Robust Direction of Arrival Estimation in the Presence of Array Faults using Snapshot Diversity

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Overview

- Background
- Snapshot Diversity Formulation
- Algorithm for DOA Estimation and Fault Identification
- Simulation Results

Applications

Vitaly V. Kuzmin [CC BY-SA 4.0]

Radar

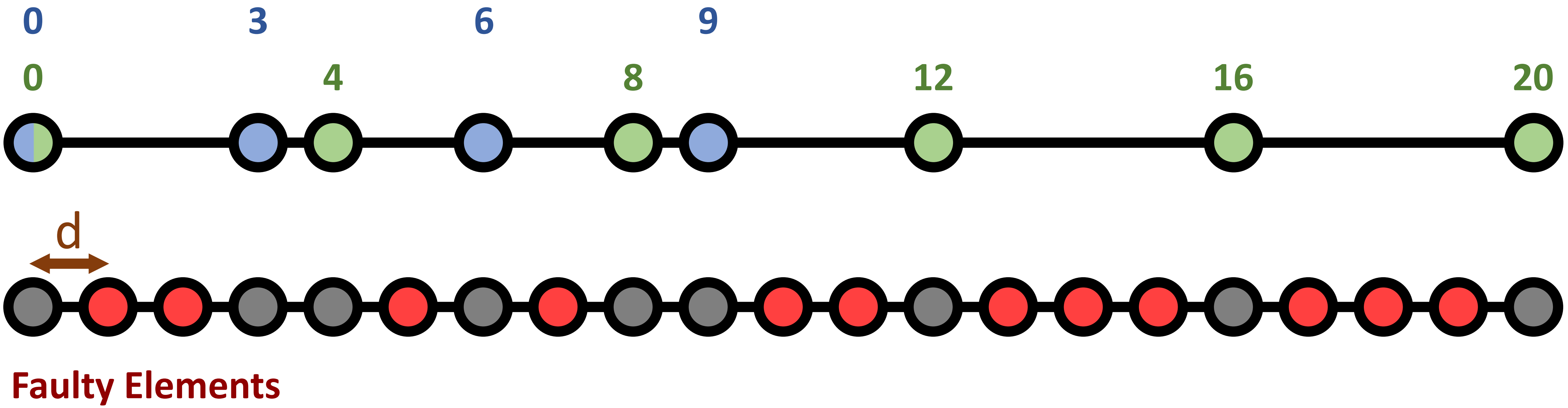
US Navy [Public domain]

Sonar

Wireless Communications

Motivation

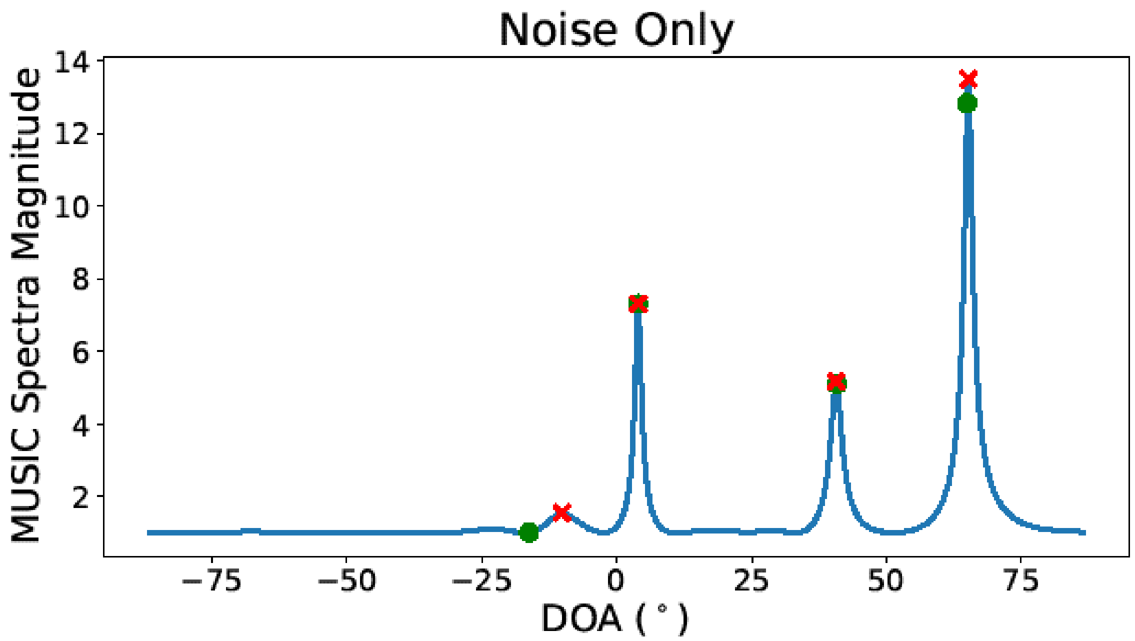
Coprime Array:



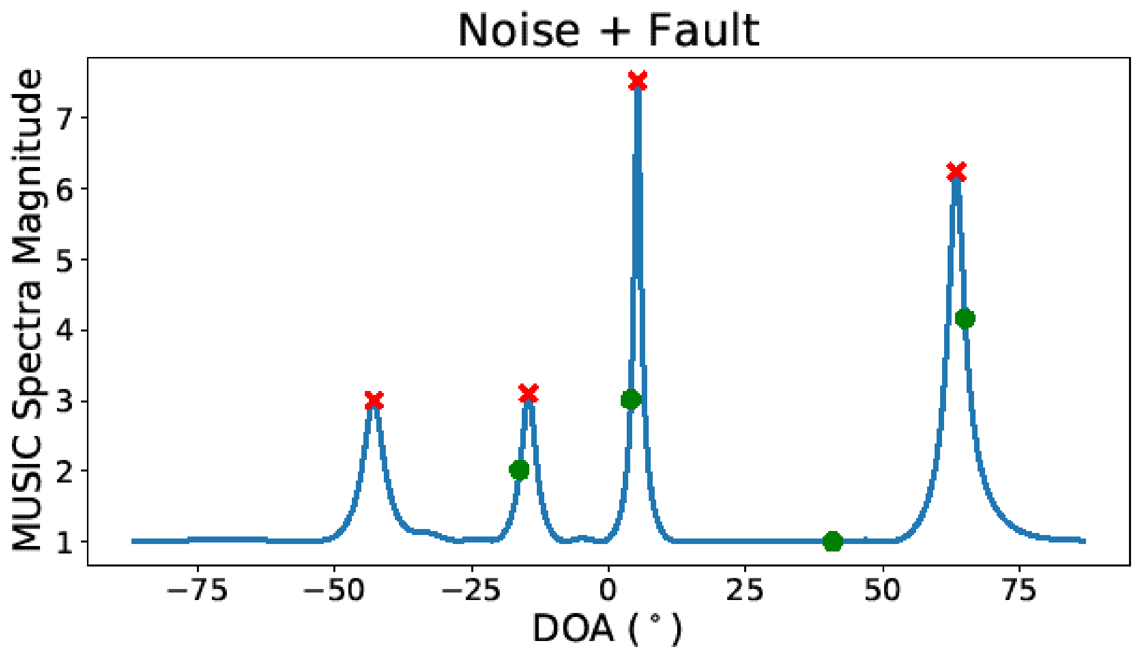
(Alexiou and Manikas, 2005; Liu and Vaidyanathan, 2018)

Single Snapshot MUSIC in the presence of fault

Legend:
×: *Actual DOA*
•: *Estimated DOA*



SNR 5 dB



SNR 30 dB

+

Fault (Random Measurements)

Problem Statement

Given an antenna array that has a few faulty components –

Can we perform robust direction-of-arrival estimation?

Faulty components:

- Arbitrary measurements; may not provide information about DOAs
- Unknown locations, but remains fixed across snapshots (time)

Related Efforts

- Analysis of array robustness
  - (Alexiou and Manikas, 2005)
  - (Liu and Vaidyanathan, 2018)

- Missing or invalid measurements
  - Matrix Completion (Yerriswamy and Jagadeesha, 2011)
  - Missing Data after some time T (Larsson and Stoica, 2001)
- Impulse noise
  - Arbitrarily large values, but outliers are randomly distributed in locations over different snapshots (Dai and So, 2018)
- Faulty snapshots
  - Robust PCA methods (Xu, Caramanis and Manoor, 2013)
- Neural network methods
  - Minimum Resource Allocation Neural Network (Vigneshwaran, Sundararajan and Saratchandran, 2007) – noise-only measurements

Main Contributions

To perform DOA estimation in the presence of faulty elements, we propose:

1. Novel **snapshot diversity** formulation, for cases where multiple-snapshot is not practical

2. Algorithm for **robust DOA estimation** and **identification of faulty components** (unknown locations, arbitrary faulty measurements)

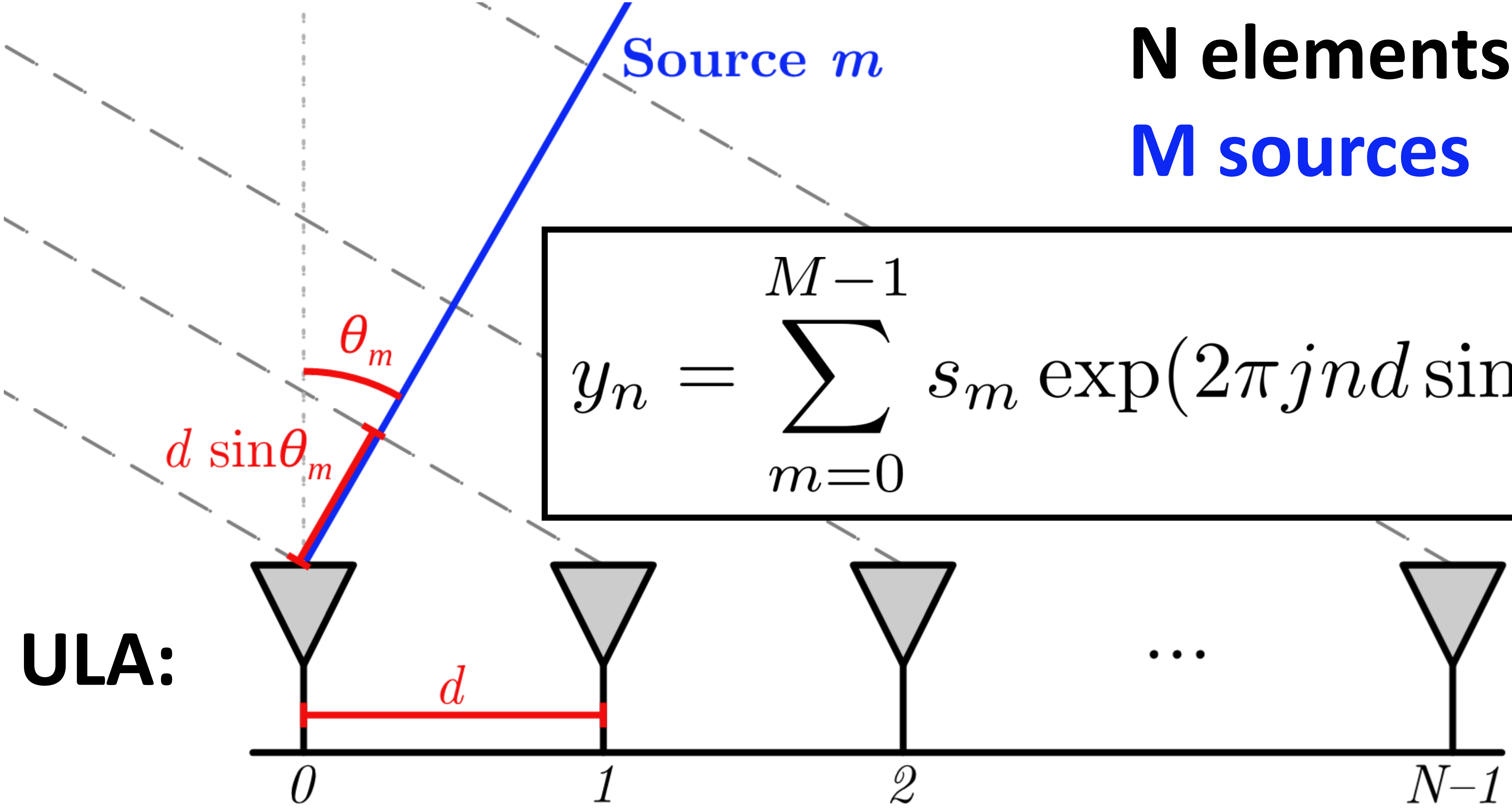
DOA Estimation – Model

N elements

M sources

$$y_n = \sum_{m=0}^{M-1} s_m \exp(2\pi j n d \sin\theta_m / \lambda_c)$$

ULA:



DOA Estimation – Mathematical Model

$$y_n(t) = \sum_{m=0}^{M-1} s_m(t) \exp(\pi j n \sin\theta_m) + \varepsilon_n(t)$$

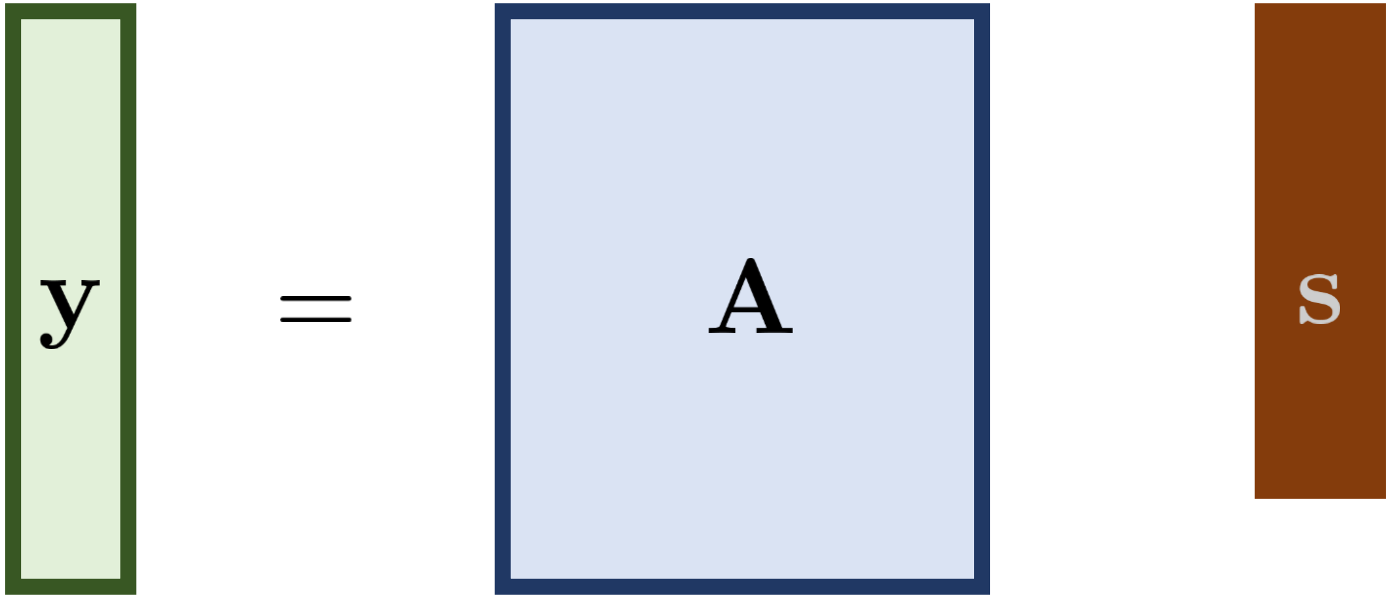
$$\varepsilon_n(t) = \begin{cases} w_n(t) \sim \mathcal{N}(0, \sigma^2) \text{ if } n \text{ is not faulty} \\ e_n(t) \text{ (Arbitrarily large), if } n \text{ is faulty} \end{cases}$$

DOA Estimation Algorithms

- Subspace Methods: MUSIC, Single Snapshot MUSIC
- Direct Data Domain: Matrix Pencil
- Sparse Representation Recovery

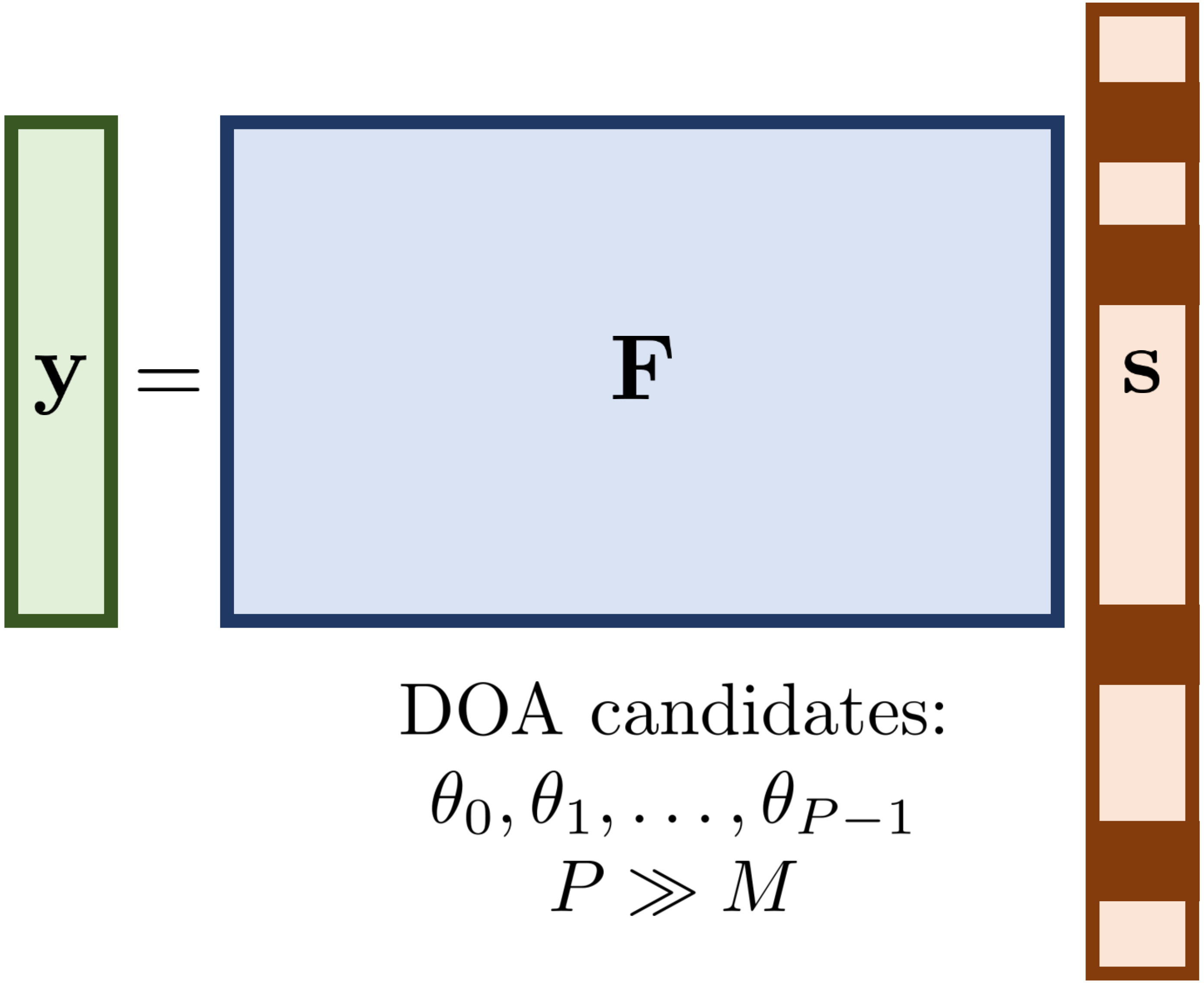
Assumes all measurements are accurate
(noise only; no array failure)

DOA Estimation



$$\theta_0, \theta_1, \ldots, \theta_{M-1}$$

Sparse Representation for DOA Estimation



DOA candidates:

$\theta_0, \theta_1, \ldots, \theta_{P-1}$

$P \gg M$

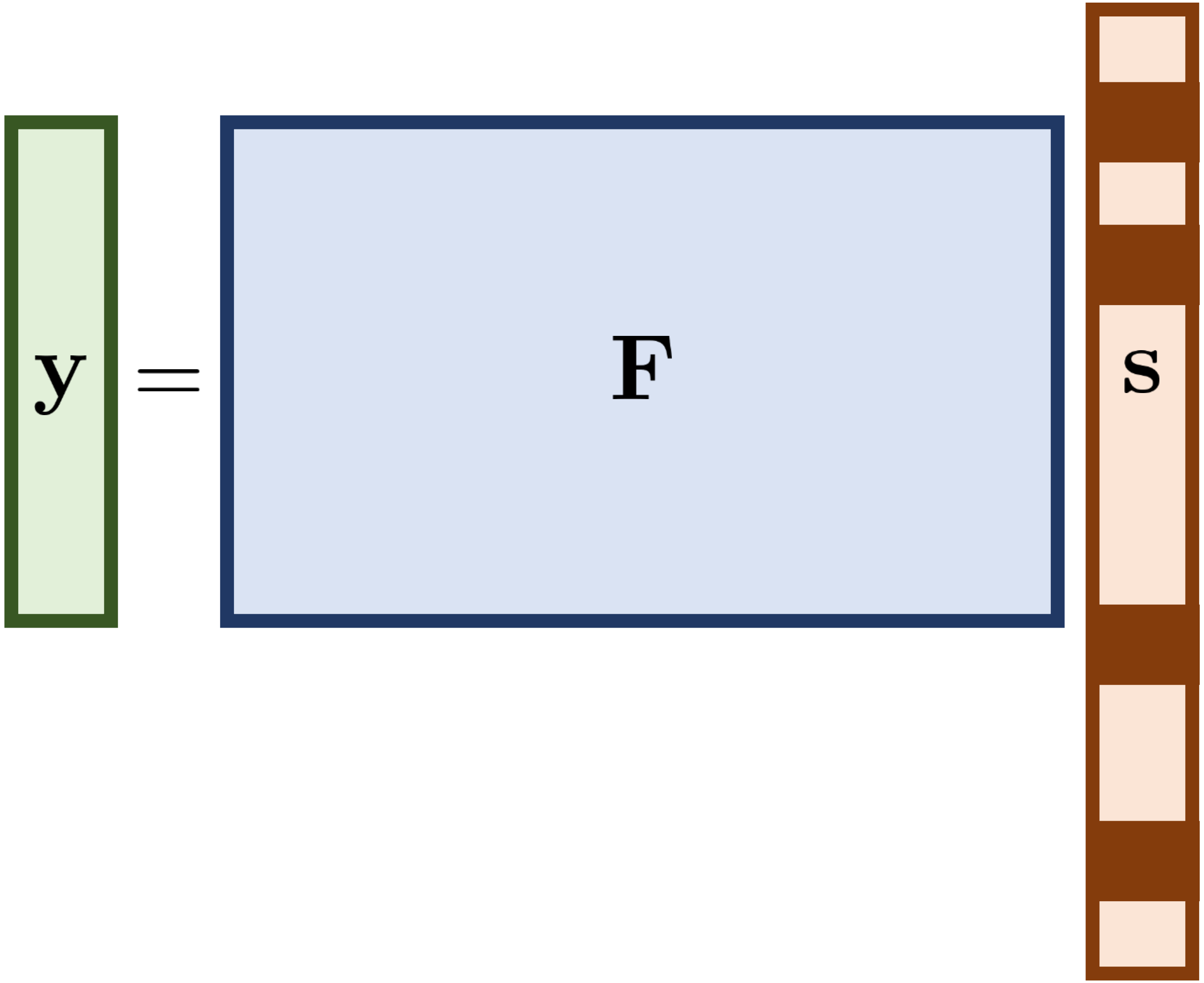
Sparse Representation for DOA Estimation

$$\widehat{\mathbf{s}} = \arg\min_{\mathbf{s}} \{\|\mathbf{s}\|_0\}$$
$$\text{subject to } \mathbf{y} = \mathbf{F}\mathbf{s}$$

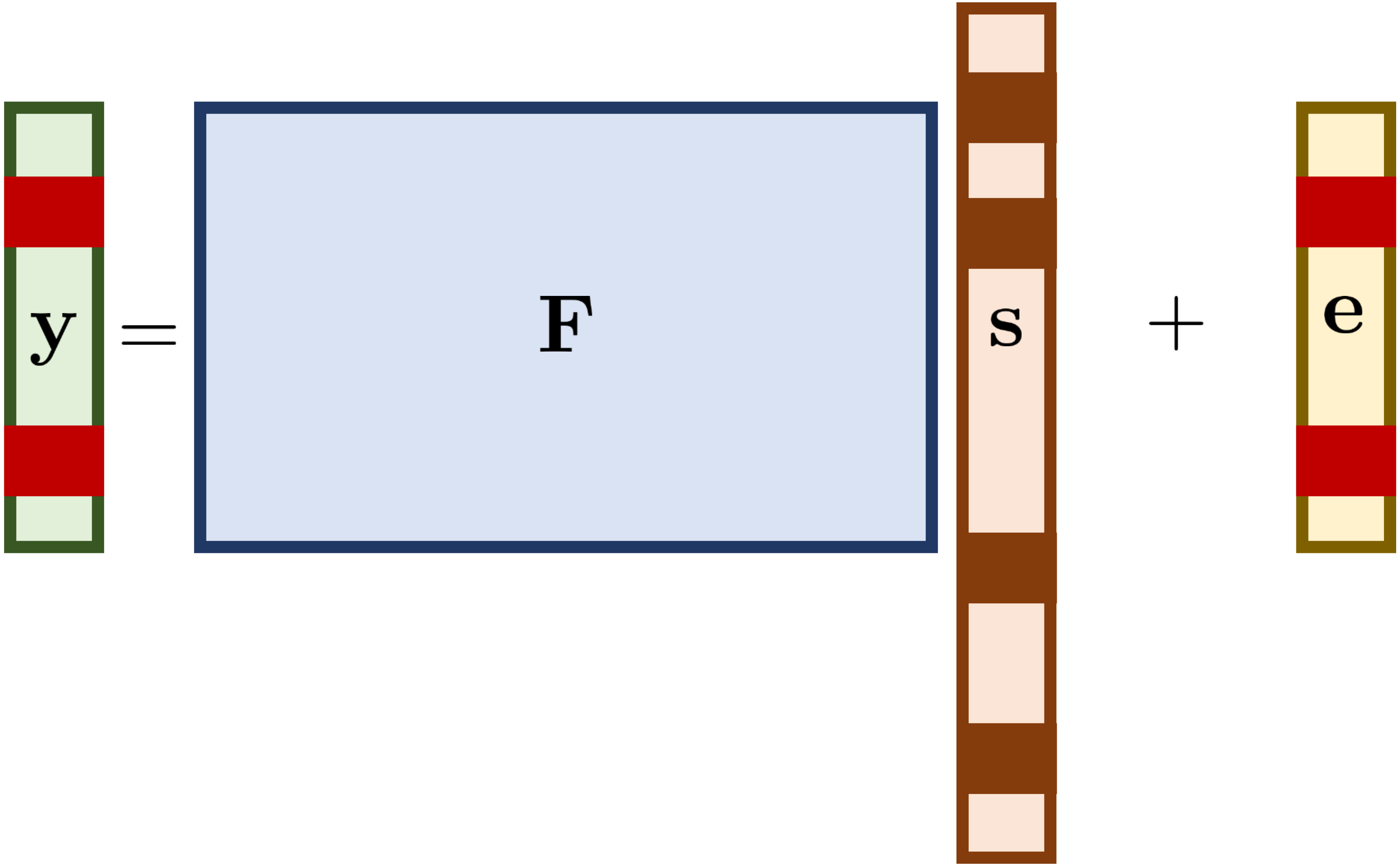
$$\widehat{\mathbf{s}} = \arg\min_{\mathbf{s}} \left\{\|(\mathbf{y} - \mathbf{F}\mathbf{s})\|_2^2 + \alpha\|\mathbf{s}\|_0\right\}$$

- Greedy Approach: Matching Pursuit
- Convex Relaxation: Basis Pursuit

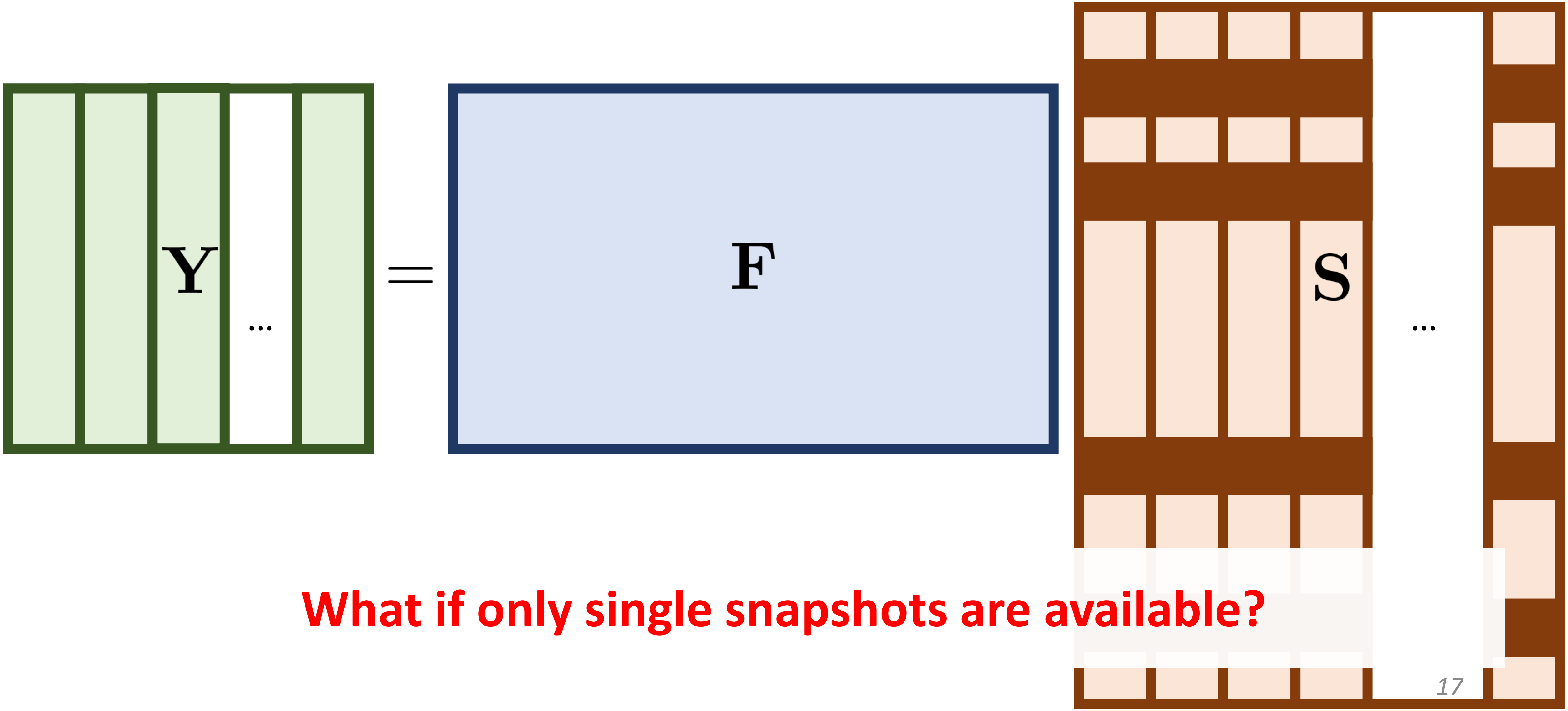
Sparse Representation for DOA Estimation



Sparse DOA Estimation + Fault

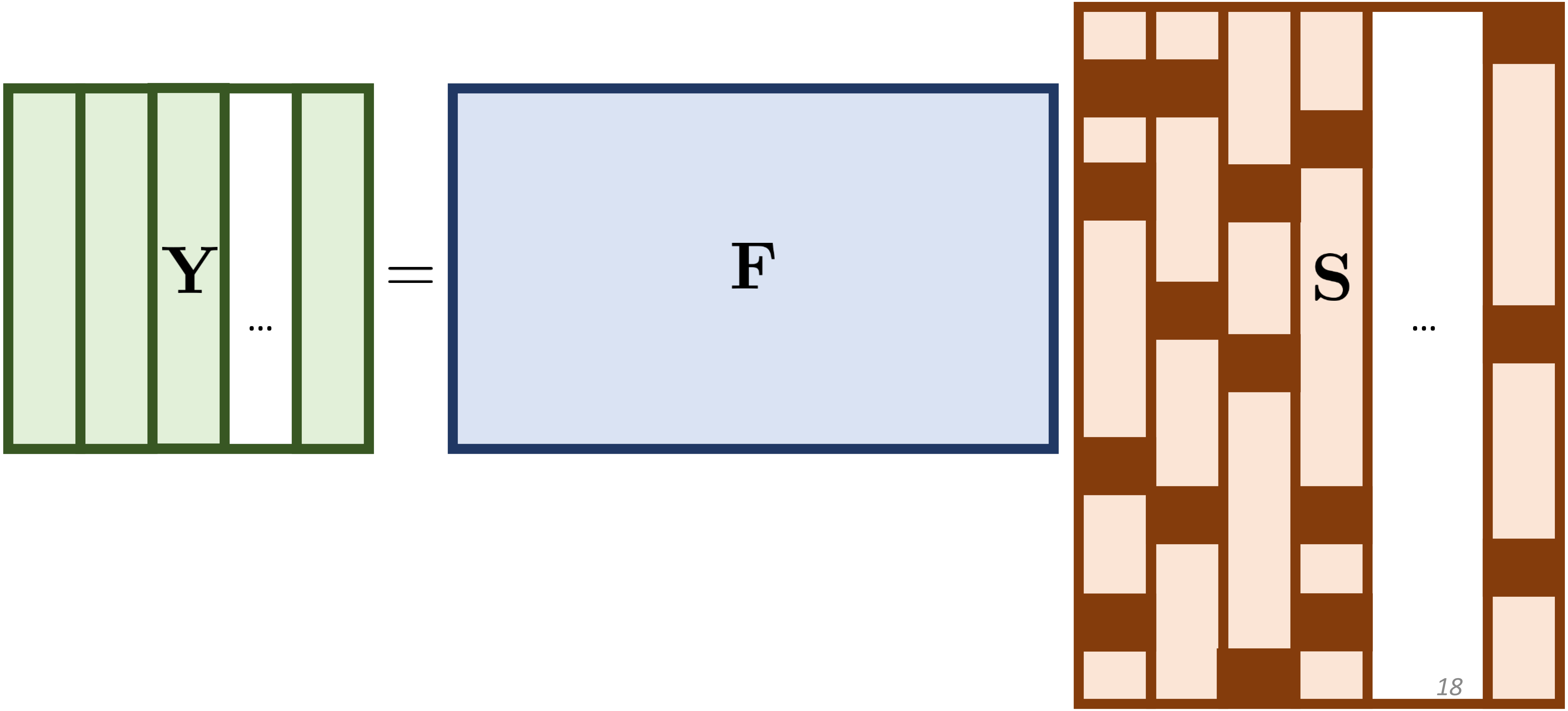


Sparse Representation for DOA Estimation

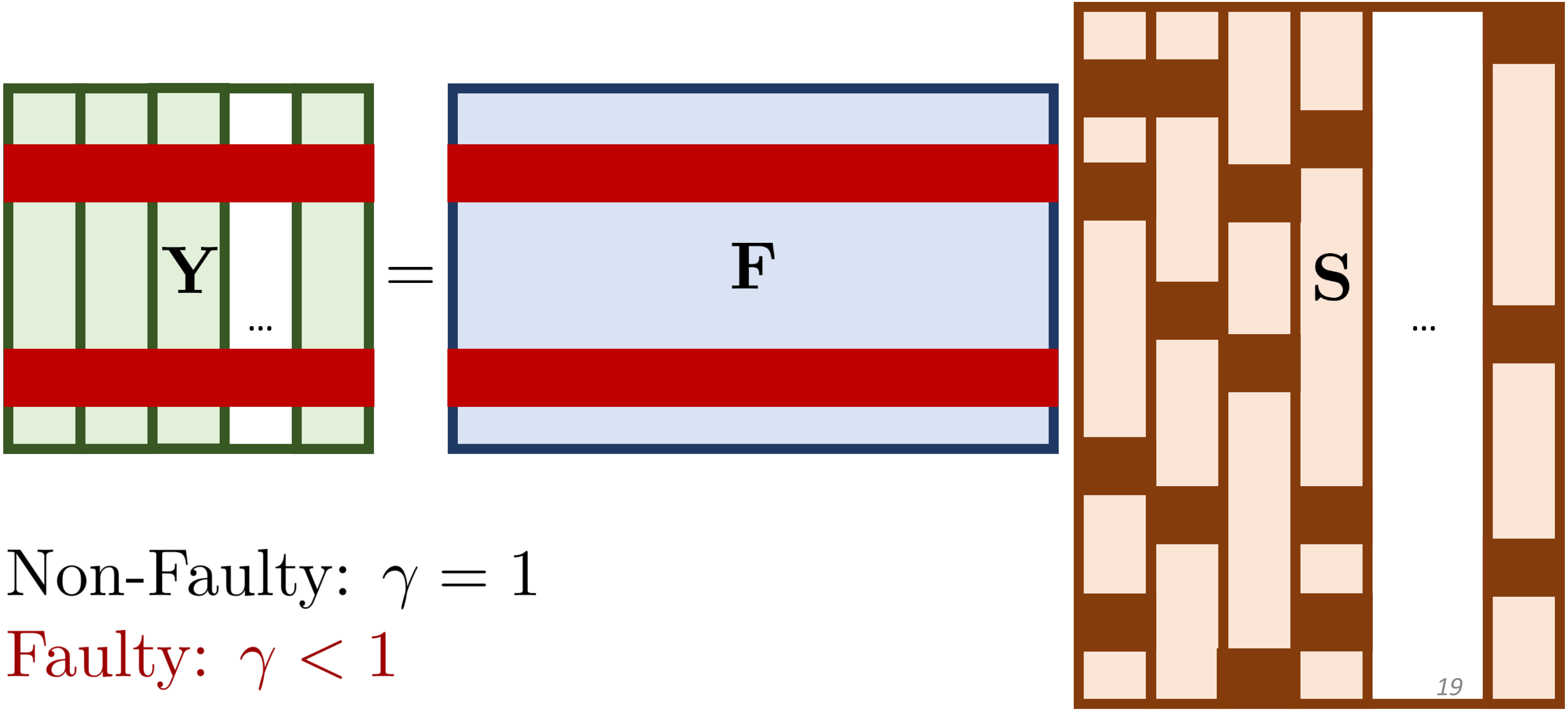


What if only single snapshots are available?

Snapshot Diversity



Snapshot Diversity + Fault



Non-Faulty: $\gamma = 1$

Faulty: $\gamma < 1$

Snapshot Diversity Formulation

$$(\widehat{\mathbf{S}}, \widehat{\boldsymbol{\gamma}}) = \arg\min_{\mathbf{S}, \boldsymbol{\gamma}} \left\{ \|\mathrm{diag}(\boldsymbol{\gamma}) \cdot (\mathbf{Y} - \mathbf{FS})\|_F^2 + \alpha \|\mathbf{S}\|_0 + J(\boldsymbol{\gamma}) \right\}$$

Snapshot Diversity Formulation

$$(\widehat{\mathbf{S}}, \widehat{\boldsymbol{\gamma}}) = \arg\min_{\mathbf{S},\boldsymbol{\gamma}} \left\{ \|\mathrm{diag}(\boldsymbol{\gamma}) \cdot (\mathbf{Y} - \mathbf{FS})\|_F^2 + \alpha\|\mathbf{S}\|_0 + \lambda\|\log(\boldsymbol{\gamma})\|_1 \right\}$$

Similar to **(Gao and Fang, 2016)**, whose work focused on **outlier detection and robust regression**

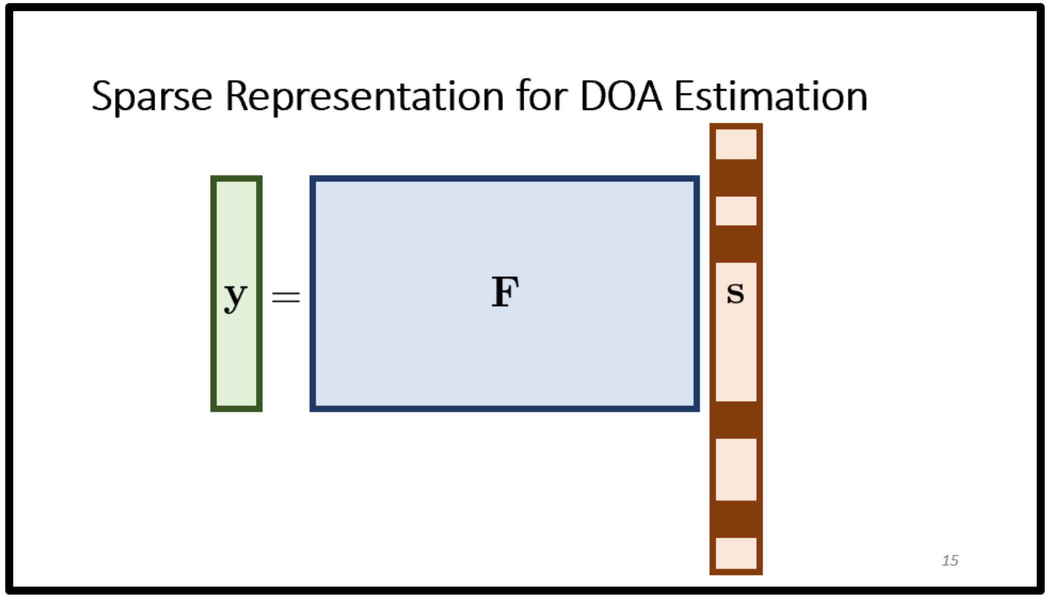
Bayesian Interpretation

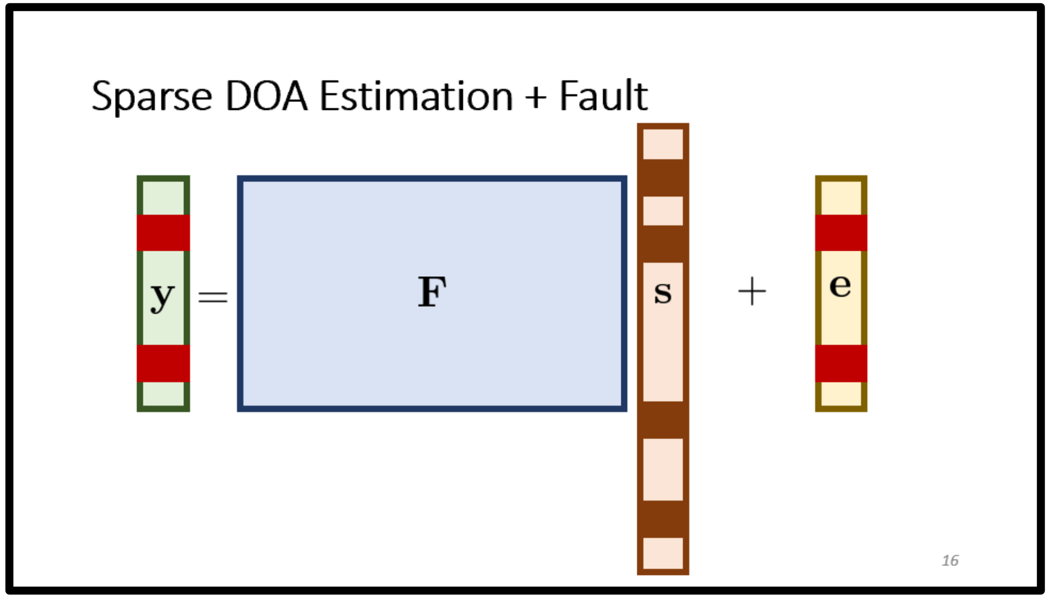
- *Gaussian prior for noise/residuals*
- *Spike and Slab prior for signal (sparse components)*
- *Inverse Power-Law prior for weights*

Algorithm Idea

Fix $\boldsymbol{\gamma}$:
Sparse Representation Recovery

$$\widehat{\mathbf{S}} = \arg\min_{\mathbf{S}} \left\{ \|\mathrm{diag}(\boldsymbol{\gamma}) \cdot (\mathbf{Y} - \mathbf{F}\mathbf{S})\|_F^2 + \alpha \|\mathbf{S}\|_0 \right\}$$





Algorithm Idea

Fix $\boldsymbol{\gamma}$:
Sparse Representation Recovery

$$\widehat{\mathbf{S}} = \arg\min_{\mathbf{S}} \left\{ \|\mathrm{diag}(\boldsymbol{\gamma}) \cdot (\mathbf{Y} - \mathbf{FS})\|_F^2 + \alpha \|\mathbf{S}\|_0 \right\}$$

Fix $\mathbf{S}$:

$$\widehat{\boldsymbol{\gamma}} = \arg\min_{\boldsymbol{\gamma}} \left\{ \|\mathrm{diag}(\boldsymbol{\gamma}) \cdot (\mathbf{Y} - \mathbf{FS})\|_F^2 + \lambda \| \log(\boldsymbol{\gamma})\|_1 \right\}$$

i.e.,

$$\widehat{\sigma}_{\mathrm{ref}} = 10 \cdot \mathrm{MAD}(\mathbf{R})$$

$$\gamma_n^* = \sqrt{\frac{\lambda/T}{\frac{2}{T}\sum_{t=1}^{T}\left(y_n(t) - \mathbf{F}_n \mathbf{s}(t)\right)^2}} = \frac{\sqrt{\frac{\lambda}{2T}}}{\widehat{\sigma}_n} = \frac{\widehat{\sigma}_{\mathrm{ref}}}{\widehat{\sigma}_n}.$$

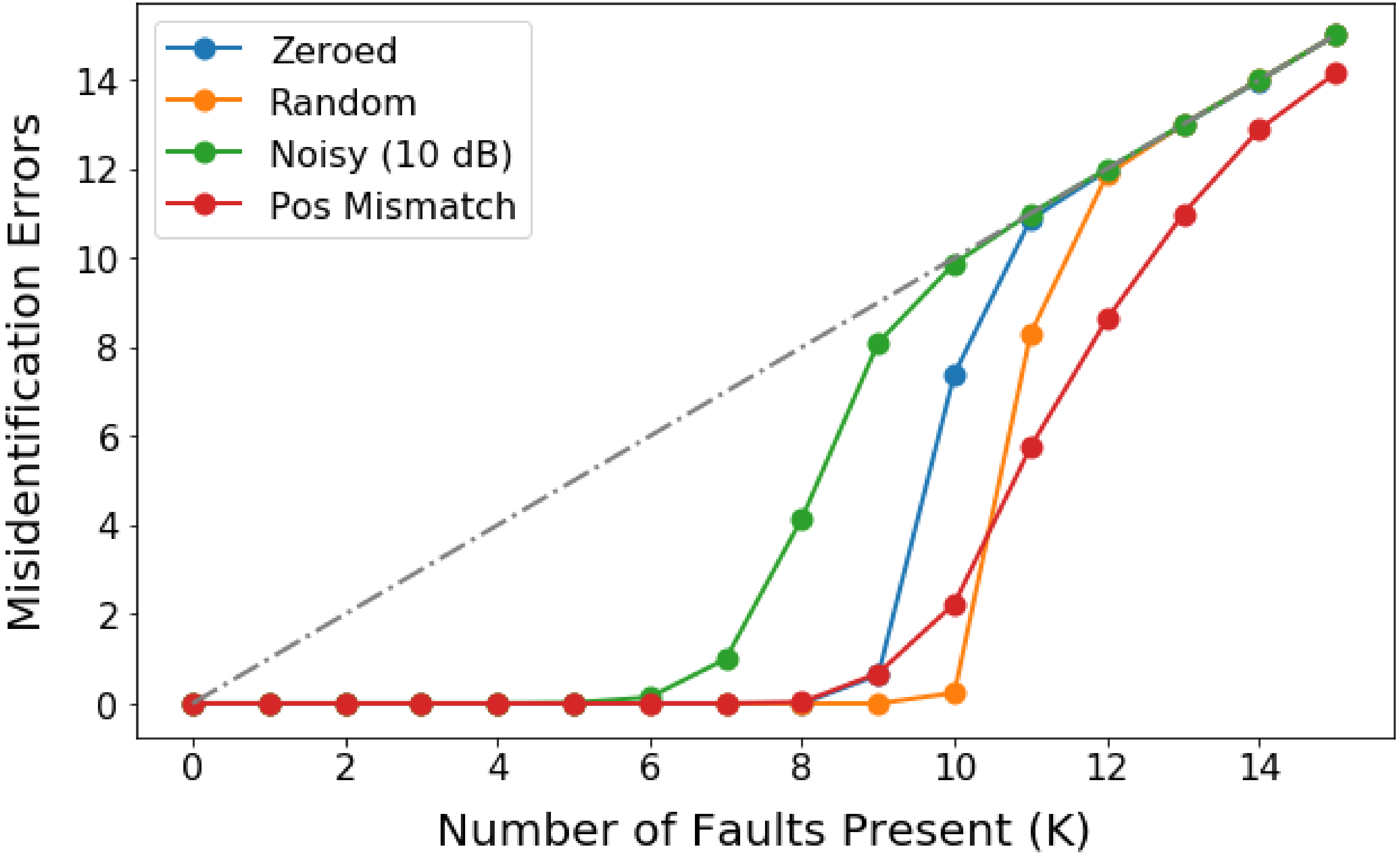
Simulation

- N=32 elements, T=100 snapshots, M sources vary (in DOA and in number)
  - Number of sources between 1 and N/4
  - Well-separated sources
- 4 fault models: Random, Zeroed, High Noise, Position Offset

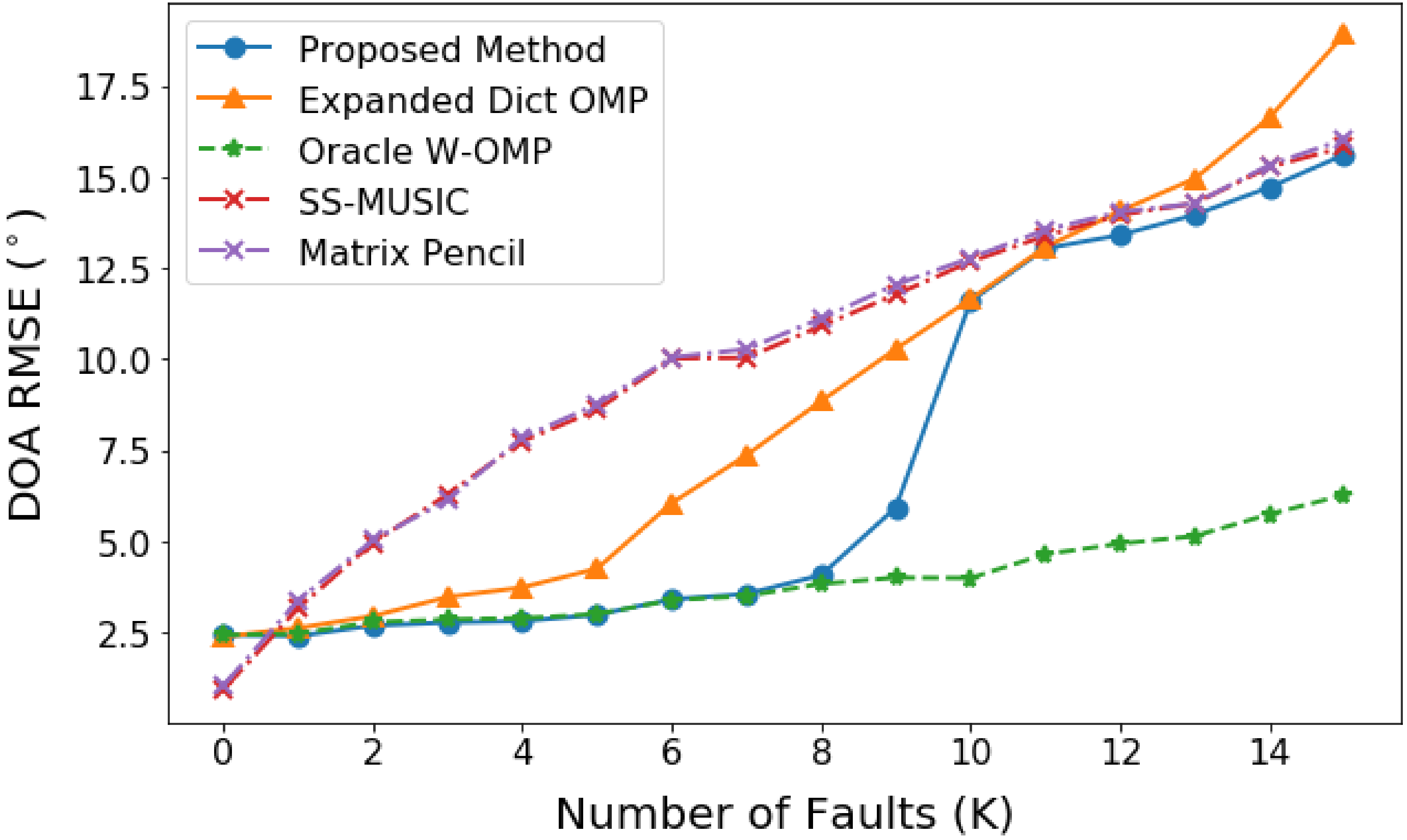
Metric:

- ***Fault Identification:*** Classify all elements with $\gamma<1$ as faulty, determine missed faults/false alarms
- ***DOA Estimation:*** Root-mean-square-error of DOAs (final weighted OMP)

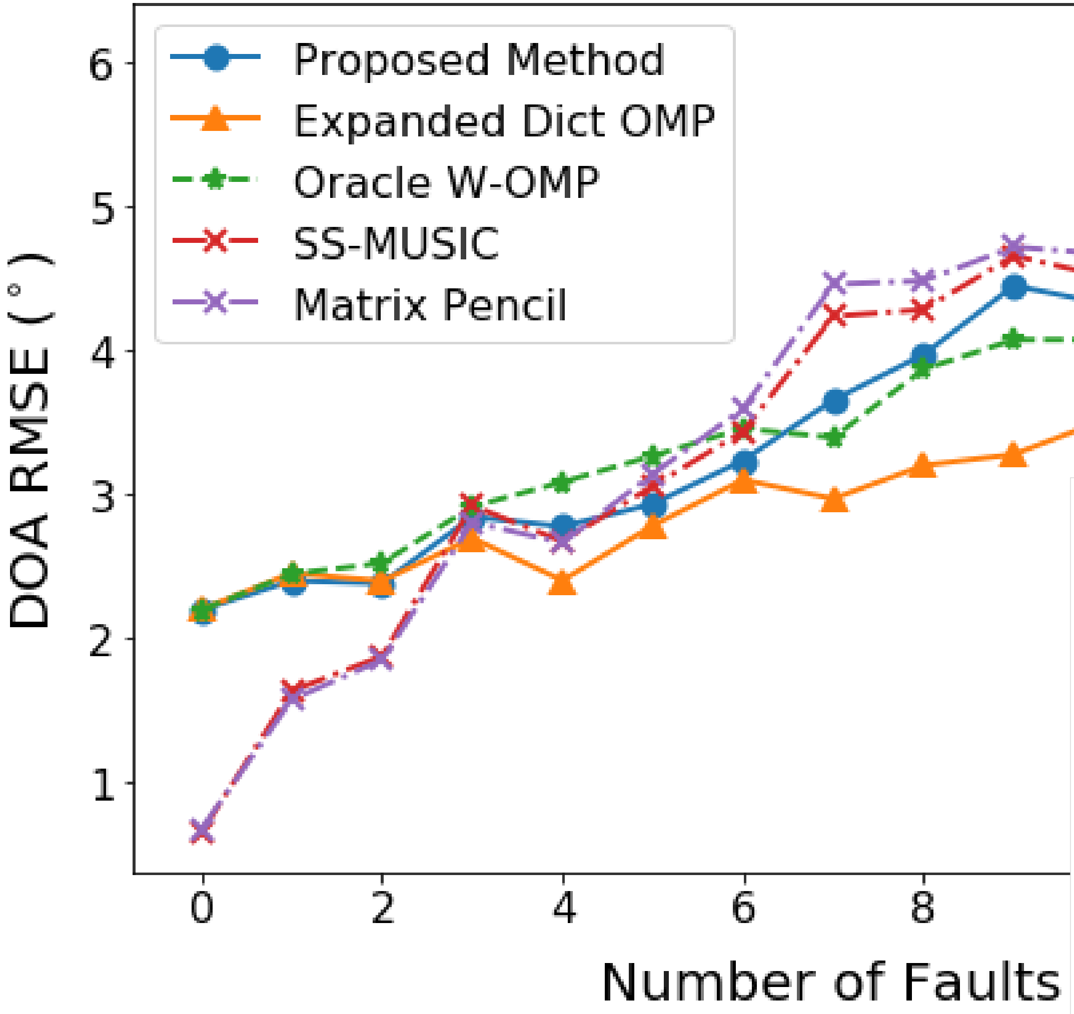
Fault Identification / Misidentification



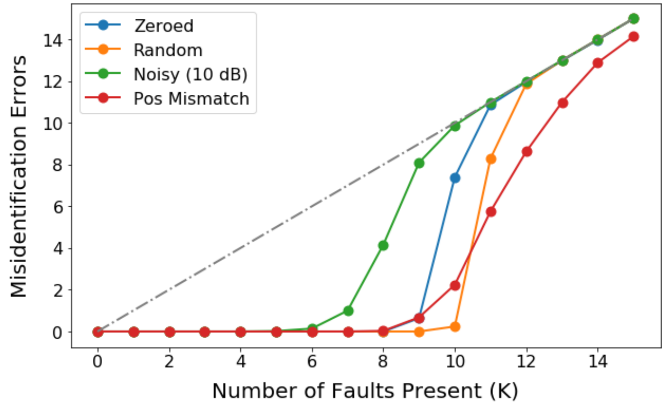
Random faulty measurements



Noisy (10 dB) faulty measurements



Fault Identification / Misidentification



Discussion

- DOA estimation under few faults is comparable to when fault locations are known
- Able to detect faults up to ~N/4
  - Upon removing faulty elements, other DOA estimation methods can be used
- Poorer performance in DOA estimation in the fault-free case
  - Sparse recovery methods may be less noise-resistant
- Performance may vary based on failure mode

Open Questions

- σ_{ref}: Choice of parameters, setting "reference" value
- Understanding the interplay between number of sources, number of faults and mode of failure (i.e., uniqueness in sparse recovery)
- Sparse DOA recovery: Gridless search (e.g. TV minimization)
- Comparing against other ways of modeling faults and outliers

Conclusion

We proposed:

1. Novel **snapshot diversity** formulation: *exploits fixed fault locations in situations where single snapshots are not available*

2. Algorithm for **robust DOA estimation** and **fault identification**: *alternating between sparse recovery and weight estimation*