account other impairments (Timing jitter, PA nonlinearities, …)
  - We can apply intended variations in the input parameters of Chalmers Massive MIMO Testbed (MATE) and measure the resulting beam patterns in the an-echoic chamber to show and validate our results in different scenarios

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