

POWER SPECTRUM OPTIMIZATION FOR CAPACITY OF THE EXTENDED SPECTRUM HYBRID FIBER COAX NETWORK

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ABSTRACT

Capacity requirements of the fixed access network keep increasing towards multi-gigabit connections. For Hybrid Fiber Coaxial (HFC) networks, aggregated rates around 30 Gbit/s can be achieved by increasing the DOCSIS spectrum to 3GHz, assuming a spectral efficiency around 10 bit/s/Hz. Replacement of spectrum limiting components such as passive taps in the HFC network is an efficient way to achieve these data rates, compared with the cost of fiber to the home (FTTH). Transmit amplifier distortion is a major issue in the extended spectrum in addition to the high spread of attenuation between low and high frequencies. Existing spectrum allocation strategies are no longer applicable. This work presents a new method of spectrum optimization for the coax channel up to 3GHz, taking transmitter distortion into account in the optimization.

Index Terms— Hybrid Fiber Coaxial Capacity Optimization

1. INTRODUCTION

Achievable data rates in the fixed access network keep increasing. While passive optical networks (PON) move to 25 or 50 Gbit/s with IEEE 802.3ca [1] and MGfast [2] targets 10 Gbit/s aggregated Point to Point rate over twisted pair, it is time to evaluate HFC technology as a successor for 10 Gbit/s DOCSIS 4.0 [3]. Following the discussion in [4], aggregated data rates around 30 Gbit/s and a bandwidth of 3 GHz are a good choice for the next generation DOCSIS technology, which is herein called Extended Spectrum DOCSIS (ESD). Hereby, it is important to understand the rate limiting factors of the ESD network and thus, the feasibility of a competitive coaxial technology. In this paper, the capacity of the ESD network is evaluated, based on transmitter distortion as the main rate limiting factor. Optimal power allocation for ESD is an open question to be investigated for this case. To evaluate transmitter distortion limited capacity for ESD, the corresponding power allocation optimization problem is solved and the mathematical dependency to water-filling power allocation is shown. Simulation results, based on amplifier circuit models and measurements of active and passive ESD network components are presented.

2. EXTENDED SPECTRUM DOCSIS NETWORK

The ESD network consists of fiber nodes, connected over coax trunk cables, taps and coax drop wires to the subscribers cable modems (CM) in a point-to-multipoint network, as shown in Fig. 1(a).

For backward compatible operation of ESD with DOCSIS 4.0, the upstream and downstream bands can be allocated as shown in Fig. 1(b), with legacy upstream at low frequencies, full duplex bands and a legacy downstream as well as the extend spectrum downstream for frequencies above 804 MHz. This will allow 25 Gbit/s downstream and 5 Gbit/s upstream rates. For a further increase of up-

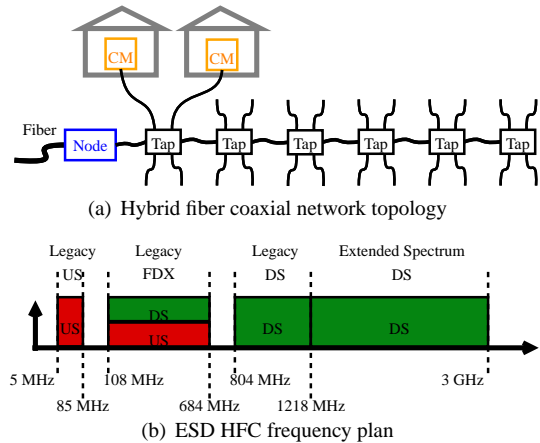


Fig. 1. Extended Spectrum DOCSIS topology and frequency plan

stream bandwidth, the full duplex band can be extended in future at the cost of losing backward compatibility to DOCSIS 4.0.

2.1. Power Allocation

One major challenge for ESD will be the transmit power allocation. For legacy DOCSIS, the spectrum allocation is done with the goal of a flat spectrum at the receiver. This is not practical for ESD, as the spread of attenuation over the used frequency band can be 50 dB and more. For 1.8 GHz bandwidth, a flat receive spectrum below and above 1 GHz can be achieved with a step at 1 GHz [3].

Another popular approach to allocate power is water-filling [5], which is optimal for a rx noise limited channel with sum power constraints. It gives an almost flat transmit spectrum.

In this work, the power allocation problem is formulated as a capacity optimization problem. The new approach presented to solve the capacity optimization for a transmit distortion limited channel is an extension of the method described in [6], chapter 3.1.6. In addition to [6], distortion is considered in the optimization objective such that the optimal sum transmit power is an outcome of the optimization. The mathematical dependency between water-filling and the transmitter distortion limited capacity is shown.

2.2. Power Amplifiers

HFC networks are powered by a dedicated power grid separate from mains powered local electric utility. This helps operators to monitor power usage as well as detect faults in the network in a more effective manner. Making changes to the existing Coax power grid is very costly and time consuming for the operators. Assuming that

the power delivery infrastructure will remain the same for ESD, the power budget of one node is around 160 W, where nearly half of that is required for the transmit amplifier itself and the remaining power is consumed by the digital circuit.

Current technology trends in power amplifiers point to similar levels of total composite power for extended spectrum amplifiers compared to currently used 1.2 GHz amplifiers. The transmitter distortion model of Sec. 3.2 is derived under the assumption that the ESD transmit amplifier power budget is the same as for the legacy node to be able to re-use the power delivery infrastructure, which is a strong requirement from cable operators. Optimal allocation of the given transmit power is a useful tool to achieve this objective.

3. CAPACITY EVALUATION

The capacity evaluation for the extended spectrum HFC network requires knowledge of the channel characteristics and capacity limiting factors. The capacity limiting factor in the transmitter is amplifier distortion. At the receiver, additive white Gaussian noise and receiver distortion due to analog-to-digital conversion limits capacity.

HFC transmission schemes such as DOCSIS 4.0 [3] use OFDM modulation, where the channel is partitioned into K narrowband subcarriers $k = 1, \dots, K$ with a subcarrier spacing Δf . Those orthogonal channels are coupled only by nonlinear distortion or a sum power constraint. The transmit power per carrier $x^{(k)}$ as well as the information rate per carrier $b^{(k)}$ can be adjusted per carrier.

The data rates for a given signal-to-noise ratio $SNR^{(k)}$ on carrier k is given by

$$R = \eta \Delta f \sum_{k=1}^K \min \left(\log_2 \left(1 + \frac{SNR^{(k)}}{\Gamma} \right), b_{\max} \right). \quad (1)$$

where limitations of modulation and coding are considered in terms of an SNR gap to capacity Γ [7] as well as with a limit b_{\max} to the number of bits transmitted per carrier and channel use. The OFDM system requires overhead for the cyclic extension to guarantee orthogonal channels, which is considered in an efficiency factor η . Using $\eta = 1$, $\Gamma = 1$ and $b_{\max} \rightarrow \infty$ gives the capacity without coding and modulation limitations.

3.1. Capacity with Power Constraints

Capacity C and achievable rate R are evaluated with respect to power constraints where the simplest case is a sum power constraint [5]. For practical systems, additional per-carrier constraints are considered, as shown in [6], Sec. 3.1.6. For this case, achievable rate R (and capacity C for the case of $\Gamma = 1$) is the solution to

$$\begin{aligned} R_l &= \max_{x^{(k)}} \sum_k \log_2 \left(1 + \frac{|H_l^{(k)}|^2 x^{(k)}}{\Gamma \sigma_l^{(k),2}} \right) \\ \text{s.t.} \quad &\sum_k x^{(k)} \leq p_{\max} \\ \text{s.t.} \quad &0 \leq x^{(k)} \leq p_{\text{mask}}^{(k)} \end{aligned} \quad (2)$$

where $H_l^{(k)}$ is the channel coefficient on carrier k (attenuation, phase) between node and CM l and $\sigma_l^{(k),2}$ is the additive white Gaussian noise variance on carrier k and CM l .

The power constraints are formulated as a sum power limit p_{\max} and a spectral mask constraint $p_{\text{mask}}^{(k)}$. Limitations of the modulation

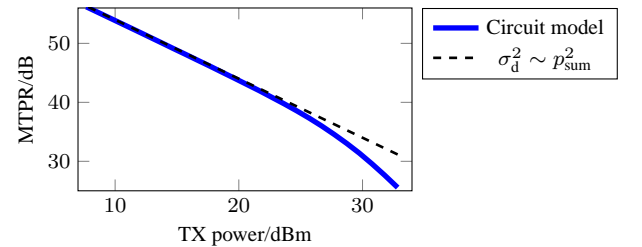
alphabet size to b_{\max} are incorporated into the spectral mask constraint, using $p_{\text{mask}}^{(k)} = \Gamma(2^{b_{\max}} - 1)\sigma_l^{(k),2}/|H_l^{(k)}|^2$. The solution to Eq. (2) is obtained by a modified water-filling algorithm as described in [6], chapter 3.1.6. Other algorithms to solve Eq. (2) have been published in [8, 9].

As there are no power constraints for ESD defined, the distortion limited capacity will be evaluated, which will show that Eq. (2) and the distortion limited capacity have the same solution.

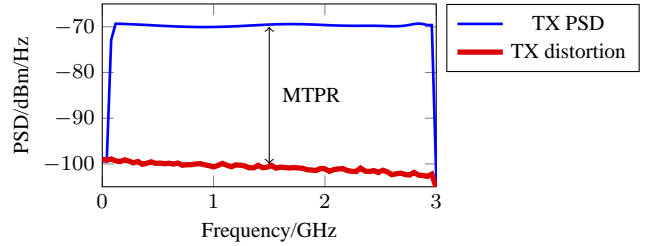
3.2. Transmitter Distortion Model

The sum power limit $\sum_k x^{(k)} \leq p_{\max}$ in Eq. (2) can be seen as a simplified model for the behavior of real transmit amplifier, where the SNR and thus the data rate is limited by distortion increasing with increasing transmit power. Transmit amplifier distortion can be seen as a transmit power dependent noise source with variance σ_d^2 . It is characterized in measurement and simulation by a missing tone power ratio (MTPR). MTPR is the ratio between signal power and distortion power, as shown in Fig. 2(b). It is determined as the signal level on one OFDM carrier which is transmitted with zero power while the others are transmitted at the desired level ($MTPR^{(k)} = x^{(k)}/\sigma_d^2$). In the following discussion, the cable modem index l is skipped without loss of generality.

Fig. 2(a) shows the increase of nonlinear distortion σ_d^2 in a 3 GHz amplifier circuit model. The MTPR decreases with increasing transmit power $p_{\text{sum}}(x^{(k)}) = \sum_{k=1}^K x^{(k)}$. In frequency domain, as shown in Fig. 2(b), distortion is approximately flat.



(a) MTPR vs. TX power for flat transmit spectrum



(b) Transmitter noise over frequency at $p_{\text{sum}} = 25$ dBm

Fig. 2. Distortion evaluation from amplifier circuit model

The dependency between distortion variance σ_d^2 and signal power $p_{\text{sum}}(x^{(k)})$ can be described by $\sigma_d^2 = \delta \left(p_{\text{sum}}(x^{(k)}) \right)^\alpha$. For the amplifier shown in Fig. 2(a), the constants are $\delta = -64$ dB and $\alpha = 2$. Following the argumentation of [10] a lower bound for the capacity of the nonlinear copper channel is derived.¹

¹Lower bounded by assuming a Gaussian distribution of the distortion.

3.3. Transmitter Distortion Limited Capacity

Introducing distortion in the SNR per carrier gives the term

$$SNR^{(k)} = \frac{|H^{(k)}|^2 x^{(k)}}{\sigma^2 + \delta^{(k)} (p_{\text{sum}}(x^{(k)}))^\alpha} \quad (3)$$

where the distortion variance is $\sigma_d^{(k),2} = \delta^{(k)} (p_{\text{sum}}(x^{(k)}))^\alpha$ with $\delta^{(k)} = \delta \frac{|H^{(k)}|^2}{K}$, assuming white distortion.² This gives the rate R (or capacity C with $\Gamma = 1$) according to

$$R = \max_{x^{(k)}} \sum_k \log_2 \left(1 + \frac{|H^{(k)}|^2 x^{(k)}}{\Gamma (\sigma^2 + \delta^{(k)} (p_{\text{sum}}(x^{(k)}))^\alpha)} \right) \quad (4)$$

s.t. $0 \leq x^{(k)} \leq p_{\text{mask}}^{(k)}$

The derivative $\frac{\partial R}{\partial x^{(k)}}$ is given by

$$\frac{\partial R}{\partial x^{(k)}} = \frac{|H^{(k)}|^2}{(\sigma^2 + \delta_k (p_{\text{sum}}(x^{(k)}))^\alpha) (\Gamma + SNR_k)} - \sum_{d=1}^K \frac{|H^{(d)}|^2 x^{(d)} \alpha \delta^{(d)} (p_{\text{sum}}(x^{(k)}))^{\alpha-1}}{(\sigma^2 + \delta^{(d)} (p_{\text{sum}}(x^{(k)}))^\alpha)^2 (\Gamma + SNR^{(d)})} \quad (5)$$

and $\partial R / \partial x^{(k)} = 0$ must hold for the optimal power allocation for all carriers with $0 < x^{(k)} < p_{\text{mask}}^{(k)}$.

The optimal power allocation can be found, e.g., by a projected gradient method with a step size ρ as given by

$$x_{t+1}^{(k)} = \min \left(\max \left(x_t^{(k)} + \rho \frac{\partial R}{\partial x^{(k)}}, 0 \right), p_{\text{mask}}^{(k)} \right). \quad (6)$$

3.4. Optimality and Dependency to Water-Filling

As Eq. (4) is a non-convex problem in general, Eq. (6) gives a local optimum rather than the global optimum solution. For the practical application, this is no concern as the algorithm is initialized at the best known p_{sum} value and will improve from there.

From a theory perspective, it is of further relevance to mention that the method is globally optimal under certain conditions. This is the case when the objective in (4) is convex or pseudo-convex (which is satisfied, e.g., for the amplifier model from Sec. 3.2) and when the $p_{\text{mask}}^{(k)}$ constraints are not active or the optimal set of carriers \mathbb{I}_{mask} where the mask constraint is active, is known, e.g., by a full search.

To show the power constrained capacity in Eq. (2) and the distortion-limited capacity in Eq. (4) to have the same solution under these conditions, the optimality conditions are compared. Eq. (2) has a water-filling solution, which is

$$\frac{|H^{(k)}|^2}{\sigma^{(k),2} (\Gamma + SNR^{(k)})} - \frac{1}{\mu} = 0 \forall k : 0 < x^{(k)} < p_{\text{mask}}^{(k)}. \quad (7)$$

with the water-filling level $1/\mu$ for nonzero carriers not limited by the spectral mask.

The optimality condition for Eq. (4), $\partial R / \partial x^{(k)} = 0 \forall k : 0 < x^{(k)} < p_{\text{mask}}^{(k)}$, has the same form when replacing σ^2 by $\sigma_{\text{nd}}^{(k),2} = (\sigma^2 + \delta^{(k)} (p_{\text{sum}}(x^{(k)}))^\alpha)$, and $\frac{1}{\mu}$ to be

$$\frac{1}{\mu} = \sum_{d=1}^K \frac{|H^{(d)}|^2 x^{(d)} \alpha \delta^{(d)} (p_{\text{sum}}(x^{(k)}))^{\alpha-1}}{(\sigma^2 + \delta^{(d)} (p_{\text{sum}}(x^{(k)}))^\alpha)^2 (\Gamma + SNR^{(d)})} \quad (8)$$

²For colored distortion, $1/K$ is replaced by the frequency dependency of interest, while the following capacity evaluation still holds true.

With the carrier index sets \mathbb{I}_{mask} for carriers with spectral mask constraint active and \mathbb{I}_{fill} for nonzero carriers, the dependency between μ and p_{sum} is given by

$$\frac{1}{\mu} = \frac{1}{|\mathbb{I}_{\text{fill}}|} \left(p_{\text{sum}}(x^{(k)}) + \sum_{k \in \mathbb{I}_{\text{fill}}} \frac{\Gamma \sigma_{\text{nd}}^{(k),2}}{|H^{(k)}|^2} - \sum_{k \in \mathbb{I}_{\text{mask}}} p_{\text{mask}}^{(k)} \right) \quad (9)$$

where $|\mathbb{I}_{\text{fill}}|$ to be the cardinality, i.e., the number of elements of the set. This implies the assumption of $p_{\text{sum}} = p_{\text{max}}$ for Eq. (2), which is satisfied when \mathbb{I}_{fill} is not empty.

Accordingly, the transmit power per carrier is given by

$$x^{(k)} = \begin{cases} \frac{1}{\mu} - \frac{\Gamma \sigma_{\text{nd}}^{(k),2}}{|H^{(d)}|^2} & \text{for } k \in \mathbb{I}_{\text{fill}} \\ 0 & \text{for } k \in \mathbb{I}_0 \\ p_{\text{mask}}^{(k)} & \text{otherwise.} \end{cases} \quad (10)$$

where \mathbb{I}_0 is the set of carriers where the positiveness constraint is active (zero power carriers).

Algorithm 1 Sum-power optimization algorithm

Initialize p_{sum}

repeat

 Identify $\mathbb{I}_0, \mathbb{I}_{\text{fill}}, \mathbb{I}_{\text{mask}}$ (water-filling, [6])

 Update $x^{(k)}$ using Eq. (10)

 Calculate μ from Eq. (8) with updated $x^{(k)}$

 Update p_{sum} from μ , using Eq. (9)

until Convergence of p_{sum}

The sum-power limit p_{sum} for Eq. (2) is derived from Eq. (8) and thus, depends on the transmitter linearity limitations. The (locally or globally) optimal value can be found, using fixed-point updates according to Alg. 1.

3.5. Transmitter and Receiver Distortion Limited Capacity

Receiver distortion, e.g., due to quantization noise of the analog-to-digital converter is another potentially capacity limiting factor. The dependency between quantization noise variance and receive power is linear, adding the noise term $\delta_{\text{rx}} p_{\text{sum,rx}}(x^{(k)})$ with $p_{\text{sum,rx}}(x^{(k)}) = \sum_{k=1}^K |H^{(k)}|^2 x^{(k)}$. In practice, receivers are distortion optimized by analog pre-equalizers to modify $H^{(k)}$ and noise shaping, where δ_{rx} will be frequency-dependent.

The rate/capacity term for transmitter and receiver distortion is

$$R = \max_{x^{(k)}} \sum_k \log_2 \left(1 + \frac{|H^{(k)}|^2 x^{(k)}}{\Gamma (\sigma^{(k),2} + \delta^{(k)} p_{\text{sum}}^\alpha + \delta_{\text{rx}} p_{\text{sum,rx}})} \right) \quad (11)$$

s.t. $x^{(k)} \geq 0$

and the optimality condition for carriers $k : x^{(k)} > 0$ is

$$\frac{\partial R}{\partial x^{(k)}} = 0 = \frac{|H^{(k)}|^2}{(\sigma^2 + \delta^{(k)} p_{\text{sum}}^\alpha + \delta_{\text{rx}} p_{\text{sum,rx}}) (\Gamma + SNR^{(k)})} - \sum_{d=1}^K \frac{|H^{(d)}|^2 x^{(d)} \left(\alpha \delta^{(d)} p_{\text{sum}}^{\alpha-1} + \delta_{\text{rx}} |H^{(k)}|^2 \right)}{(\sigma^2 + \delta^{(d)} p_{\text{sum}}^\alpha + \delta_{\text{rx}} p_{\text{sum,rx}})^2 (\Gamma + SNR^{(d)})}. \quad (12)$$

This is no longer a water-filling solution, because the sum term in Eq. (12) is no longer independent of k .

For practical implementation, it is recommended to design the digital-to-analog converter (DAC) such that the quantization noise is below the distortion caused by the transmit amplifier and to design the analog-to-digital converter (ADC) such that it is capable to support the maximum constellation size b_{\max} . With these design rules, the optimal power allocation is determined by Alg. 1.

3.6. Implementation Aspects

To implement the optimization scheme of Alg. 1, the distortion parameters, δ and α must be known from an amplifier characterization. During operation, the noise conditions must be known from an SNR measurement. The algorithm performs multiple water-filling steps. As the optimization can be done by software in background during operation of the link, there is no issue with computation time.

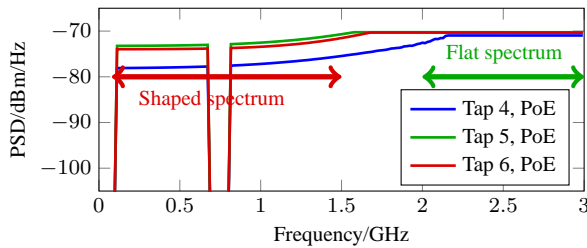


Fig. 3. Optimal transmit PSD shapes in downstream

Still, there are cases where a static power allocation is preferred. In networks operating up to 1.2 GHz bandwidth, this was done with to goal of a flat receive power spectral density (PSD), which is not practical for 3 GHz bandwidth. A flat transmit PSD is closer to the optimal choice, but with the disadvantage that transmit power at low frequencies may be too high to be beneficial for the receiver. Fig. 3 shows optimal PSD shapes from Alg. 1. There is a shaped PSD region where the optimal PSD follows the spectral mask constraint, which is followed by a flat (water-filling) region. The point of transition between the shaped and the flat region depends on the attenuation of the line. For loops where the SNR drops below the SNR required for the smallest (4 bit) constellation, no power will be allocated in this low SNR region.

4. TRANSMISSION TECHNOLOGY

OFDM channels with 192 MHz bandwidth per OFDM, channel, 4096 carriers and 50 kHz tone spacing give a good trade-off between cyclic extension overhead and complexity. A generic overhead of 10% is assumed for cyclic prefix, pilot signals and overhead channels. Digital mixers are used to bring the OFDM channels to the desired spectrum. Residual echo after echo cancellation is assumed to be negligible. The DOCSIS 3.1 FEC (Rate 8/9 LDPC with outer BCH code) and modulation is assumed.

5. SIMULATION RESULTS

Simulation results focus on Node+0 as in Fig. 1(a) with 6 taps, 50m trunk cable of type QR540 and 30m drop cable of type RG-6. Two in-home wiring scenarios are compared, where the cable modem is either connected at the point of entry (PoE) or with a 30 m additional

RG-6 cable (Deep home run) [11]. Following the topology of Fig. 1(a), the tap values are 29 dB for taps 1 and 2, 23 dB for tap 3, 20 dB for tap 4, 16 dB for tap 5 and 7 dB for the last tap. The frequency plan of Fig. 1(b) is used. Accordingly, the aggregated rate is distributed approximately 25 Gbit/s downstream and 5 Gbit/s upstream.

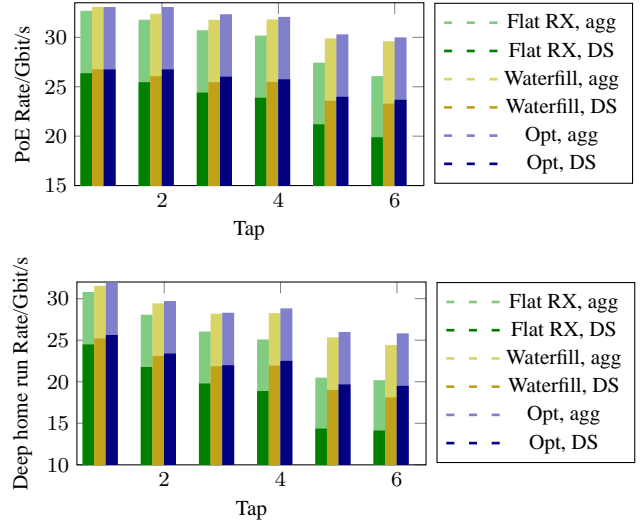


Fig. 4. ESD data rates for two in-home wiring scenarios in the passive HFC network, comparing different power allocation strategies

Fig. 4 shows the data rates achieved in ESD with flat receive spectrum, water-filling and the proposed optimization. Simulations show the aggregate rate achieved when all transmit time and spectrum is allocated a CM at the corresponding tap 1-6 where tap 1 is closest to the node. Overhead due to OFDM modulation (cyclic prefix), FEC overhead, guard bands, overhead channels and pilot tones are considered. Data rates around 25 Gbit/s downstream are achievable, which is the rate shared by the individual subscribers.

The results in Fig. 4 show that a power allocation strategy targeting a flat RX spectrum in the extended band does not perform well, while water-filling achieves reasonable data rates. The proposed spectrum optimization method described in Sec. 3 gives 1-2.5% rate improvement over water-filling for the PoE case. With in home wiring (Fig. 4, bottom), downstream rates drop to 18 Gbit/s with flat spectrum. The proposed optimization method brings it back to 20 Gbit/s for the last tap, achieving 8% gain over water-filling.

6. CONCLUSION

Extended spectrum DOCSIS with 3 GHz bandwidth allows data rates of 20 – 25 Gbit/s downstream and 5 – 6 Gbit/s upstream in the considered scenarios. The presented approach to capacity evaluation and spectrum optimization for a distortion limited channel can be used to optimize the power allocation dynamically with respect to channel and noise conditions or derive static PSDs. Spectrum optimization is always beneficial and can give up to 8% rate increase in some cases. On distortion limited loops, power saving may be achieved without loss of data rate. Improvements in the modulation and forward error correction coding may give a further performance increase. The evaluation shows 3 GHz extended spectrum DOCSIS to be able to fulfill future access network requirements.

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