Estimating Correlation Coefficients for Quantum Radar and Noise Radar

A Simulation Study

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What is a quantum radar?

- Any radar that exploits **phenomena from quantum physics** to **improve detection performance**
- But quantum physics includes everything in classical physics!
- We look at a distinctively quantum phenomenon called **entanglement**

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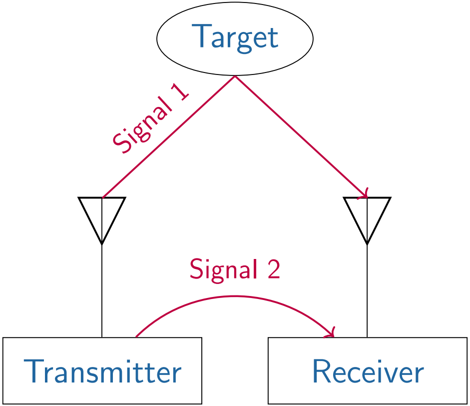
- Correlation $\implies$ probability theory. Does entanglement involve probability, statistics, random variables?

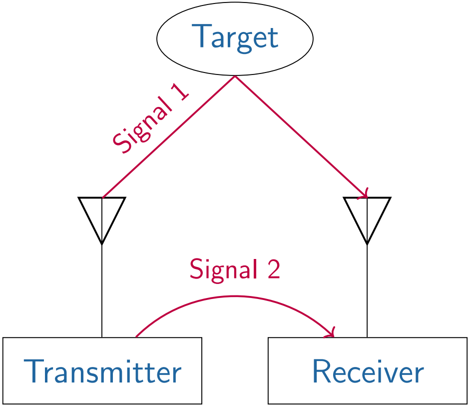
YES!

Quantum radar: the basic idea



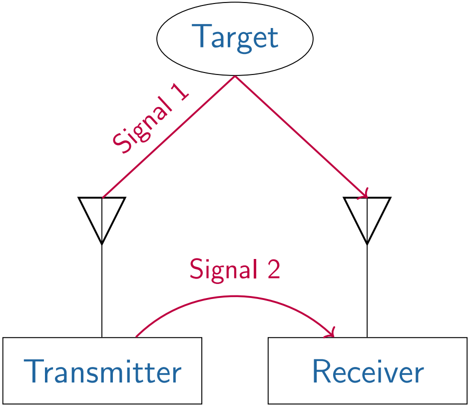
1. Produce a pair of entangled microwave beams.
2. Transmit one of the beams. Keep the other.
3. Receive and measure the signal.
4. Correlate the received and retained signals. Declare a detection if the correlation exceeds a certain threshold.

Quantum radar: the basic idea



- Entangled signals are highly **correlated** at transmitter
- **High** correlation at receiver $\Longrightarrow$ target **present**
- **Low** correlation at receiver $\Longrightarrow$ target **absent**

But you don't need quantum for this, right?



- Isn't this just **matched filtering**?
- Can't we generate 100% correlated signals?
- Why bother with all this quantum stuff?

The bad news: quantum noise

- Conventional matched filtering assumes **a perfect copy of the signal is available**
- Quantum mechanics says a perfect copy is **impossible**
- There will **always be noise** in I and Q voltage measurements, even in an **theoretically ideal** system
- **Quantum noise** exists even at absolute zero and in a perfect vacuum

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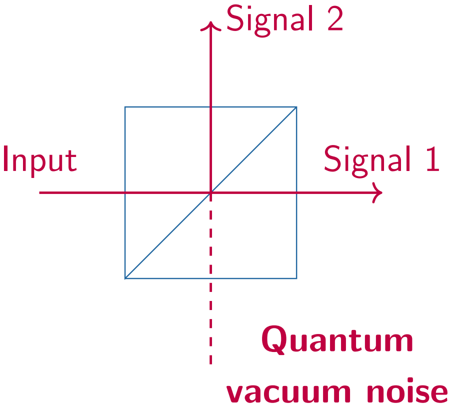
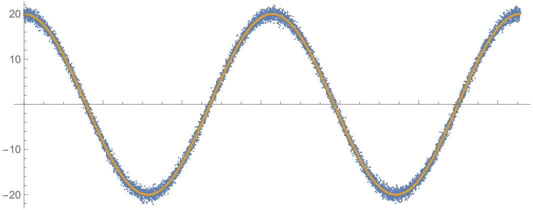
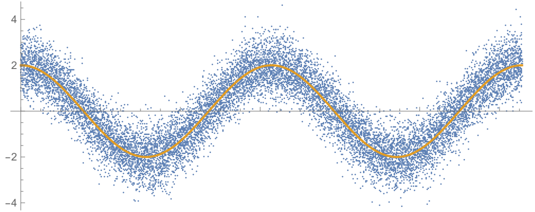


- **Vacuum noise** will creep into the beamsplitter, even at absolute zero and in a perfect vacuum

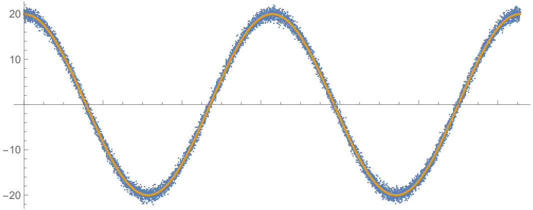
Quantum noise

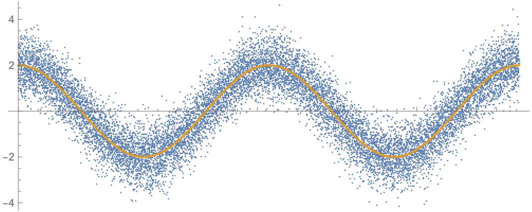




- Classically ideal signal: $I(t) = A\cos(\omega t)$
- Quantum ideal signal: $I(t) \sim A\cos(\omega t) + \mathcal{N}(0, \sigma^2)$

Quantum noise



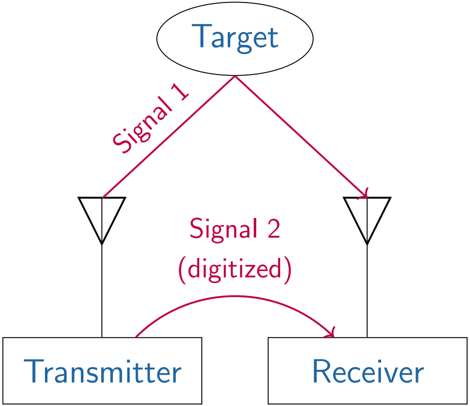


- Classically ideal signal: $I(t) = A\cos(\omega t)$
- Quantum ideal signal: $I(t) \sim A\cos(\omega t) + \mathcal{N}(0, \sigma^2)$
  - Gaussian noise with power depending only on ω

Quantum noise and entanglement

- 100% correlation is **impossible** between signals with uncorrelated quantum noise
  - No such thing as perfect matched filtering
- Quantum noise cannot be eliminated, but **can be correlated** between two signals
  - Better "matched filtering"

Quantum two-mode squeezing radar



1. Produce a pair of **entangled** microwave beams.
2. Transmit one of the beams. Immediately record a time series of I/Q voltages for the other beam.
3. Receive and record I/Q voltages.
4. Perform matched filtering as usual.

Note: **a prototype QTMS radar has been built!**

The QTMS radar covariance matrix

$$\begin{bmatrix} \sigma_1^2 & 0 & \rho\sigma_1\sigma_2\cos\phi & \rho\sigma_1\sigma_2\sin\phi \\ 0 & \sigma_1^2 & \rho\sigma_1\sigma_2\sin\phi & -\rho\sigma_1\sigma_2\cos\phi \\ \rho\sigma_1\sigma_2\cos\phi & \rho\sigma_1\sigma_2\sin\phi & \sigma_2^2 & 0 \\ \rho\sigma_1\sigma_2\sin\phi & -\rho\sigma_1\sigma_2\cos\phi & 0 & \sigma_2^2 \end{bmatrix}$$

- I_1, Q_1, I_2, Q_2 are **Gaussian random variables** characterized by this covariance matrix
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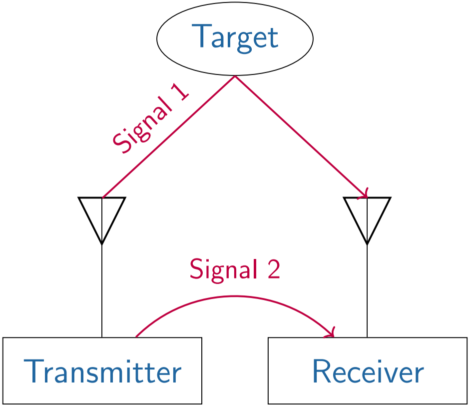
- I_1, Q_1, I_2, Q_2 are **Gaussian random variables** characterized by this covariance matrix
- σ_1^2, σ_2^2 are signal powers for the received and recorded signals; ϕ is the phase between them
- ρ characterizes the **correlation** between the two signals

ρ as a detector function



- $\rho > 0$ at receiver $\implies$ target **present**
- $\rho = 0$ at receiver $\implies$ target **absent**
- Note: entanglement improves ρ at **transmitter**
  - Only need to distinguish between $\rho > 0$ and $\rho = 0$ at receiver

Estimation of ρ

$$\begin{bmatrix} \sigma_1^2 & 0 & \rho\sigma_1\sigma_2\cos\phi & \rho\sigma_1\sigma_2\sin\phi \\ 0 & \sigma_1^2 & \rho\sigma_1\sigma_2\sin\phi & -\rho\sigma_1\sigma_2\cos\phi \\ \rho\sigma_1\sigma_2\cos\phi & \rho\sigma_1\sigma_2\sin\phi & \sigma_2^2 & 0 \\ \rho\sigma_1\sigma_2\sin\phi & -\rho\sigma_1\sigma_2\cos\phi & 0 & \sigma_2^2 \end{bmatrix}$$

- Can estimate the covariance matrix from measurement data using the **sample covariance matrix**

$$\hat{S} = \frac{1}{N}\sum_{n=1}^{N} x_n x_n^T$$

- Problem: **no guarantee** that $\hat{S}$ is of the above form

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Estimation of ρ

- One way to estimate ρ from the sample covariance matrix:

$$\min_{\sigma_1,\sigma_2,\rho,\phi} \left\| R_{\mathrm{QTMS}}(\sigma_1, \sigma_2, \rho, \phi) - \hat{S} \right\|_F$$

- $R_{\mathrm{QTMS}}(\sigma_1, \sigma_2, \rho, \phi)$ is the theoretical covariance matrix
- This gives us an **estimate** $\hat{\rho}$ of the underlying, "true" correlation ρ

Probability distribution of $\hat{\rho}$

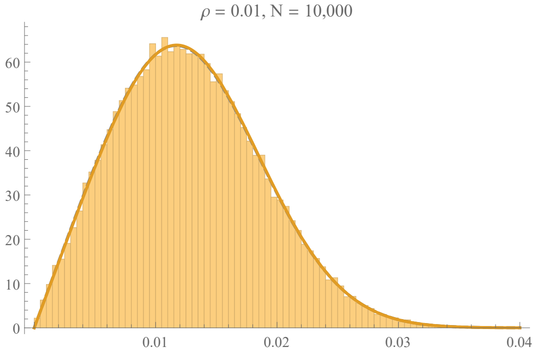
- We have found through simulations that the distribution of $\hat{\rho}$ can be **approximated by the Rice distribution**

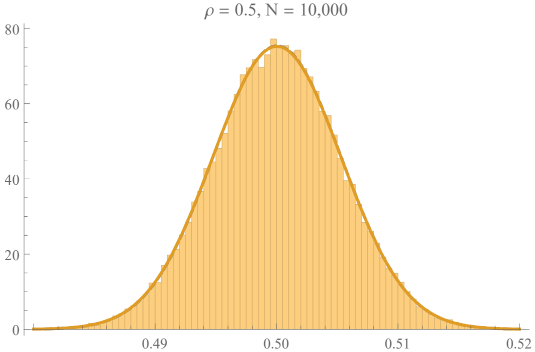
$$f(x|\alpha, \beta) = \frac{x}{\beta^2} \exp\left(-\frac{x^2 + \alpha^2}{2\beta^2}\right) I_0\left(\frac{x\alpha}{\beta^2}\right)$$

- In terms of the underlying, "true" ρ and the number of integrated samples N:

$$\alpha = \rho$$
$$\beta = \frac{1 - \rho^2}{\sqrt{2N}}.$$

Simulated data





- Orange bars: histograms of $\hat{\rho}$ obtained from simulations of QTMS radar measurements
- Solid curves: Rice distribution approximation

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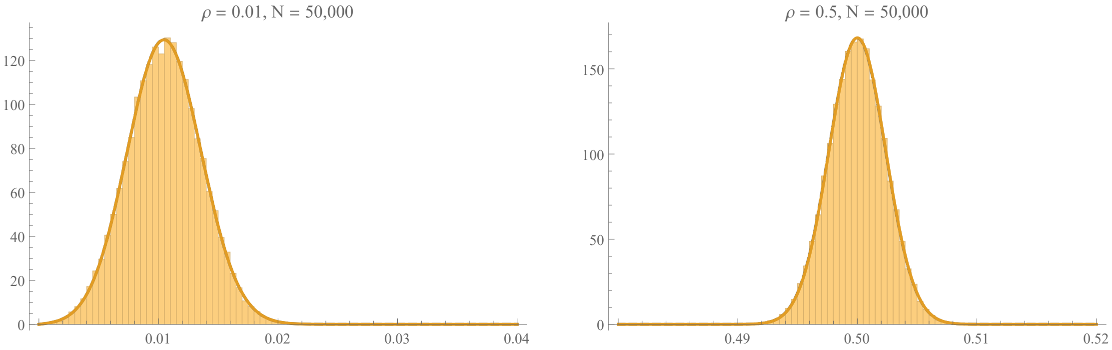

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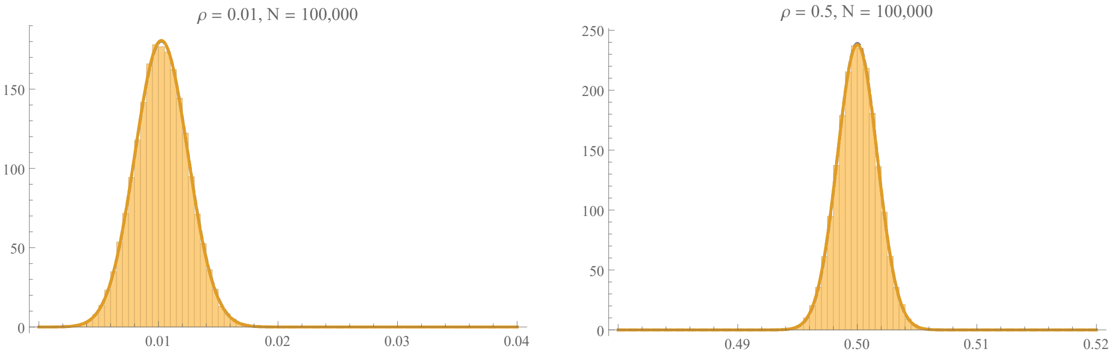

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Explicit ROC curve formula

- Based on our Rice distribution approximation, we can obtain an explicit **ROC curve formula** for the detection performance of a QTMS radar:

$$p_{\mathrm{D}}(p_{\mathrm{FA}}|\rho, N) = Q_1\left(\frac{\rho\sqrt{2N}}{1-\rho^2}, \frac{\sqrt{-2\ln p_{\mathrm{FA}}}}{1-\rho^2}\right)$$

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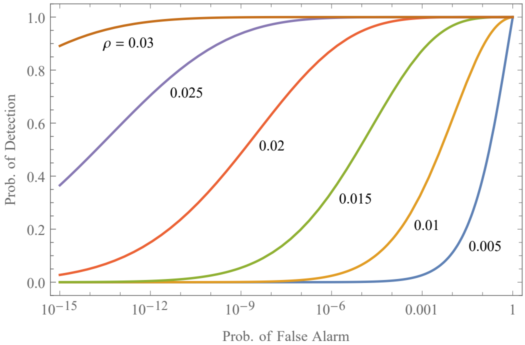
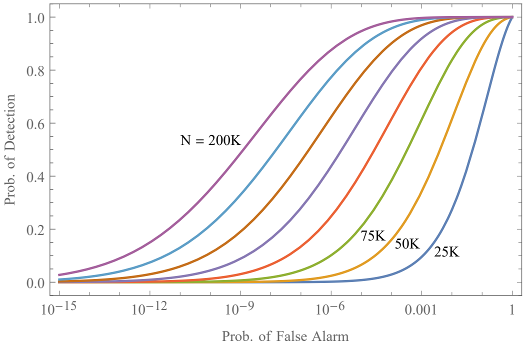
ROC curve plots





- $\rho = 0.005, 0.01, 0.015, 0.02, 0.025, 0.03$
- $N = 50{,}000$

- $\rho = 0.01$
- $N = 25\text{k}, 50\text{k}, 75\text{k}, 100\text{k}, 125\text{k}, 150\text{k}, 175\text{k}, 200\text{k}$

Conclusion

- Quantum two-mode squeezing (QTMS) radars involve **correlating two signals**
- Can extract a single **correlation coefficient** ρ that depends on whether target is present/absent
- The **Rice distribution** is a good approximation to the distribution of $\hat{\rho}$
- This is a big step toward performance prediction for QTMS radars: we need only focus on determining ρ

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