A Worst-Case Performance Optimization Based Design Approach to Robust Symbol-Level Precoding for Downlink MU-MIMO

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Alireza Haqiqatnejad[1]
alireza.haqiqatnejad@uni.lu

Shahram Shahbazpanahi[1,2]
Shahram.shahbazpanahi@uoit.ca

Björn Ottersten[1]
bjorn.ottersten@uni.lu

[1] Interdisciplinary Centre for Security, Reliability and Trust (SnT), University of Luxembourg
[2] Department of Electrical, Computer, and Software Engineering, University of Ontario Institute of Technology

Outline

- Symbol-level multiuser precoding
  - Precoding under design uncertainty
  - Distance-preserving constructive interference regions
  - SINR-constrained power minimization problem
- Worst-case performance design approach
- Proposed solution algorithm
- Simulation results

Symbol-Level Multiuser Precoding

- The idea is to convert multiuser interference (MUI) into a source of desired signal component, i.e. constructive interference (CI), based on a symbol-by-symbol transmit processing.

- Symbol-level processing of the transmit signal can potentially lead to improvements in spectral/energy efficiency, at the price of increased transmitter complexity.

- In this non-linear design approach, the precoded transmit signal is optimized as a function of the instantaneous channel as well as the instantaneous users' data symbols.

- A key consideration in designing the symbol-level precoder is to properly define the CI regions based on the received signal constellation, typically with the aim of preserving (or enhancing) the detection accuracy.

Precoding under Design Uncertainty

- The design process is highly sensitive to inaccuracies in several parameters, such as the available channel state information at the transmitter (CSIT), and any succeeding operation on the transmit signal which is not perfectly known to the precoder.

- The problem of robust design has been widely studied in the literature for scenarios where our knowledge about the environment is subject to uncertainty.

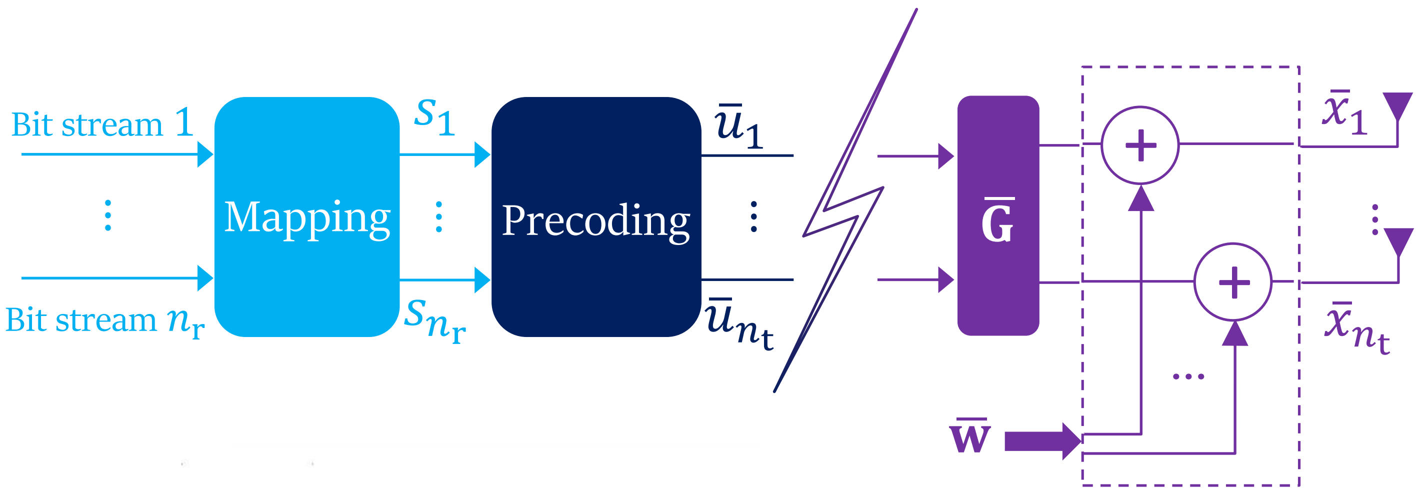
- We assume that our design process is subject to uncertainty, e.g., due to finite precision of the underlying design and implementation technology.
  Example: Low-resolution digital-to-analog converter (DAC)

- Under a linearly distorted signal model with bounded additive distortion, we aim to design a symbol-level precoding scheme such that the performance gain offered by the CI-based design is preserved.

Downlink MU-MIMO Model (I)

- An array of n_t transmit antennas, n_r users.
- Independent data symbols $\{s_i\}_{i=1}^{n_\mathrm{r}}$, taken from identical equiprobable constellation sets, to be transmitted to multiple users.
- The ith user, with a minimum SINR requirement of γ_i, detects its desired symbol s_i based on the optimal single-user maximum-likelihood (ML) decision rule.
- The precoded signal $\bar{\mathbf{u}}$ is subject to linear distortion before transmission, i.e., the actual transmitted signal is given by

$$\bar{\mathbf{x}} = \underbrace{\bar{\mathbf{G}}}_{\text{Known distortion matrix}}\,\bar{\mathbf{u}} + \underbrace{\bar{\mathbf{w}}}_{\text{Additive white noise}}, \quad \text{such that} \quad \|\bar{\mathbf{w}}\| \leq \boldsymbol{\varepsilon}$$



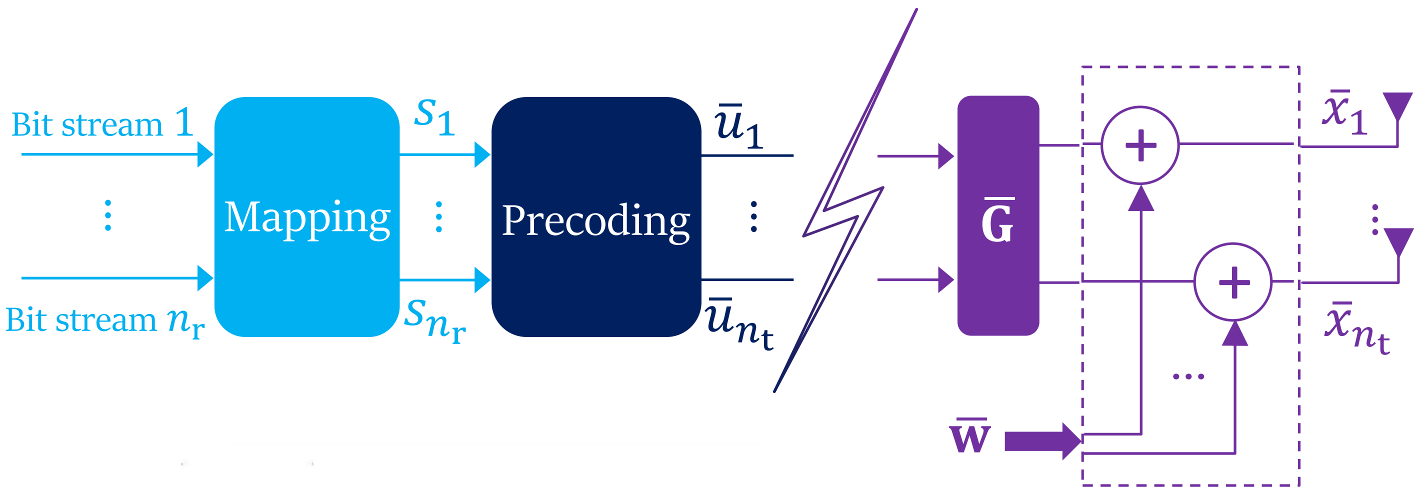
Downlink MU-MIMO Model (II)

- The baseband representation of the signal received by the ith user is

$$r_i = \underbrace{\mathbf{h}_i^T}_{\text{Instantaneous fading coefficients of the } i\text{th channel}} \bar{x} + z_i = \mathbf{h}_i^T (\bar{\mathbf{G}}\, \bar{\mathbf{u}} + \bar{\mathbf{w}}) + z_i, \qquad z_i \sim \mathcal{CN}\left(0, \sigma_i^2\right)$$

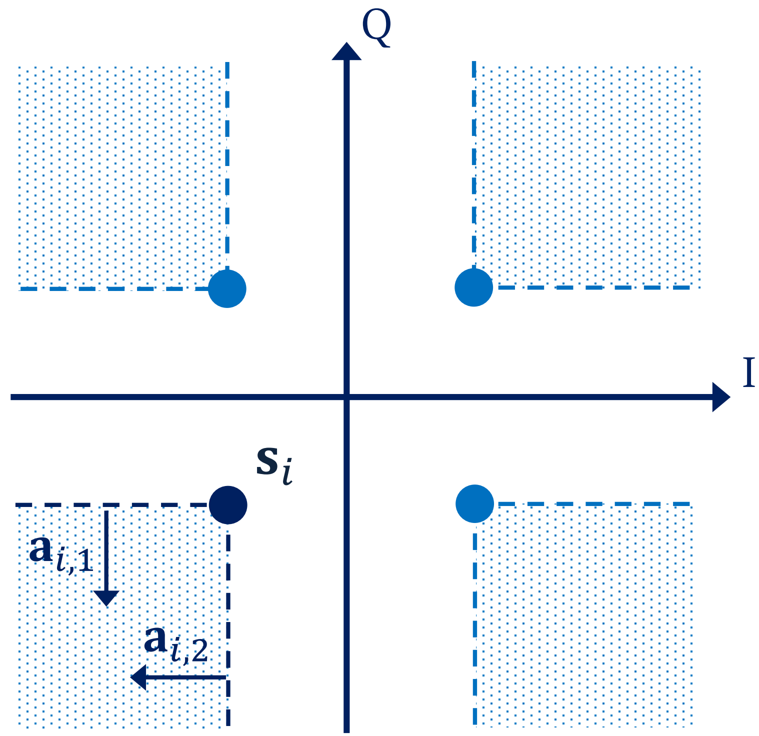
- Equivalent real-valued notations:

$$\mathbf{u} \triangleq \begin{bmatrix} \mathrm{Re}\{\bar{\mathbf{u}}\} \\ \mathrm{Im}\{\bar{\mathbf{u}}\} \end{bmatrix}_{2n_\mathrm{t}\times 1}, \boldsymbol{s}_i \triangleq \begin{bmatrix} \mathrm{Re}\{s_i\} \\ \mathrm{Im}\{s_i\} \end{bmatrix}_{2\times 1}, \mathbf{H}_i \triangleq \begin{bmatrix} \mathrm{Re}\{\mathbf{h}_i\} & -\mathrm{Im}\{\mathbf{h}_i\} \\ \mathrm{Im}\{\mathbf{h}_i\} & \mathrm{Re}\{\mathbf{h}_i\} \end{bmatrix}_{2\times 2n_\mathrm{t}}$$



Distance Preserving CI Regions

Definition: Any two points belonging to two distinct CI regions are distanced by at least the distance between the corresponding constellation points.



$$\mathcal{D}_i \triangleq \{\mathbf{s} | \mathbf{A}_i(\mathbf{s} - \mathbf{s}_i) \succcurlyeq \mathbf{0}\},\ \mathbf{A}_i = \begin{pmatrix} \mathbf{a}_{i,1}^T \\ \mathbf{a}_{i,2}^T \end{pmatrix}$$

- The CI constraint for the ith user:

$$\mathbf{A}_i\left(\mathbf{H}_i\mathbf{u} - \sigma_i\sqrt{\gamma_i}\mathbf{s}_i\right) = \mathbf{t}_i,\ \text{where} \begin{cases} \mathbf{t}_i \succcurlyeq \mathbf{0}, & \text{if } \mathbf{s}_i \text{ is an outer symbol} \\ \mathbf{t}_i = \mathbf{0}, & \text{if } \mathbf{s}_i \text{ is an interior symbol} \end{cases}$$

Symbol-Level Precoding Design Formulation

- The SINR-constrained instantaneous (i.e., per-symbol) power minimization problem with CI constraints:

$$\boxed{\underset{\mathbf{x},\mathbf{t}\succcurlyeq\mathbf{0}}{\text{minimize}} \quad \mathbf{x}^{\mathrm{T}}\mathbf{x} \qquad \text{subject to} \quad \mathbf{H}_i\mathbf{x} \in \mathcal{D}_i(\mathbf{s}_{\mathrm{i}}, \gamma_i, \sigma_i)}$$

- The above problem can be formulated, in a compact form, as a linearly-constrained quadratic program (QP):

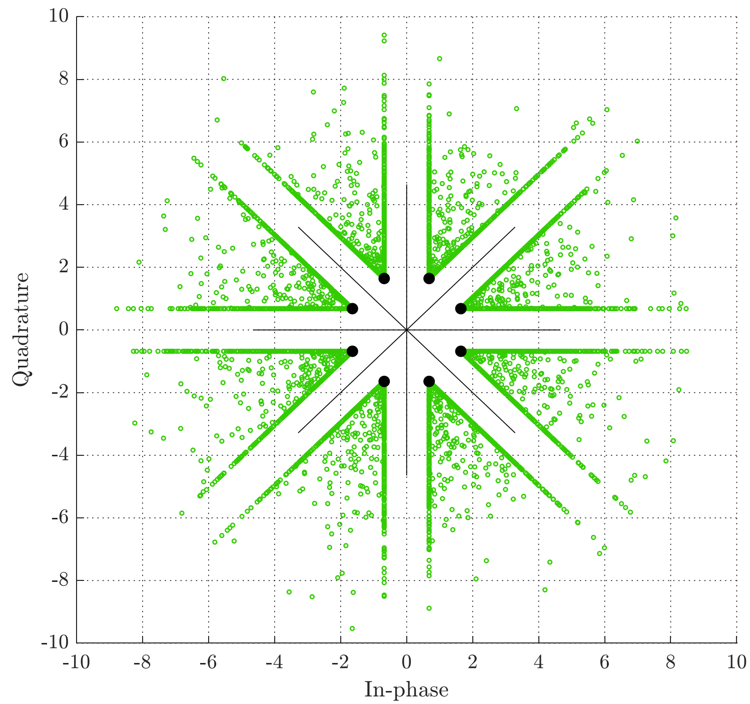
$$\boxed{\underset{\mathbf{x},\mathbf{t}\succcurlyeq\mathbf{0}}{\text{minimize}} \quad \mathbf{x}^{\mathrm{T}}\mathbf{x} \qquad \text{subject to} \quad \mathbf{H}\mathbf{x} = \mathbf{D}\mathbf{s} + \mathbf{A}^{-1}\mathbf{W}\mathbf{t}}$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{A}_{n_{\mathrm{r}}} \end{pmatrix}, \mathbf{H} = \begin{pmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_{n_{\mathrm{r}}} \end{pmatrix}, \mathbf{s} = \begin{pmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_{n_{\mathrm{r}}} \end{pmatrix}, \mathbf{t} = \begin{pmatrix} \mathbf{t}_1 \\ \vdots \\ \mathbf{t}_{n_{\mathrm{r}}} \end{pmatrix}, \mathbf{D} = \operatorname{diag}\left(\sigma_1\sqrt{\gamma_1}, \dots, \sigma_{n_{\mathrm{r}}}\sqrt{\gamma_{n_{\mathrm{r}}}}\right) \otimes \mathbf{I}_2$$

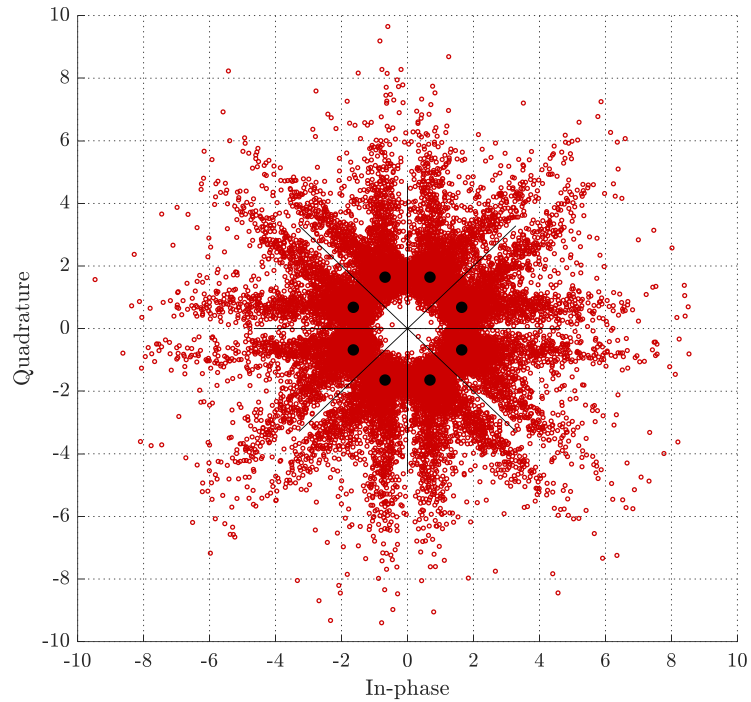
$$\mathbf{W} = \operatorname{diag}(w_1, \dots, w_{n_{\mathrm{r}}}) \otimes \mathbf{I}_2, \qquad \text{where} \quad w_i = \begin{cases} 1, & \text{if } \mathbf{s}_i \text{ is an outer symbol} \\ 0, & \text{if } \mathbf{s}_i \text{ is an interior symbol} \end{cases}$$

Distortion from Receiver Viewpoint

- Scatter plot of the noise-free received signals with 8PSK signaling:



Perfect SLP



Distorted SLP

Worst-Case Performance Design Approach

- We cast a new design formulation based on the penalty method by introducing the linear equality CI constraints as an ℓ2-norm penalty into the objective function, i.e.,

$$\boxed{\min_{\mathbf{x},\mathbf{t}\succcurlyeq\mathbf{0}} \quad \|\mathbf{x}\|^2 + \beta\|\mathbf{Hx} - \mathbf{Ds} - \mathbf{A}^{-1}\mathbf{t}\|^2}$$

- Unlike the original problem, this new formulation does not strictly impose the CI constraints, but penalizes any deviation from the intended CI regions.

- Replacing $\mathbf{x}$ with $\mathbf{Gu} + \mathbf{w}$, the worst-case design formulation can be written as

$$\boxed{\min_{\mathbf{x},\mathbf{t}\succcurlyeq\mathbf{0}} \ \max_{\|\mathbf{w}\|\leq\varepsilon} \quad \|\mathbf{Gu} + \mathbf{w}\|^2 + \beta\|\mathbf{H}(\mathbf{Gu} + \mathbf{w}) - \mathbf{\Phi}(\mathbf{t})\|^2}$$

where $\mathbf{\Phi}(\mathbf{t}) \triangleq \mathbf{Ds} + \mathbf{A}^{-1}\mathbf{t}$

Challenge: The above optimization problem is nonconvex, and thus, may not be amenable to a computationally efficient solution.

Solution: We propose a three-step iterative block coordinate ascent-descent algorithm.

Solution Approach (I)

First step (updating $\mathbf{w}$): Given $\mathbf{u}$ and $\mathbf{t} \succcurlyeq \mathbf{0}$, the main difficulty comes from the inner maximization over $\mathbf{w}$.
Clue: The norm constraint on $\mathbf{w}$ is active at the optimum.

- Applying the method of Lagrange multipliers:

$$\mathbf{w}^* = -(\mathbf{P} - \mu^* \mathbf{I})^{-1} \mathbf{H}^T \big(\mathbf{GHu} - \mathbf{\Phi}(\mathbf{t})\big)$$

- Imposing the norm constraint:

$$f(\mu) = \big(\mathbf{PGu} - \mathbf{H}^T \mathbf{\Phi}(\mathbf{t})\big)^T (\mathbf{H}^T\mathbf{H} - \mu \mathbf{I})^{-2} \big(\mathbf{PGu} - \mathbf{H}^T \mathbf{\Phi}(\mathbf{t})\big) = \varepsilon^2$$

where $\mathbf{P} \triangleq \mathbf{H}^T\mathbf{H} + \frac{1}{\beta}\mathbf{I}$

- No closed-form solution is known in general for $f(\mu) = 0$.

Solution Approach (II)

- Function $f(\mu)$ has a finite number of roots bounded as

Lemma 1: Let z denote the number of roots of $f(\mu)$, then z is always an even number bounded as

$$2 \leq z \leq rank(\mathbf{H})$$

among which the unique maximizer of interest lies in the interval given by

Theorem: The value of μ^* is equal the largest positive root of $f(\mu)$ and is bounded as

$$\bar{\lambda}_{\max} \leq z \leq \frac{1}{\varepsilon}\|\mathbf{PGu} - \mathbf{H}^T\mathbf{\Phi}(\mathbf{t})\| + \bar{\lambda}_{\max}$$

with $\bar{\lambda}_{\max} \triangleq \|\mathbf{H}\|^2 + \frac{1}{\beta}$

Solution Approach (III)

- One can search for μ^* in the given bounded interval via numerical methods, e.g., a simple bisection search.
- For relatively small values of ε (compared to $\bar{\lambda}_{\text{max}}$), one can also use quite an accurate approximation for μ^* with a closed-form expression given below.

Lemma 2: For small ε, the value of μ^* can be well approximated by

$$\mu^* \approx 2\sqrt[3]{\frac{\left\|\mathbf{P}\left(\mathbf{PGu}-\mathbf{H}^T\boldsymbol{\Phi}(\mathbf{t})\right)\right\|^2}{\varepsilon^2}}$$

Solution Approach (IV)

Second step (updating $\mathbf{t}$): For given $\mathbf{w}$ and $\mathbf{u}$, the value of $\mathbf{t}$ can be updated as the solution to the following optimization problem:

$$\min_{\mathbf{t} \geqslant \mathbf{0}} \quad \|\mathbf{H}(\mathbf{G}\mathbf{u} + \mathbf{w}) - \boldsymbol{\Phi}(\mathbf{t})\|^2$$

which is a standard non-negative least squares (NNLS) problem and can efficiently be solved using, e.g., accelerated projected gradient descent (APGD) algorithm.

Third step (updating $\mathbf{u}$): Given $\mathbf{w}$ and $\mathbf{t} \geqslant \mathbf{0}$, the minimization over u is an unconstrained QP and hence is amenable to the following closed-form solution:

$$\mathbf{u} = \mathbf{G}^{-1}\mathbf{P}^{-1}\mathbf{H}^T\boldsymbol{\Phi}(\mathbf{t}) - \mathbf{G}^{-1}\mathbf{w}$$

- The optimal worst-case robust transmit signal is then given by

$$\mathbf{x}^* = \mathbf{G}\mathbf{u}^* + \mathbf{w}^* = \underbrace{\left(\mathbf{H}^T\mathbf{H} + \frac{1}{\beta}\mathbf{I}\right)^{-1} \mathbf{H}^T(\mathbf{D}\mathbf{s} + \mathbf{A}^{-1}\mathbf{t}^*)}$$

applying a (regularized) channel inversion to the constructively-interfered symbols

Pseudocode of the Proposed Algorithm

- The block coordinate ascent-descent algorithm iterates between finding the worst-case additive distortion vector, the slack vector-variable **t**, and the optimal precoded signal.

Algorithm 1: Block coordinate ascent-descent algorithm

input: $\mathbf{A}, \mathbf{H}, \mathbf{D}, \mathbf{s}, \varepsilon$

output: $\mathbf{u}$

initialize: $\mathbf{t}^{(0)} = \mathbf{z}^{(0)} \in \mathbb{R}_{+}^{2n_{\mathrm{r}} \times 1}, \mathbf{u}^{(0)} \in \mathbb{R}^{2n_{\mathrm{t}} \times 1}, k = 0$

set: $\varphi = \frac{1-\sqrt{\kappa}}{1+\sqrt{\kappa}}, \kappa = \frac{\sigma_{\max}}{\sigma_{\min}}, \mathbf{B} = \mathbf{I} - \sigma_{\min}^2 \times (\mathbf{A}\mathbf{A}^T)^{-1}$, *where $\sigma_{\max}$ and $\sigma_{\min}$ respectively denote the maximum and the minimum singular value of matrix* $\mathbf{A}$.

do

$k = k + 1$

compute $\mu^{(k)}$ *by solving* $f(\mu) = 0$

$\mathbf{w}^{(k)} = -\left(\mathbf{P} - \mu^{(k)}\mathbf{I}\right)^{-1}\left(\mathbf{P}\mathbf{G}\mathbf{u}^{(k-1)} - \mathbf{D}\mathbf{s} - \mathbf{A}^{-1}\mathbf{t}^{(k-1)}\right)$

$\mathbf{t}^{(k)} = \max\left\{\mathbf{B}\mathbf{z}^{(k-1)} + \sigma_{\min}^2 \mathbf{A}^{-T}\left(\mathbf{H}\left(\mathbf{G}\mathbf{u}^{(k-1)} + \mathbf{w}^{(k)}\right) - \mathbf{D}\mathbf{s}\right), 0\right\}$

$\mathbf{z}^{(k)} = \mathbf{t}^{(k)} + \varphi\left(\mathbf{t}^{(k)} - \mathbf{t}^{(k-1)}\right)$

$\mathbf{u}^{(k)} = \mathbf{G}^{-1}\mathbf{P}^{-1}\mathbf{H}^T\left(\mathbf{D}\mathbf{s} + \mathbf{A}^{-1}\mathbf{t}^{(k)}\right) - \mathbf{G}^{-1}\mathbf{w}^{(k)}$

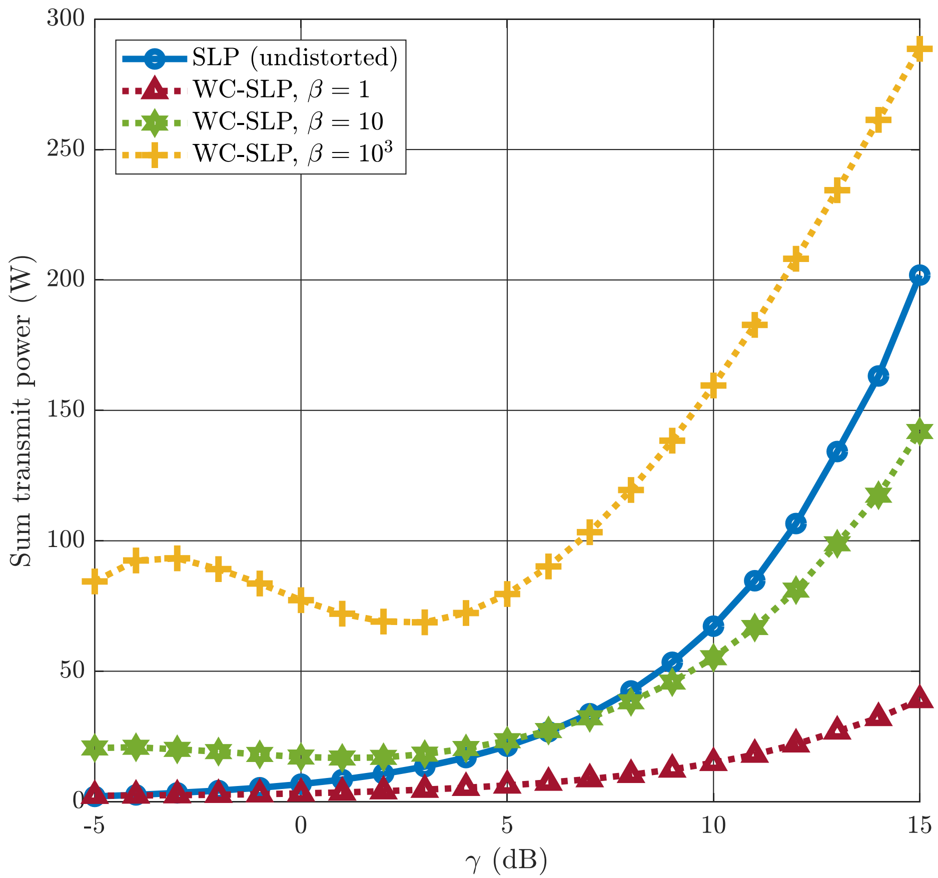
until *the terminating condition is met*;

SIMULATION SETUP

- Downlink MU-MIMO with $n_{\mathrm{t}} = n_{\mathrm{r}} = 8$
- QPSK symbols
- Rayleigh block fading channel, $h_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}), \quad i = 1, \dots, n_{\mathrm{r}}$
- Unit noise variances $\sigma_i^2 = 1, \quad i = 1, \dots, n_{\mathrm{r}}$
- Equal target SINRs $\gamma_i \triangleq \gamma, \quad i = 1, \dots, n_{\mathrm{r}}$
- Randomly generated (Gaussian) $\mathbf{w}$ with variance 0.1
- $\varepsilon = 0.56$, corresponding to a confidence level of 0.99
- $\mathbf{G} = \mathbf{I}$
- The results are averaged over 500 fading blocks each of 500 symbols.

Simulation Results

- Total transmission power versus target SINR



Simulation Results

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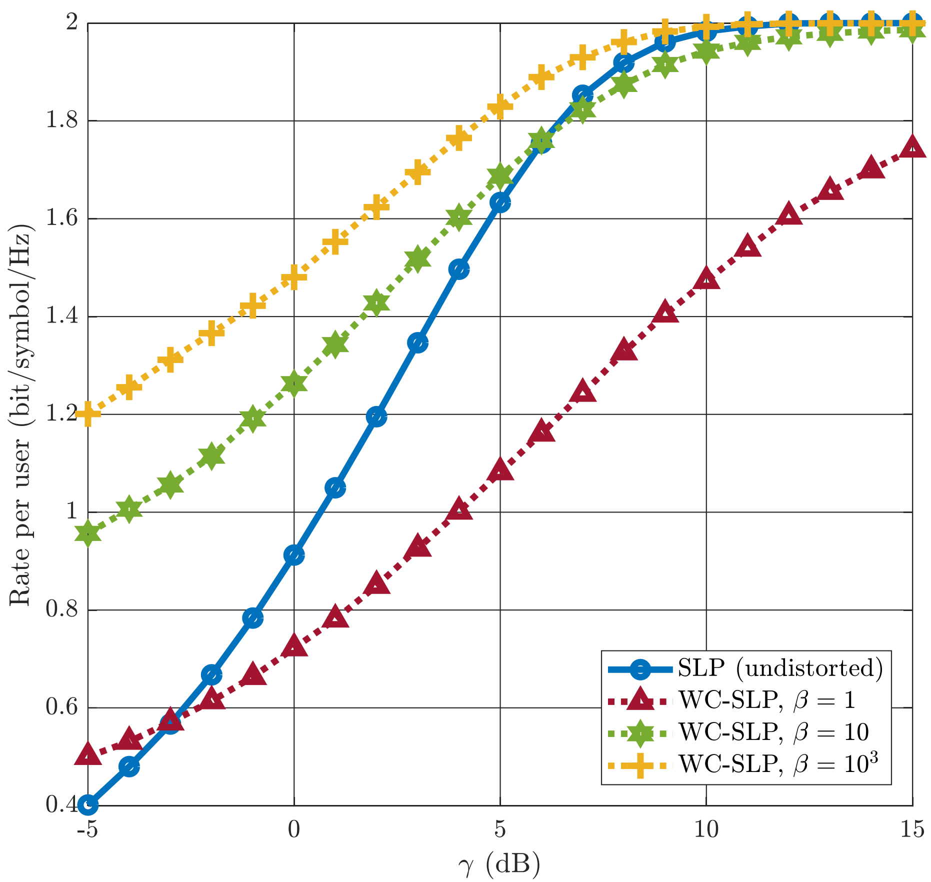
- Per-user transmission rate versus target SINR: $I(s_i; r_i) = \mathbb{E}_{s_i, r_i, \mathbf{H}} \left\{ \log_2 \frac{P_{r_i|s_i,\mathbf{H}}(r_i|s_i, \mathbf{H})}{P_{r_i|\mathbf{H}}(r_i|\mathbf{H})} \right\}$



Simulation Results

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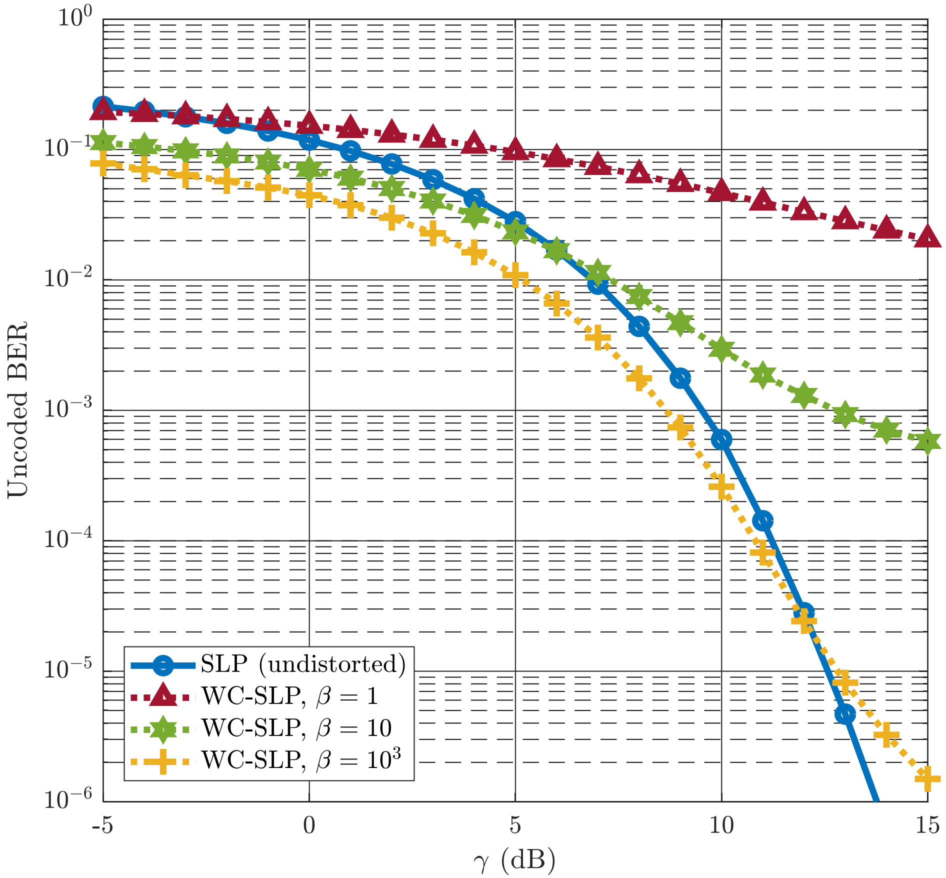
- Uncoded bit error rate versus target SINR



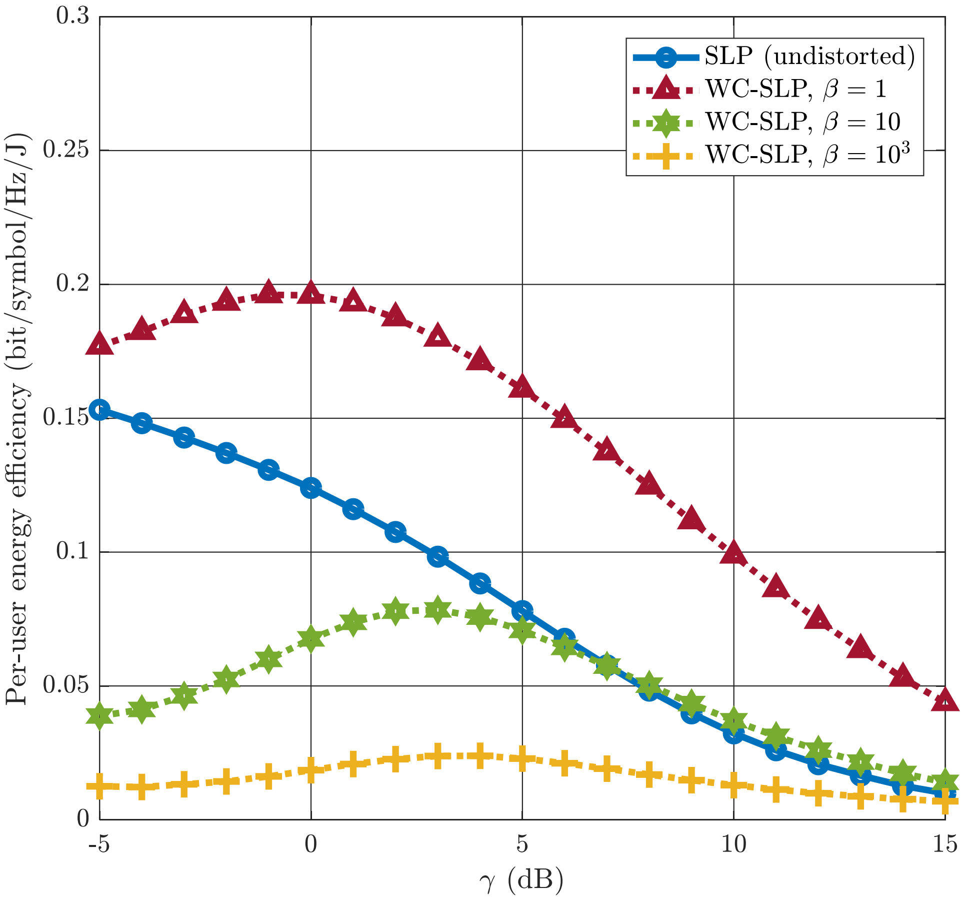
Simulation Results

- We define Energy efficiency as

$$\frac{\left(\frac{1}{n_\mathrm{r}}\sum_{i=1}^{n_\mathrm{r}} \mathrm{R}_i\right)\left(1-\frac{1}{n_\mathrm{r}}\sum_{i=1}^{n_\mathrm{r}} \mathrm{BER}_i\right)}{\|\mathbf{u}\|^2}$$

  as an overall performance measure that incorporates sum transmit power, bit error rate and per-user transmission rate.

- Having $\beta = 1$ results in a higher energy efficiency, even compared to the ideal undistorted SLP; this is a consequence of relaxing the CI constraints in the SLP problem, leading to a lower transmit power in exchange for a higher bit error rate.



Summary and Conclusions

- We proposed a worst-case design formulation with relaxed CI constraints for the QoS-constrained SLP problem minimizing the total transmit power in a scenario where the precoder's output undergoes linear distortion with bounded additive noise.
- We tackled this problem using an iterative coordinate ascent-descent algorithm to obtain the worst-case robust precoded signal.
- Finding the precoded signal involves solving a non-negative least squares problem, while obtaining the worst-case distortion vector led us to a semi-closed form solution with only one scalar parameter which has to be calculated numerically/approximately.
- Our simulation results showed that the proposed worst-case approach can outperform the undistorted SLP method, for small penalty parameters, in terms of energy efficiency.
- The penalty parameter can be adjusted in a more sophisticated way, e.g., it can be updated at each iteration based on a specific update rule.

SUMMARY AND CONCLUSIONS

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Thank You!

Alireza Haqiqatnejad

alireza.haqiqatnejad@uni.lu

Shahram Shahbazpanahi

Shahram.shahbazpanahi@uoit.ca

Björn Ottersten

bjorn.ottersten@uni.lu