Bitvectors with runs and the successor/predecessor problem

Data
Compression
Conference

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Outline

- ☐ **Introduction**
- ☐ Background
- ☐ zombit-vector
- ☐ Experimental evaluation
- ☐ Conclusions
- ☐ Future work

Introduction

- **Search engines:** looking for “*data compression*”

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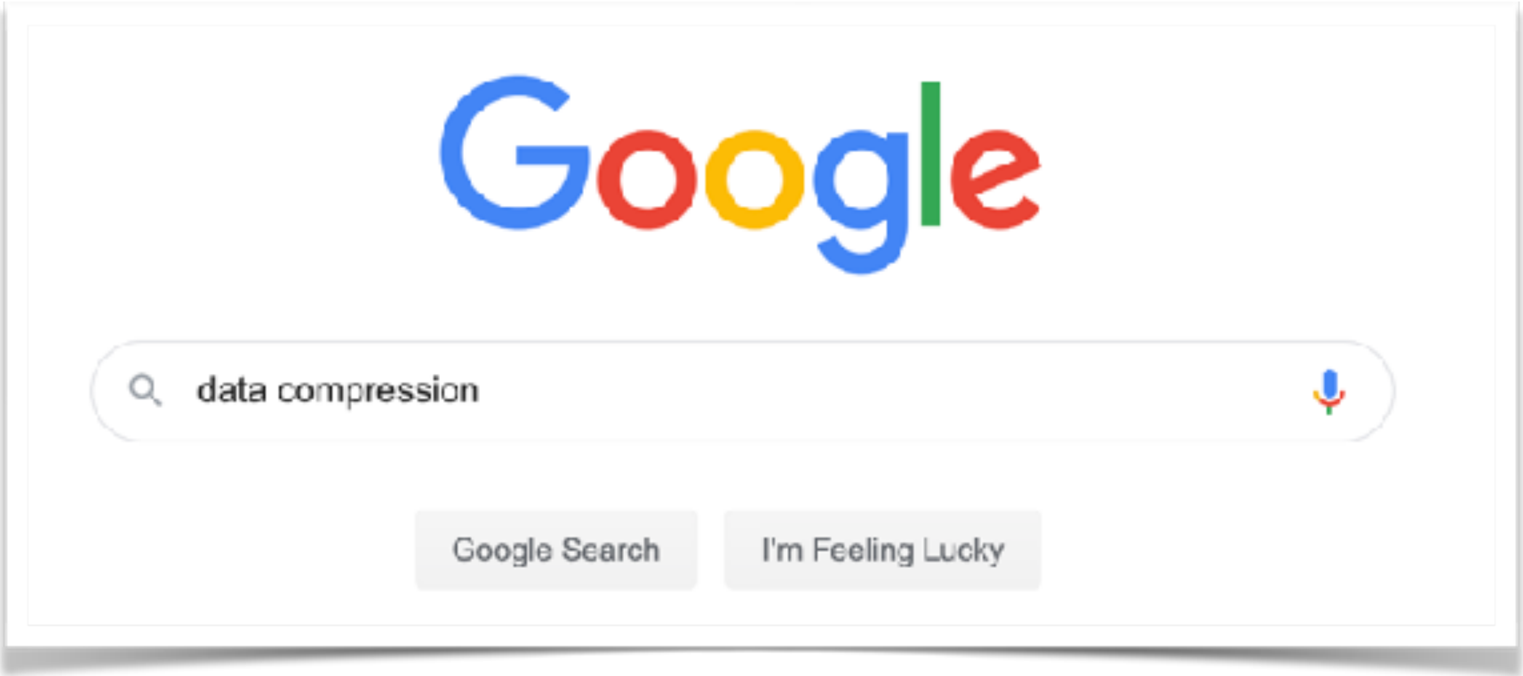

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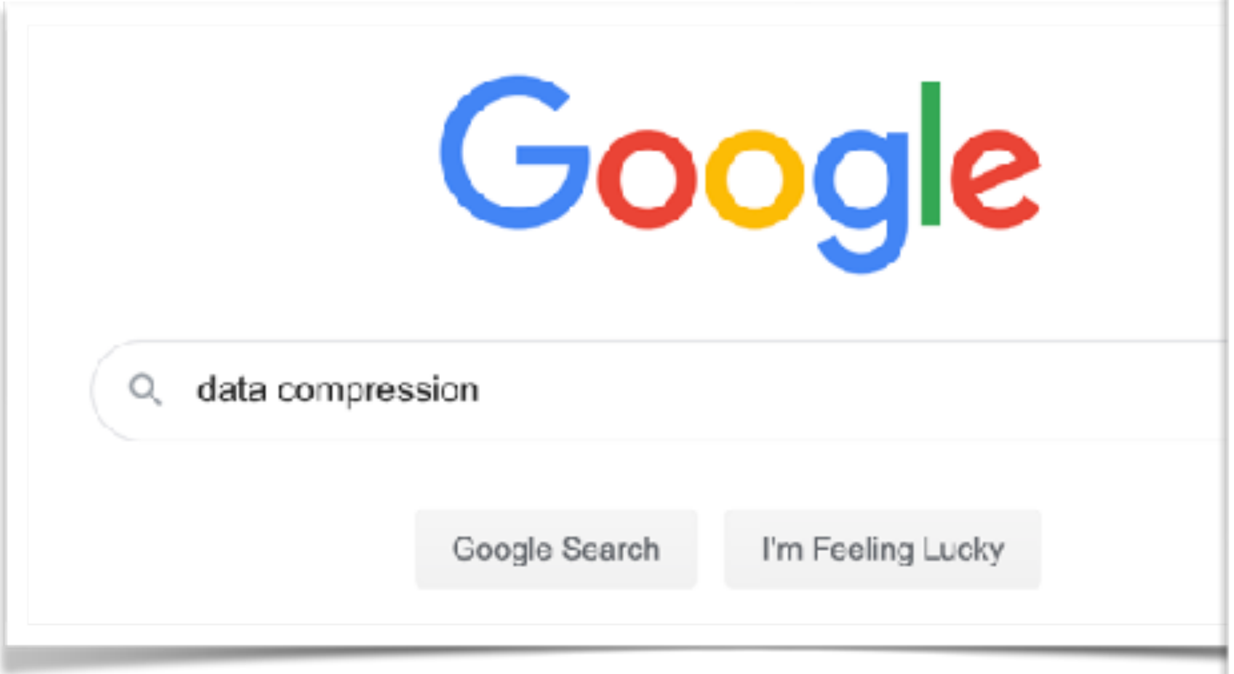

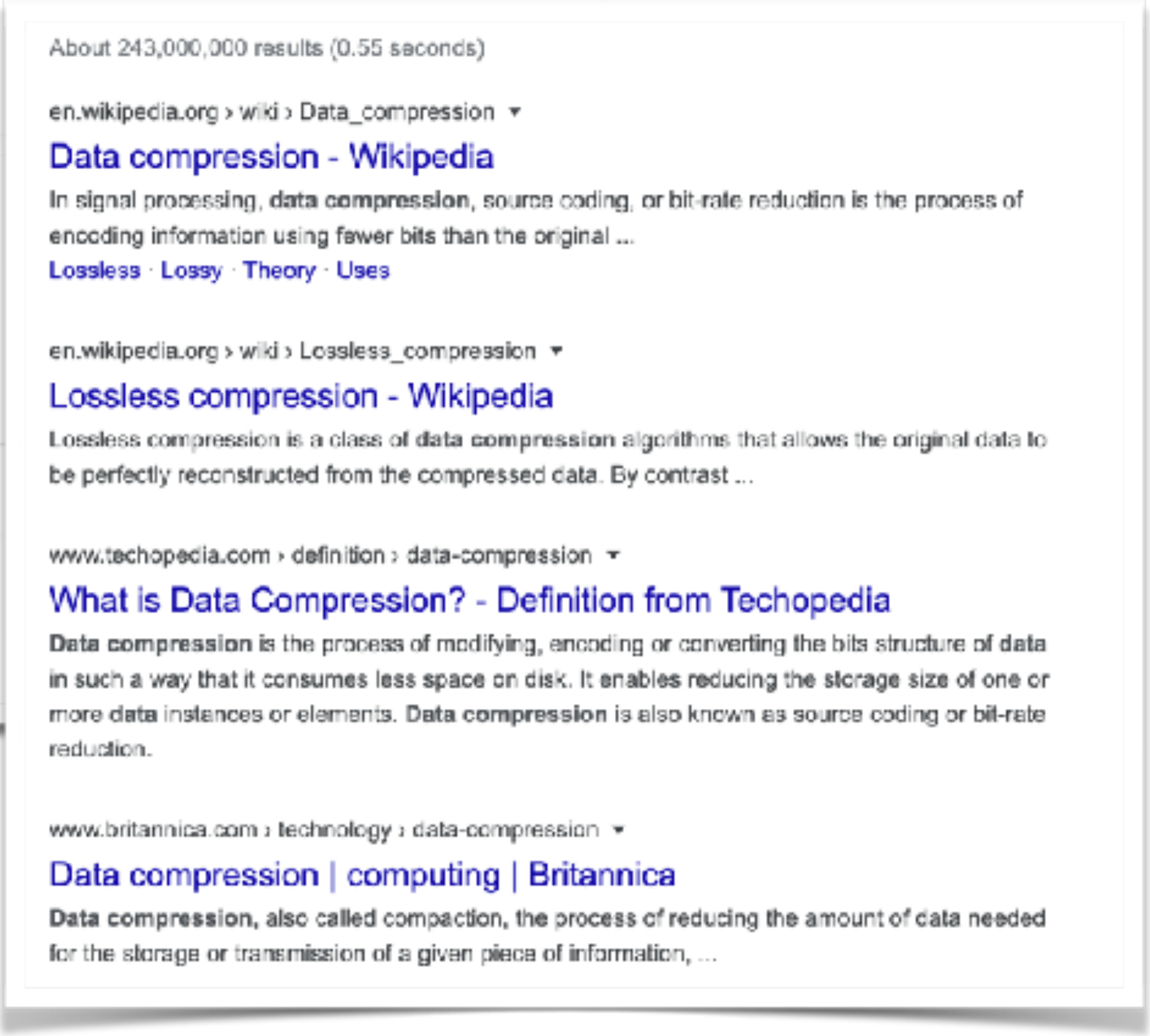

About 243,000,000 results (0.55 seconds)

en.wikipedia.org › wiki › Data_compression

Data compression - Wikipedia

In signal processing, **data compression**, source coding, or bit-rate reduction is the process of encoding information using fewer bits than the original ...

Lossless · Lossy · Theory · Uses

en.wikipedia.org › wiki › Lossless_compression

Lossless compression - Wikipedia

Lossless compression is a class of **data compression** algorithms that allows the original data to be perfectly reconstructed from the compressed data. By contrast ...

www.techopedia.com › definition › data-compression

What is Data Compression? - Definition from Techopedia

Data compression is the process of modifying, encoding or converting the bits structure of data in such a way that it consumes less space on disk. It enables reducing the storage size of one or more data instances or elements. **Data compression** is also known as source coding or bit-rate reduction.

www.britannica.com › technology › data-compression

Data compression | computing | Britannica

Data compression, also called compaction, the process of reducing the amount of data needed for the storage or transmission of a given piece of information, ...

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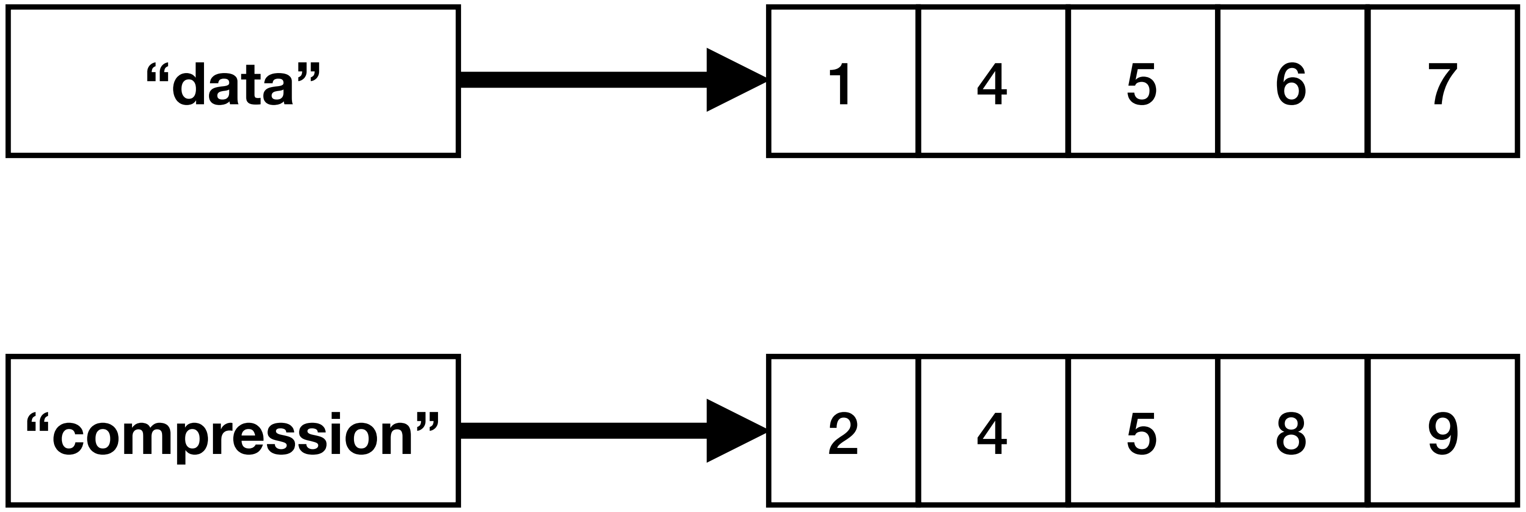

“data” → 1 4 5 6 7

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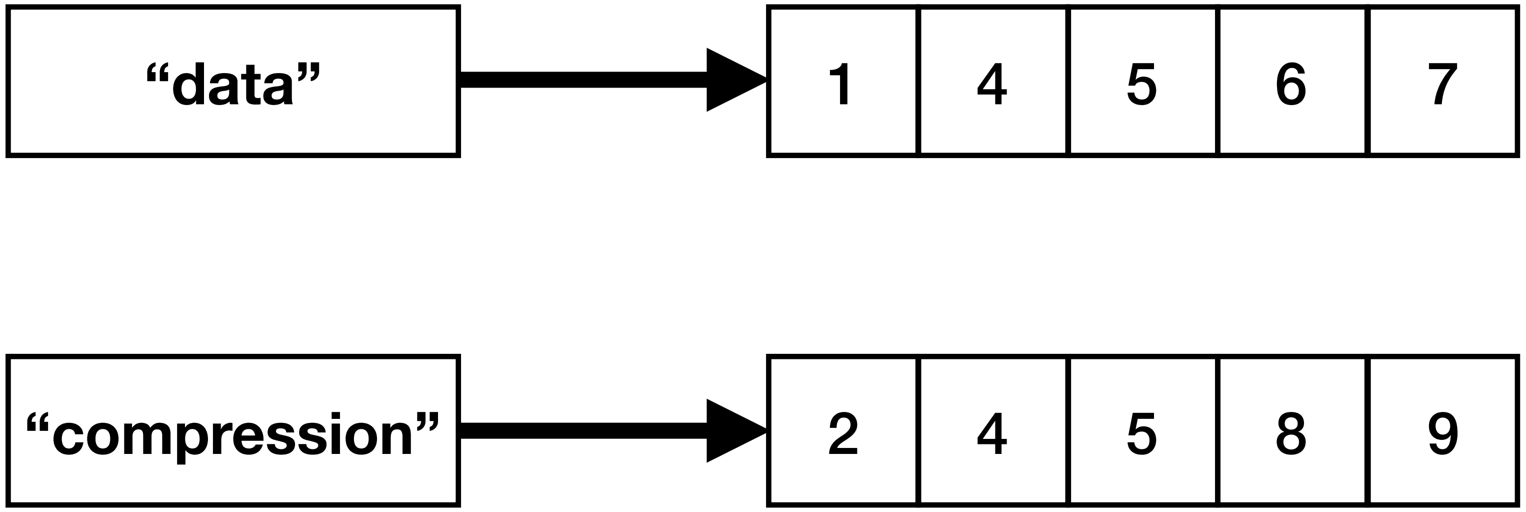

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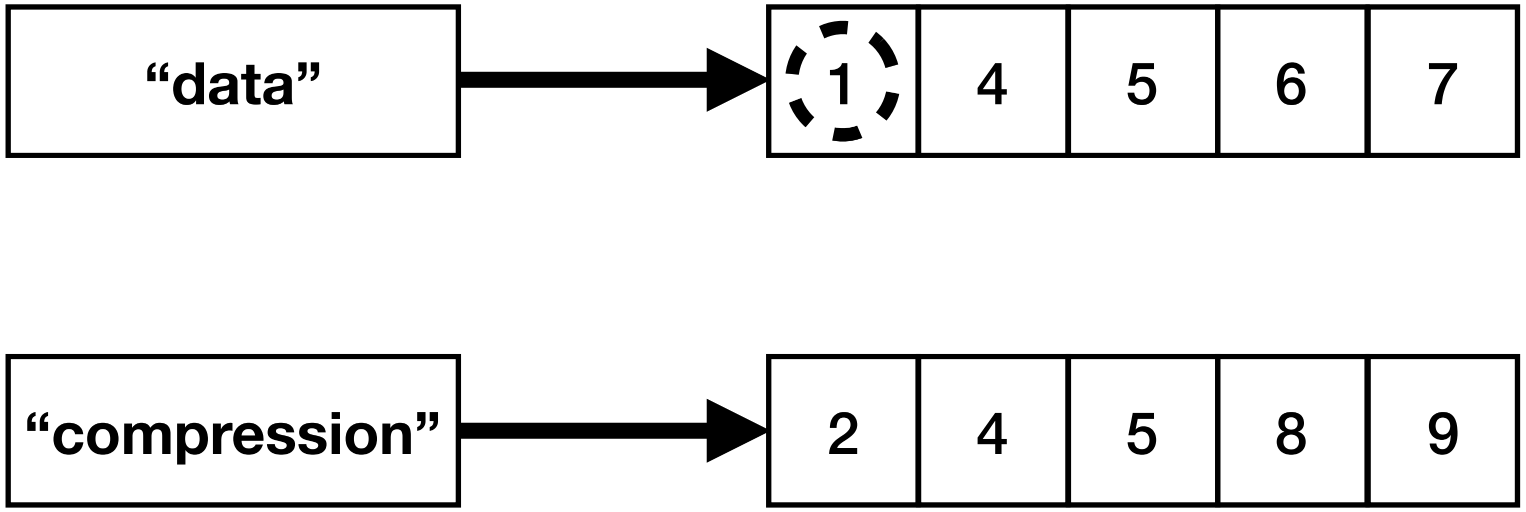

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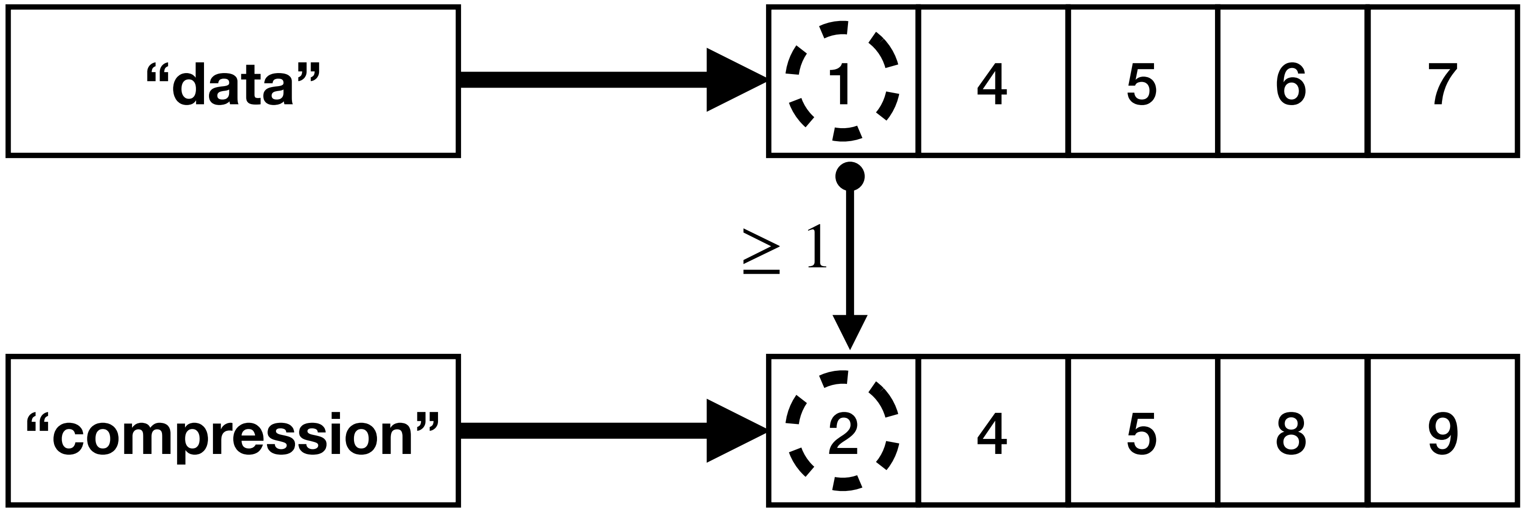

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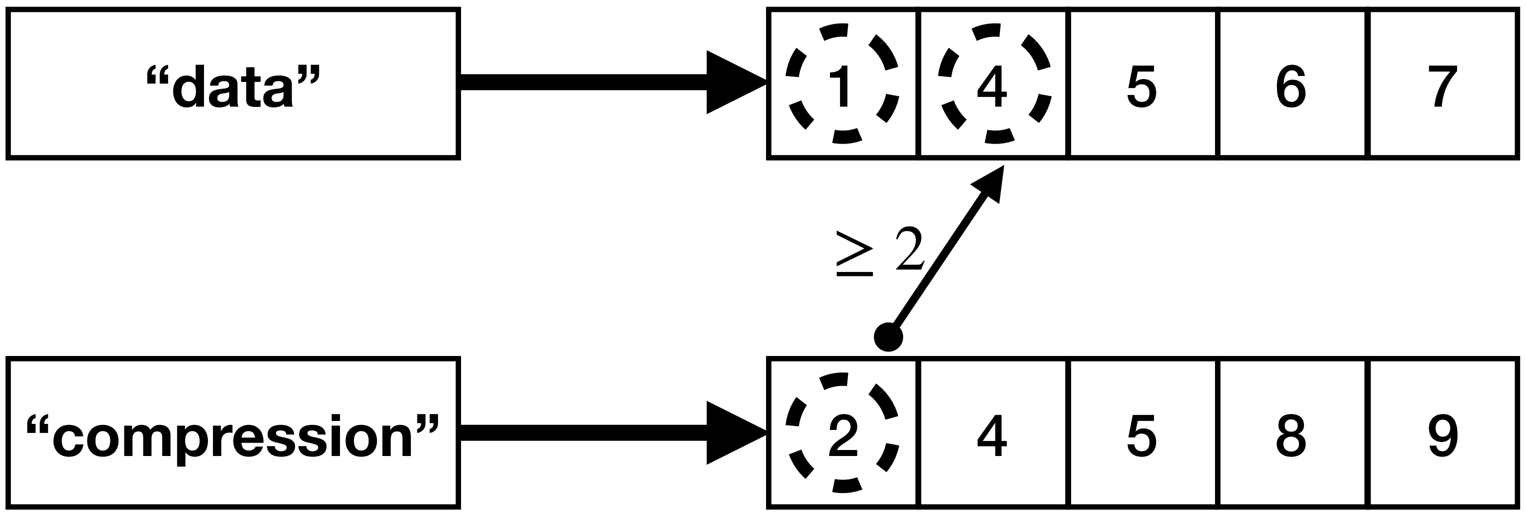

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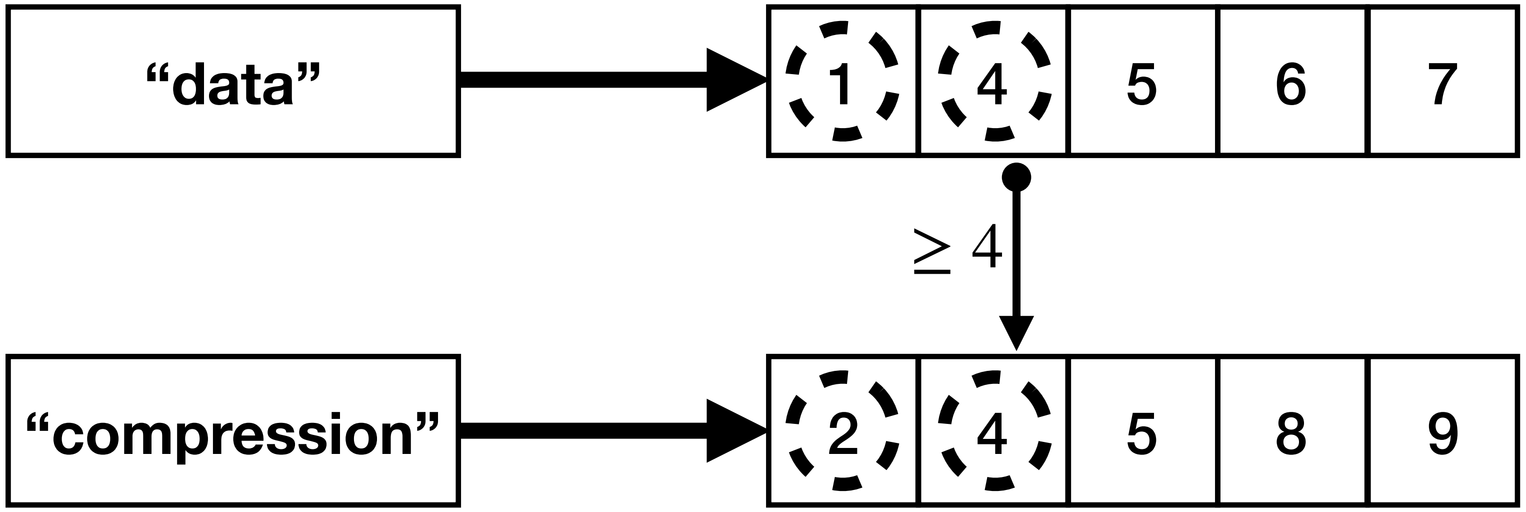

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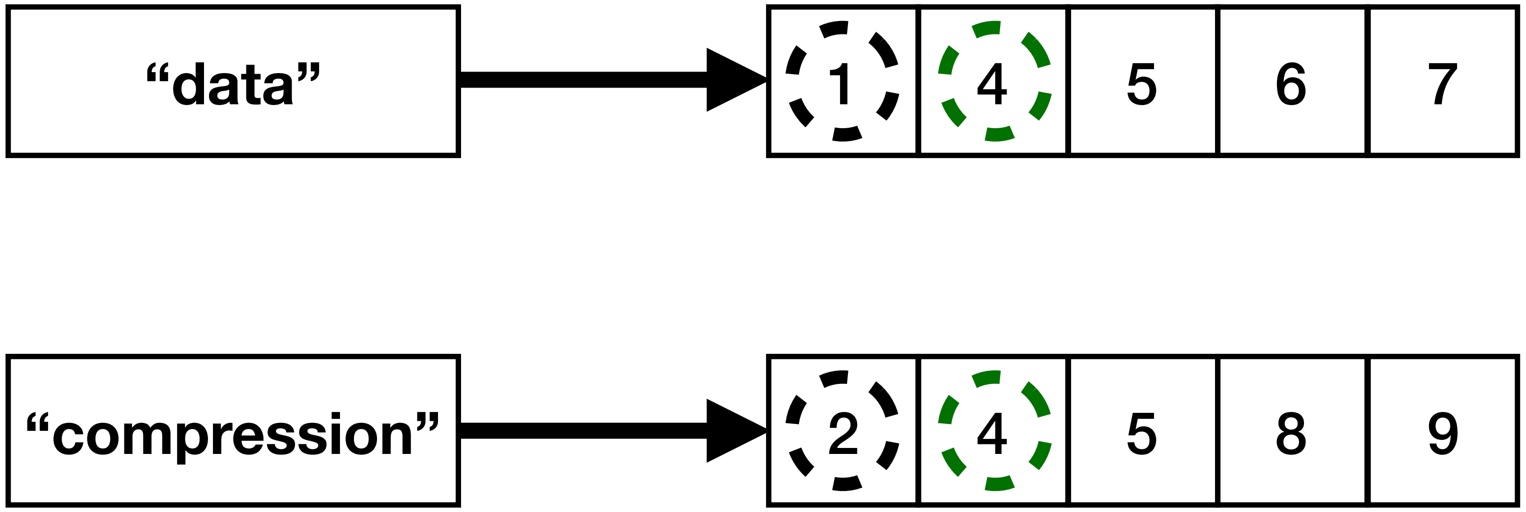

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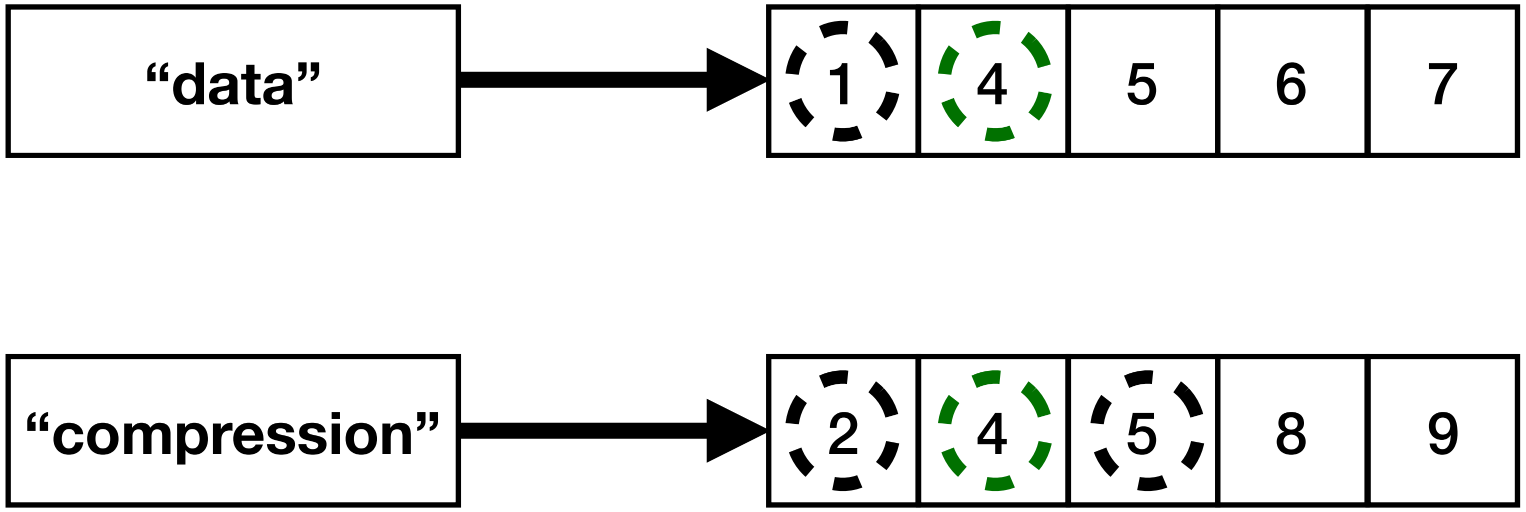

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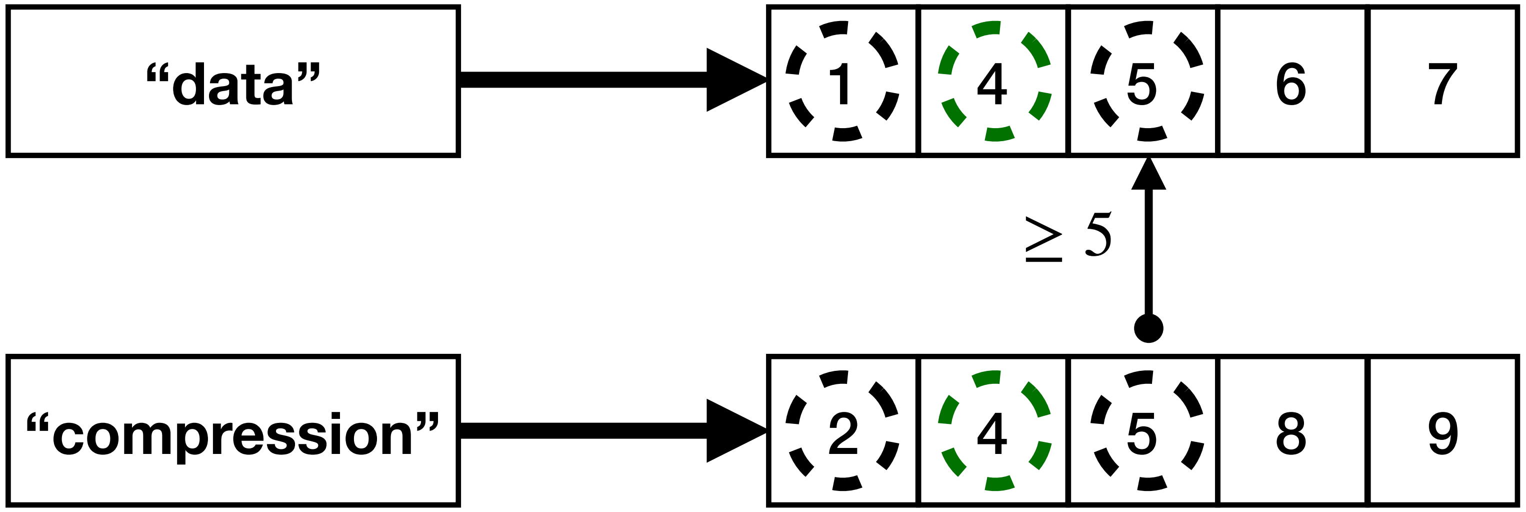

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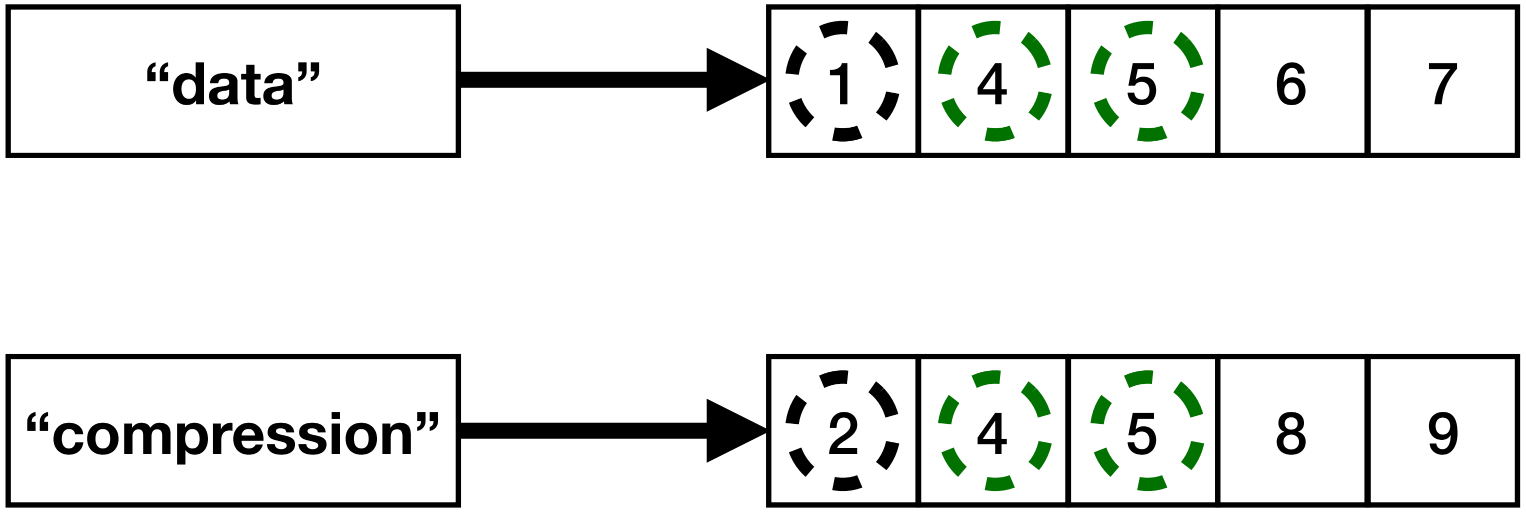

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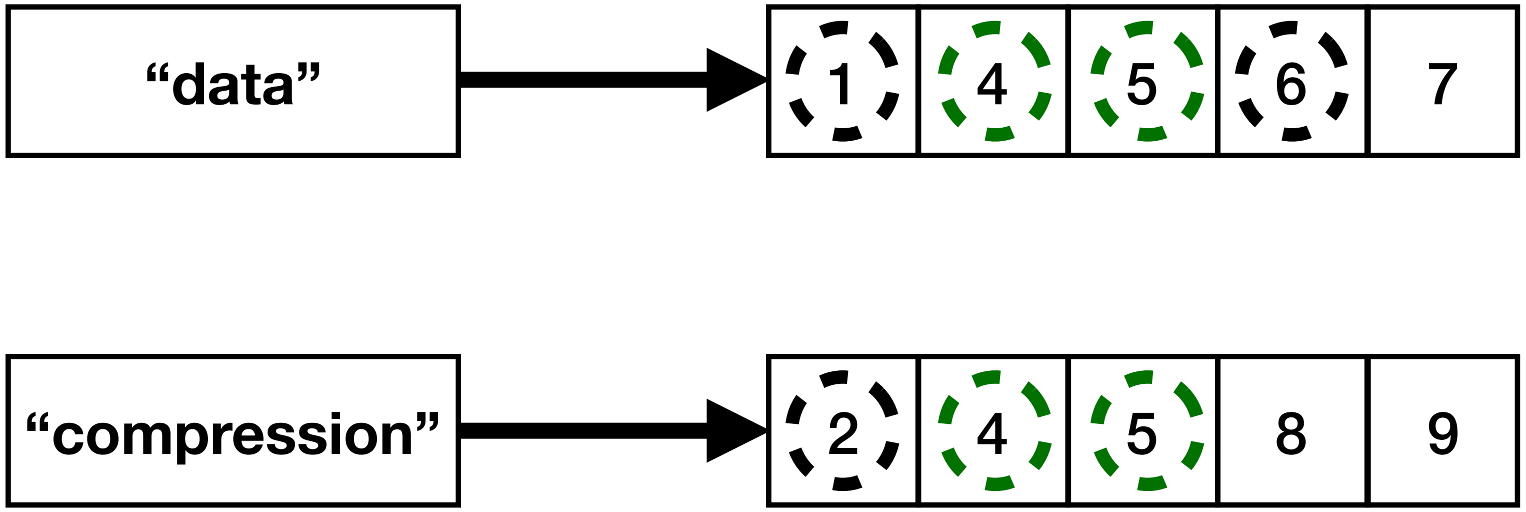

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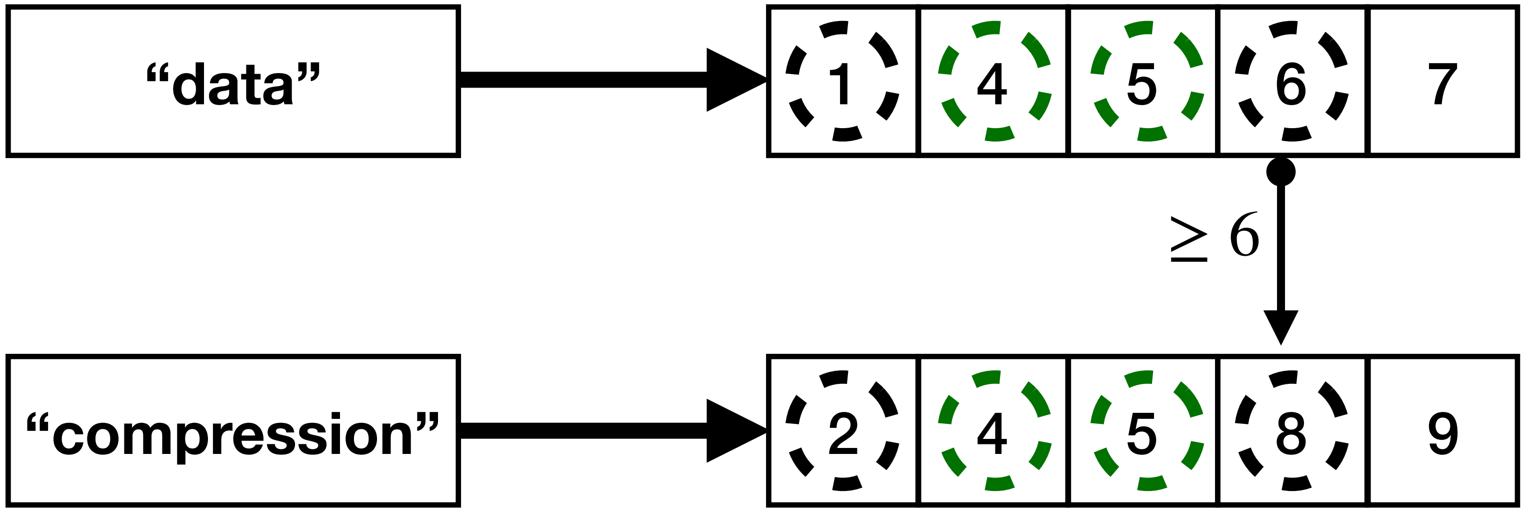

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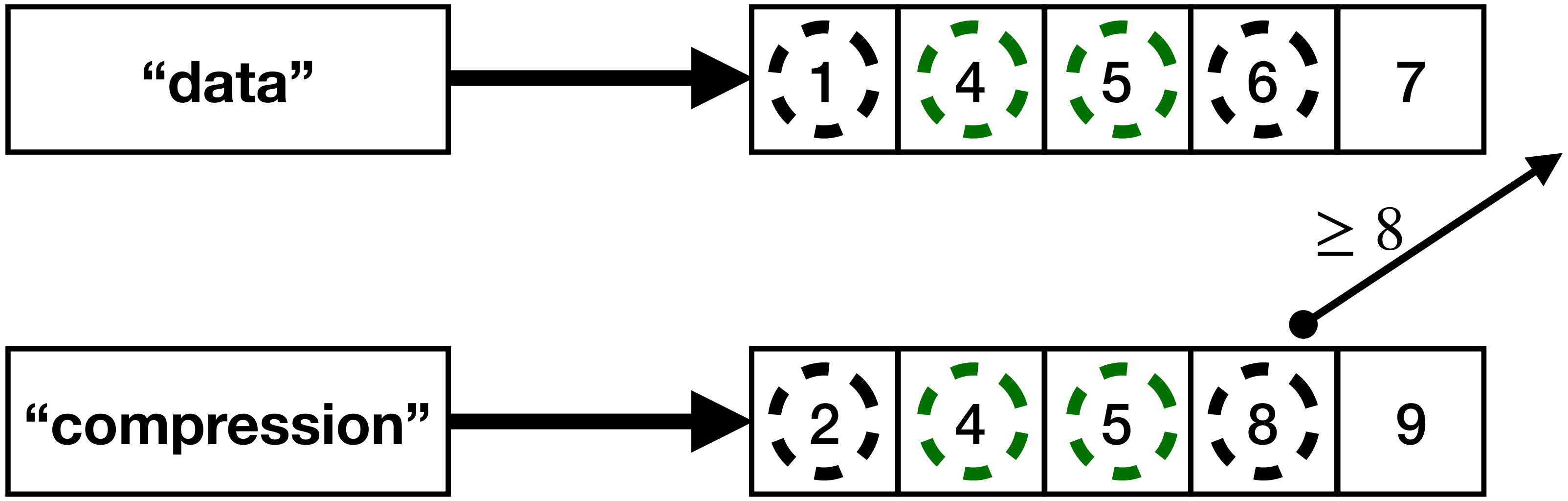

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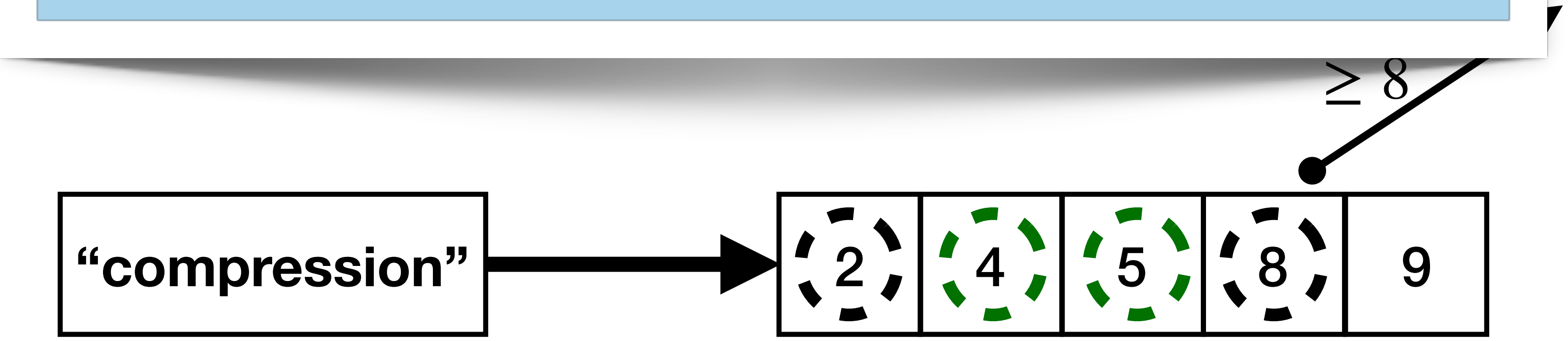
- Searc **of lists**

An efficient mechanism for finding an equal or higher value in the other list is necessary

Successor and predecessor problem



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- **Successor and predecessor**

$$S = \{x_1 < x_2 < \ldots < x_m\}$$

$succ(x) = x_i$ *minimum value* $x_i \geq x$ *of* S

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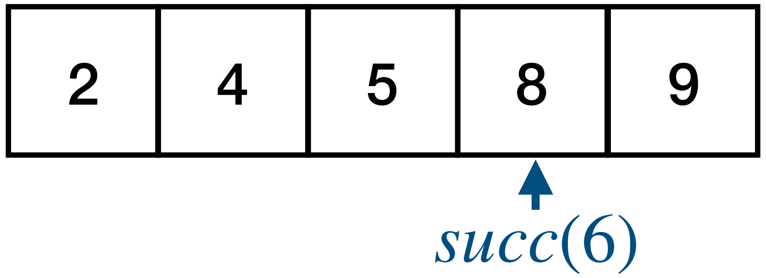

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$succ(6)$

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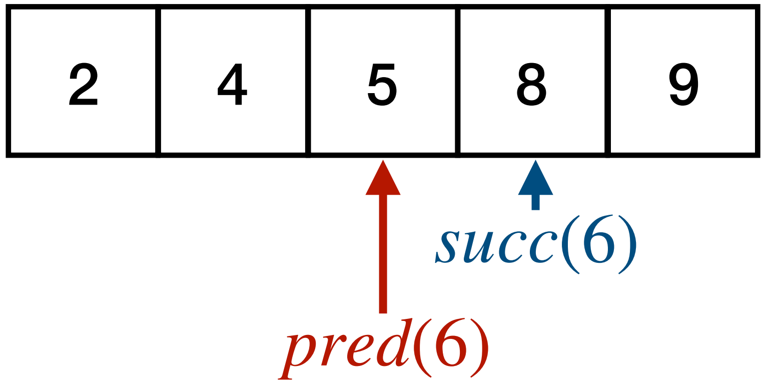

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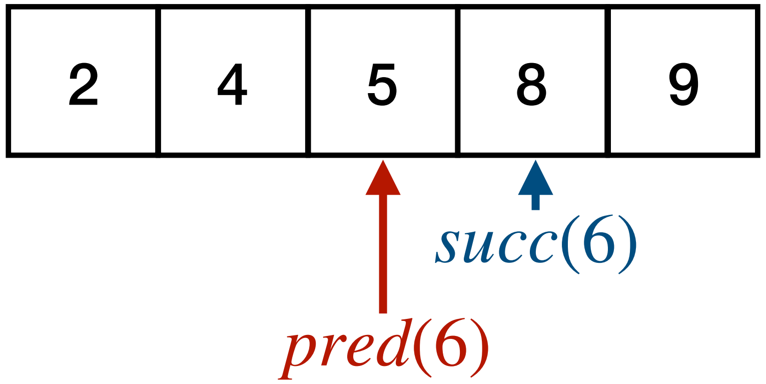

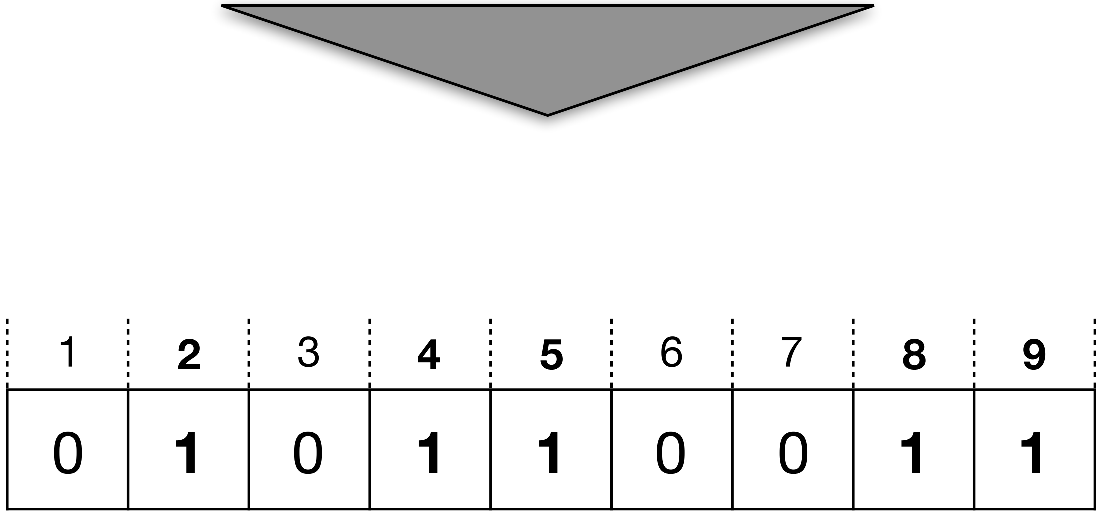

1	**2**	3	**4**	**5**	6	7	**8**	**9**
0	**1**	0	**1**	**1**	0	0	**1**	**1**

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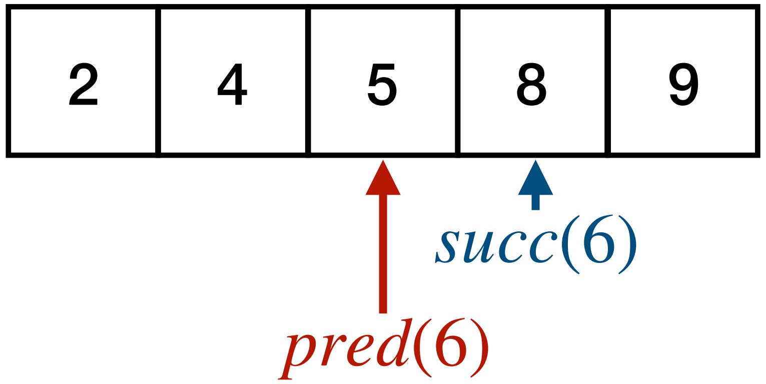

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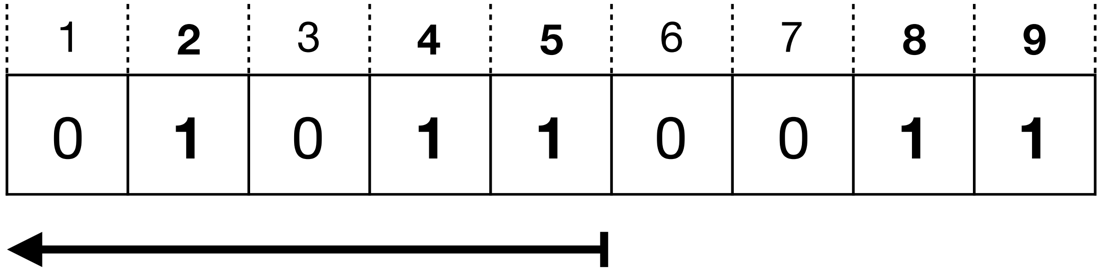

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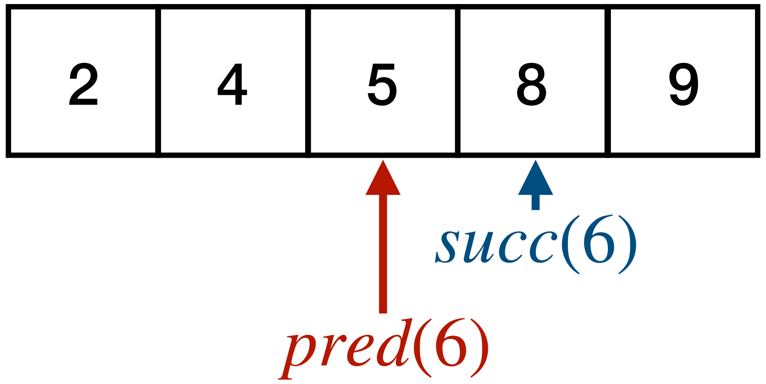

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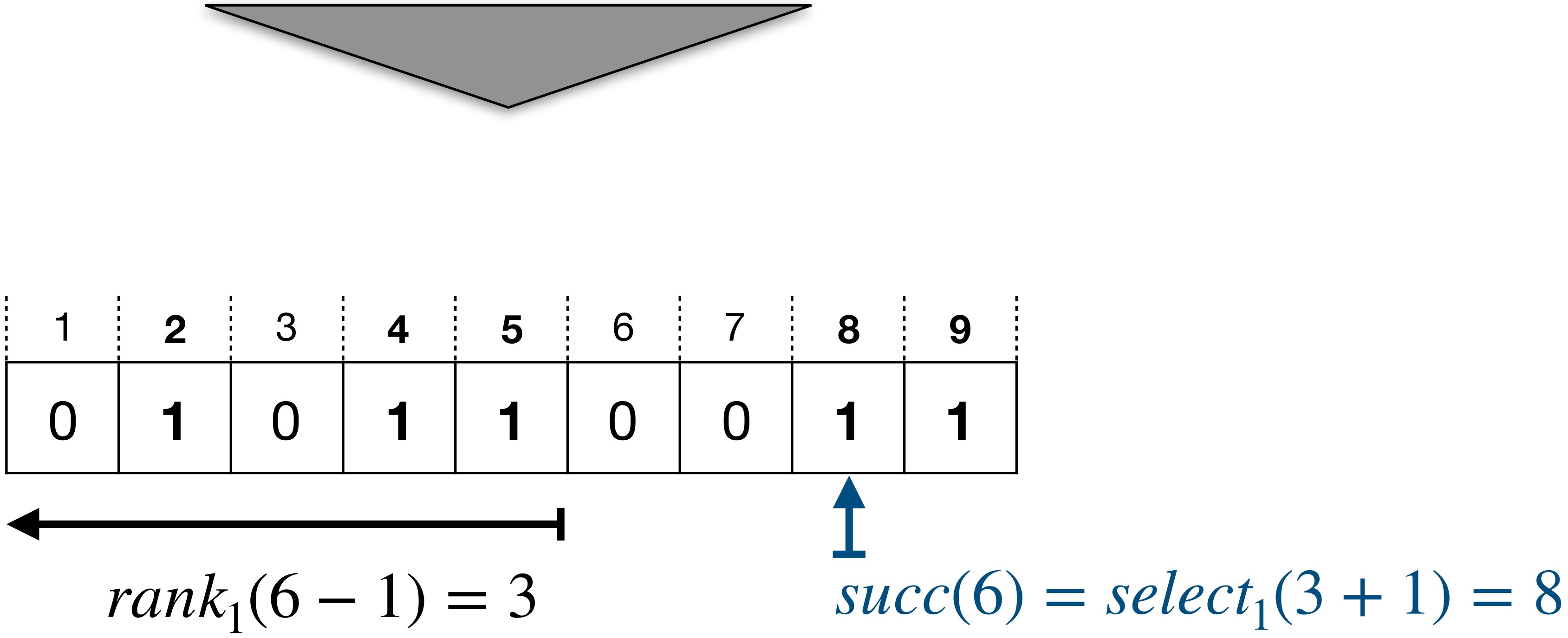

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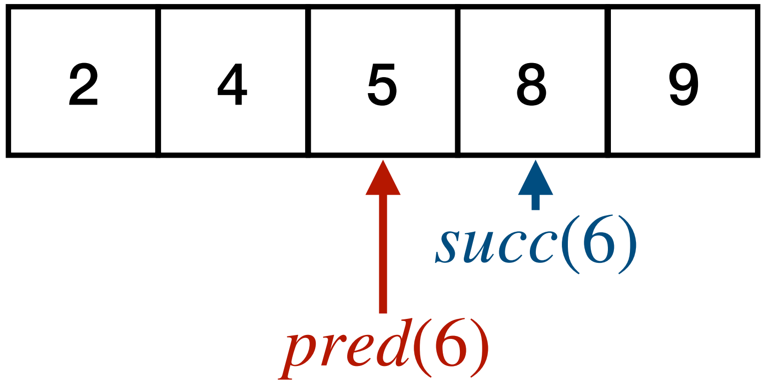

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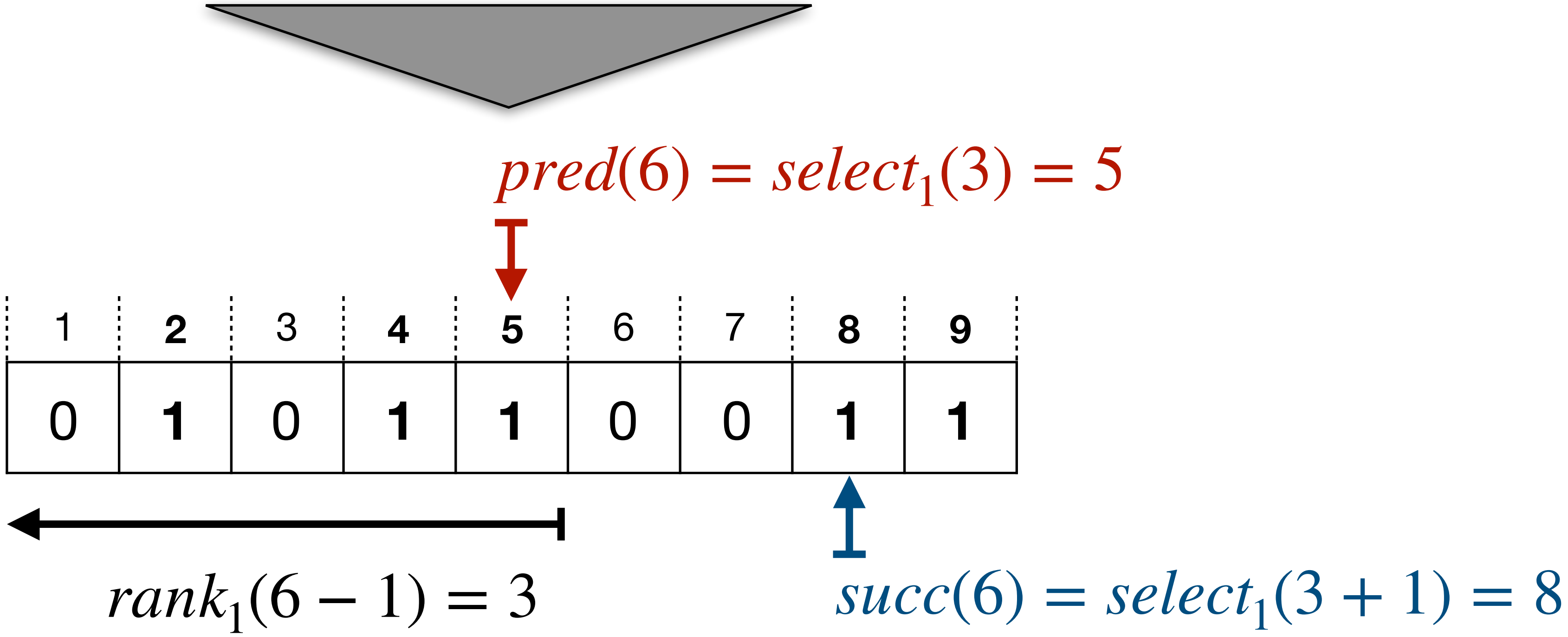

$pred(6) = select_1(3) = 5$

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0	**1**	0	**1**	**1**	0	0	**1**	**1**

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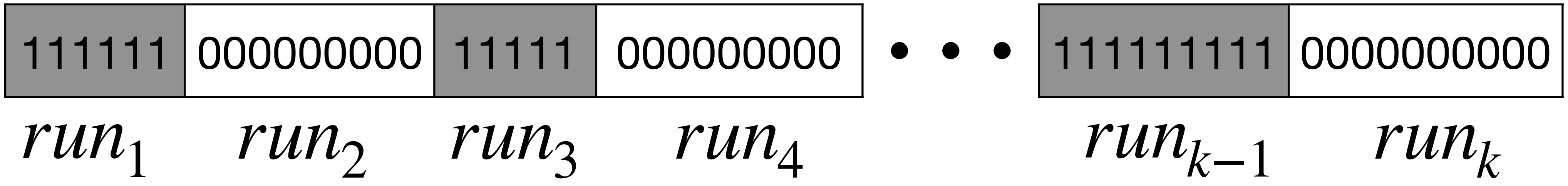

111111	000000000	11111	000000000	• • •	111111111	0000000000
run_1	run_2	run_3	run_4		run_{k-1}	run_k

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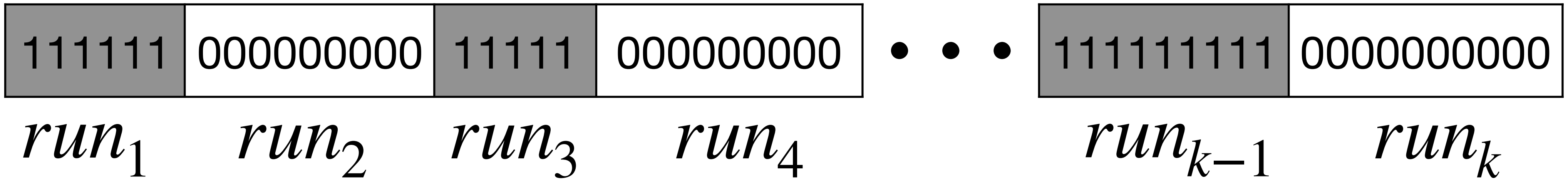

111111 000000000 11111 000000000 • • • 111111111 0000000000

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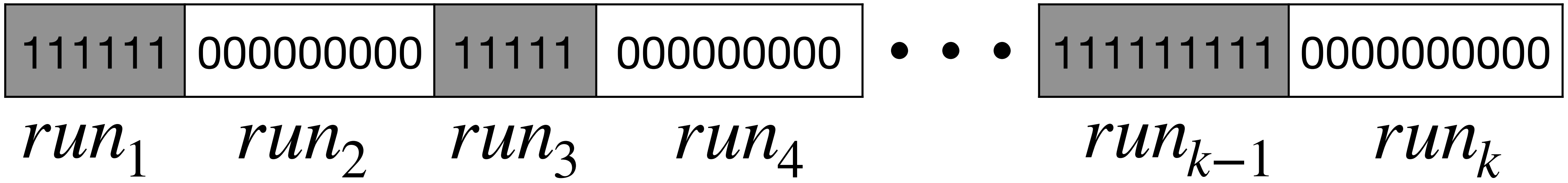

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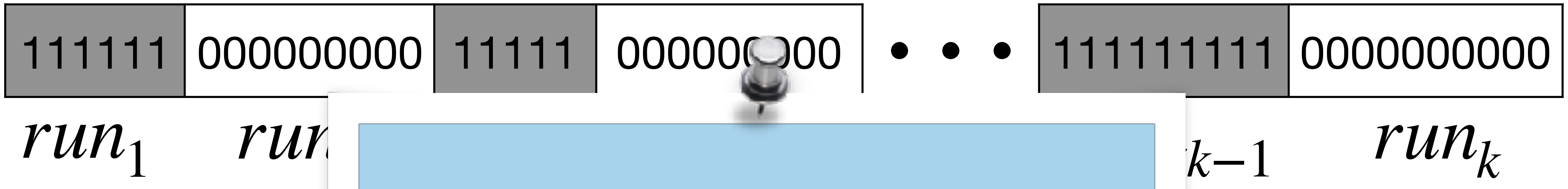

Successor and predecessor in *O(1)* time and exploiting the *k* runs

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- **Hybrid bitvector:** splits the bitvector into different parts and choose the best technique (*sparse*, *runs*, *plain*) for each individual division. [hybrid]

Background

Bitvector	**Time**	**Space**
plain	$O(1)$	$n + o(n)$
rrr	$O(1)$	$nH_0 + o(n)$
rec-rank	$O(\log \frac{n}{m})$	$\log \frac{n}{m} + m + o(n)$
sd-array	$O(\log \frac{n}{m})$	$m \log \frac{n}{m} + O(m)$
oz-vector	$O(\log k)$	$k \log \frac{2n}{k} + O(k)$
hybrid	$O(\log n)$	$min(k, m) \lceil \log b \rceil + o(n)$

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zombit-vector

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zombit-vector

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- **Partitions X_i of β size** and classification of them in three sets:
  - $\mathbb{Z}$: uniform blocks full of zeroes
  - $\mathbb{O}$: uniform blocks full of ones
  - $\mathbb{M}$: mixed blocks, which contain both bits

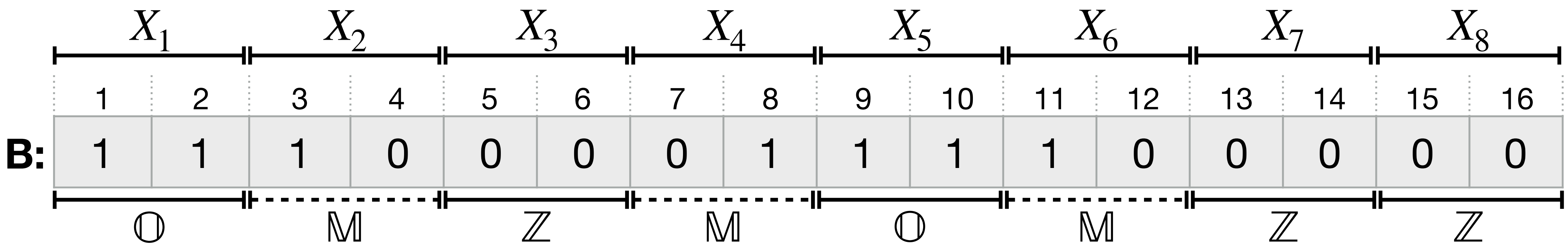
zombit-vector

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

zombit-vector

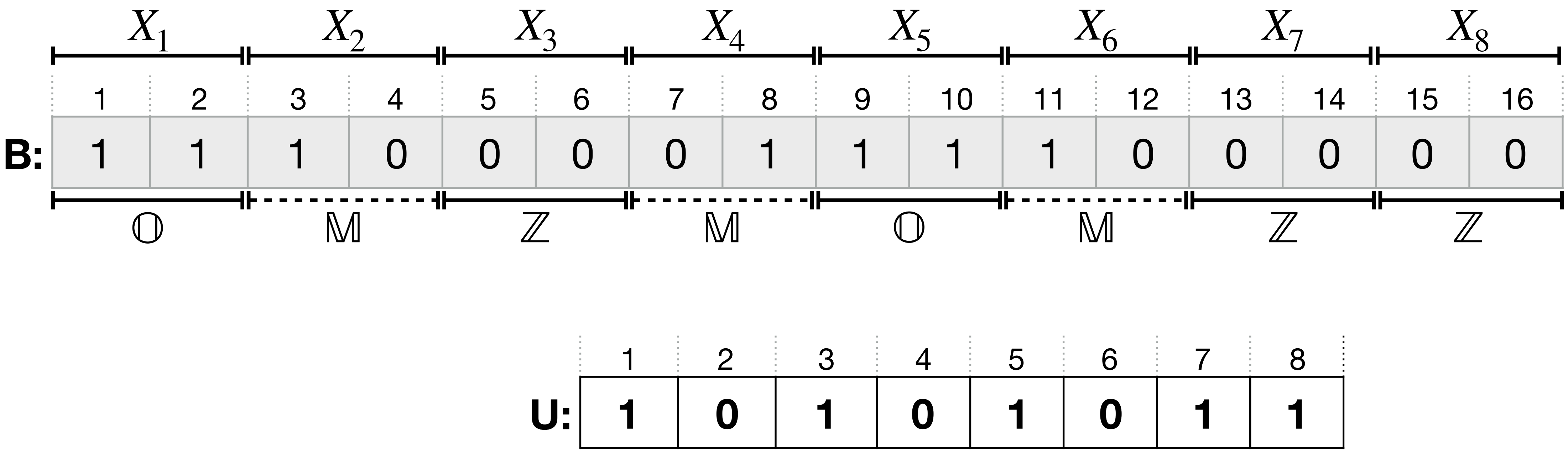
	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

zombit-vector

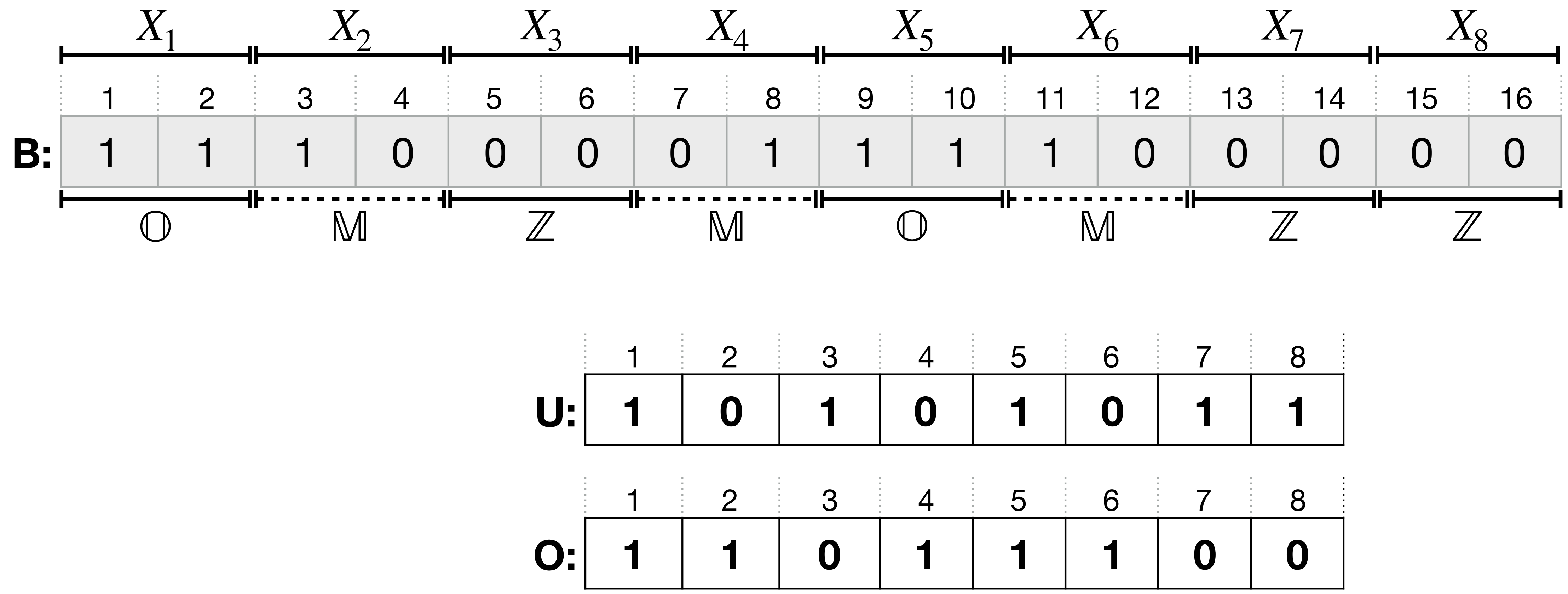


X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
𝕆	𝕄	ℤ	𝕄	𝕆	𝕄	ℤ	ℤ

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

zombit-vector

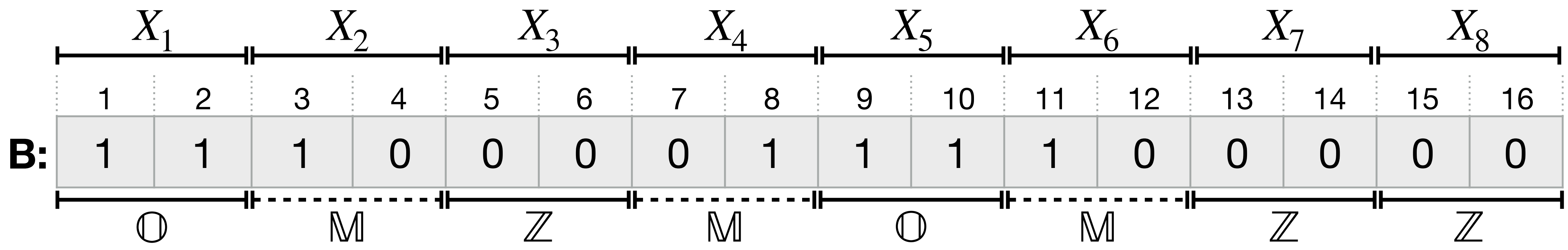


	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

	1	2	3	4	5	6
M:	**1**	**0**	**0**	**1**	**1**	**0**

zombit-vector

	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

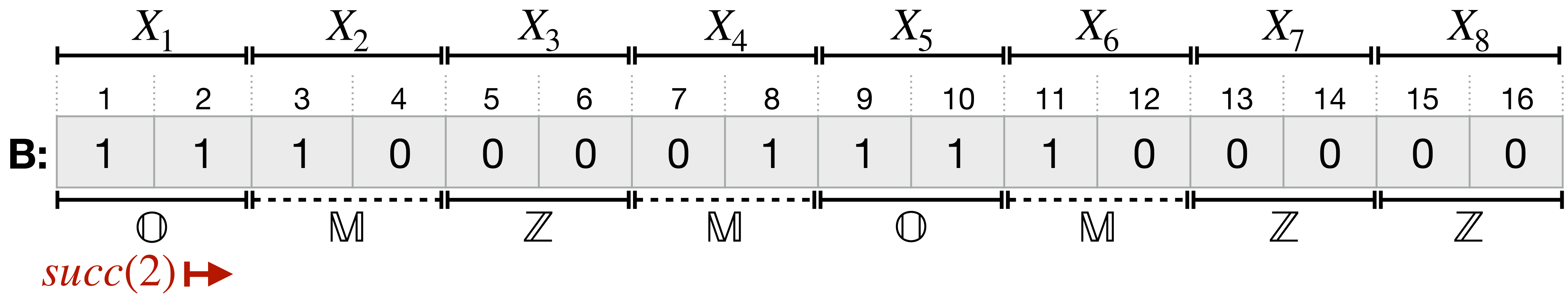
zombit-vector

	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

	1	2	3	4	5	6
M:	**1**	**0**	**0**	**1**	**1**	**0**

zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(2) \mapsto$

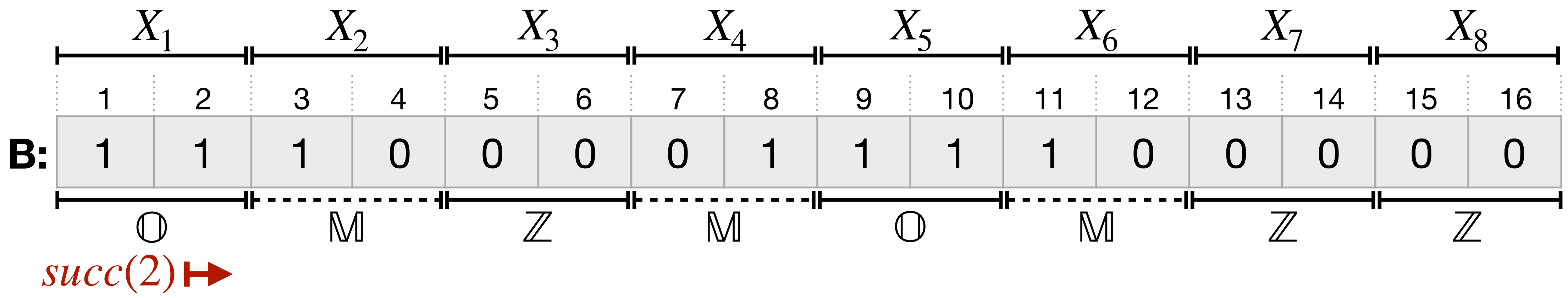
zombit-vector

	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

	1	2	3	4	5	6
M:	**1**	**0**	**0**	**1**	**1**	**0**

zombit-vector



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

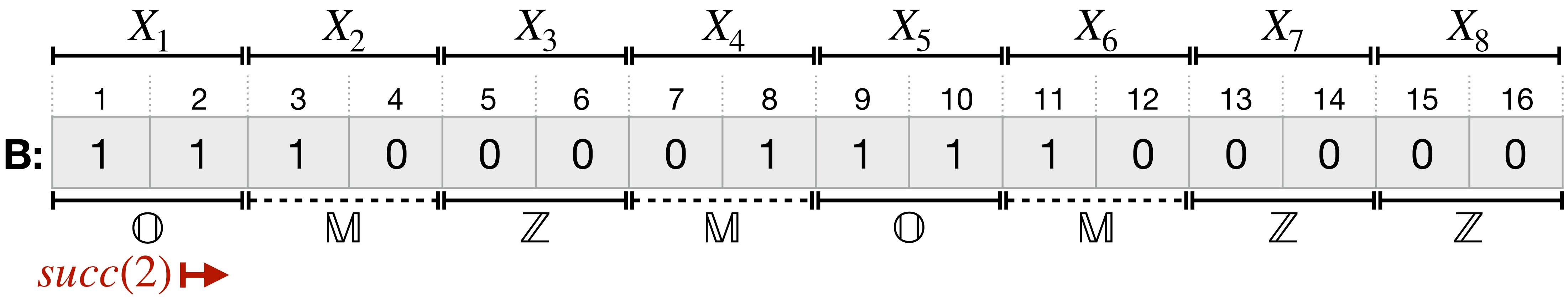
zombit-vector

	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

	1	2	3	4	5	6
M:	**1**	**0**	**0**	**1**	**1**	**0**

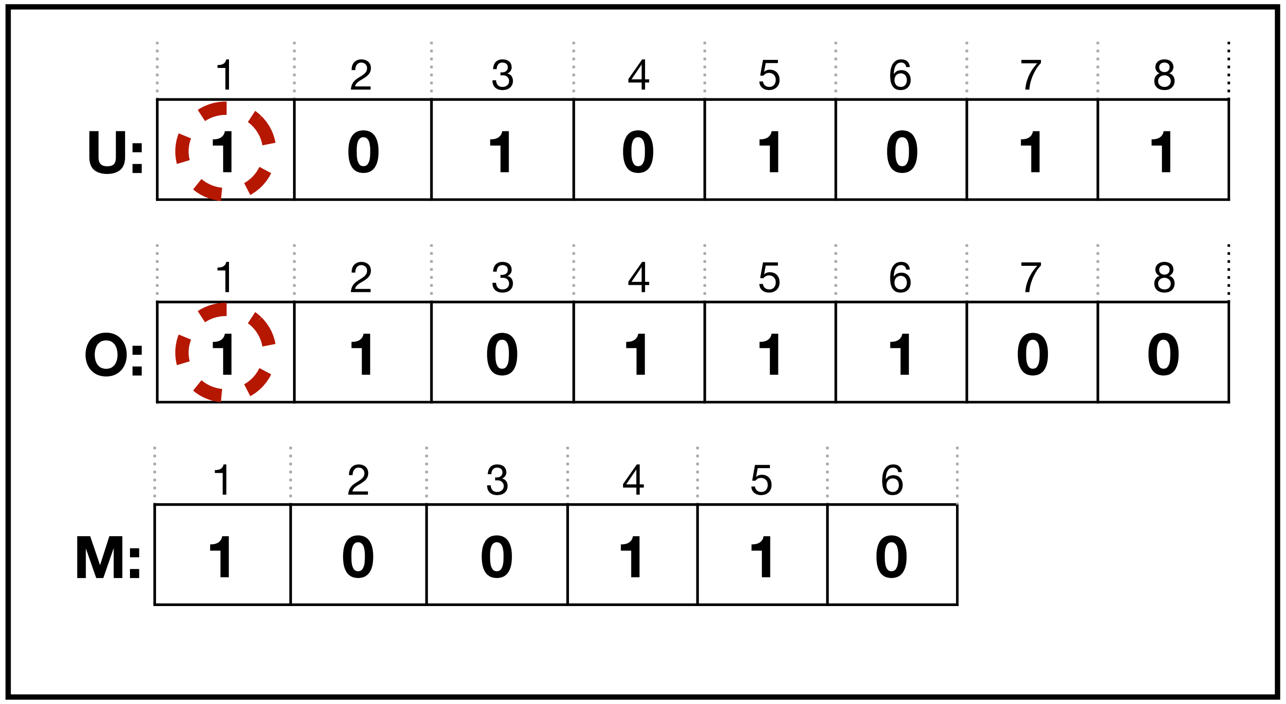
zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(2) \mapsto$

zombit-vector

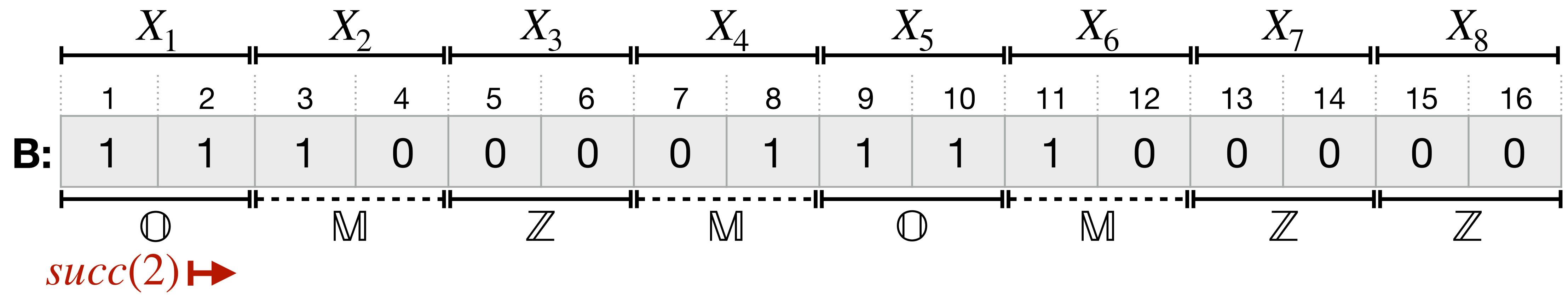


	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

	1	2	3	4	5	6
M:	1	0	0	1	1	0

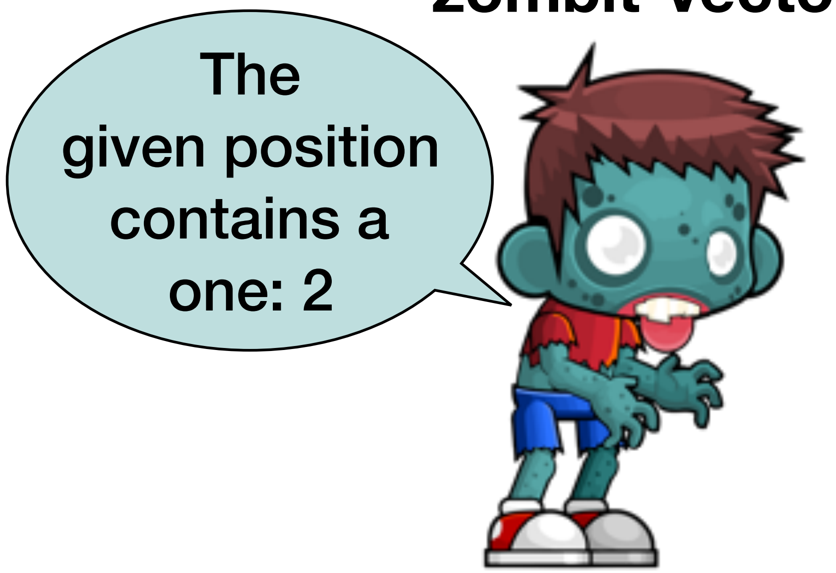
zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(2) \mapsto$

zombit-vector

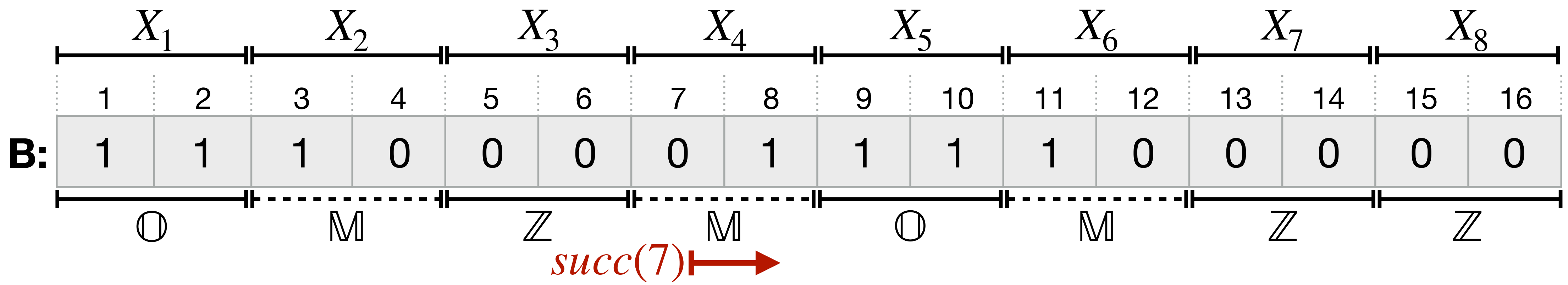


	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

	1	2	3	4	5	6
M:	1	0	0	1	1	0

zombit-vector



	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
	$\mathbb{O}$	$\mathbb{M}$	$\mathbb{Z}$	$\mathbb{M}$	$\mathbb{O}$	$\mathbb{M}$	$\mathbb{Z}$	$\mathbb{Z}$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

$succ(7)$

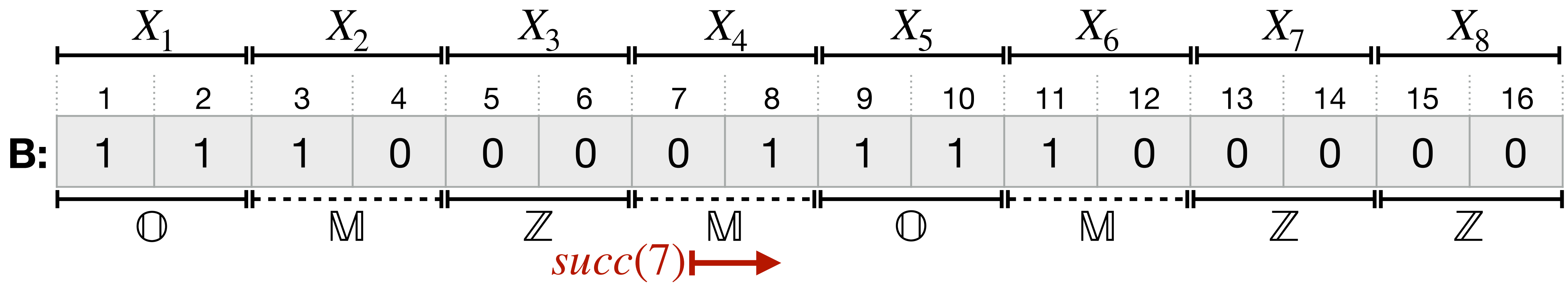
zombit-vector

	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

	1	2	3	4	5	6
M:	1	0	0	1	1	0

zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$\mathit{succ}(7) \mapsto$

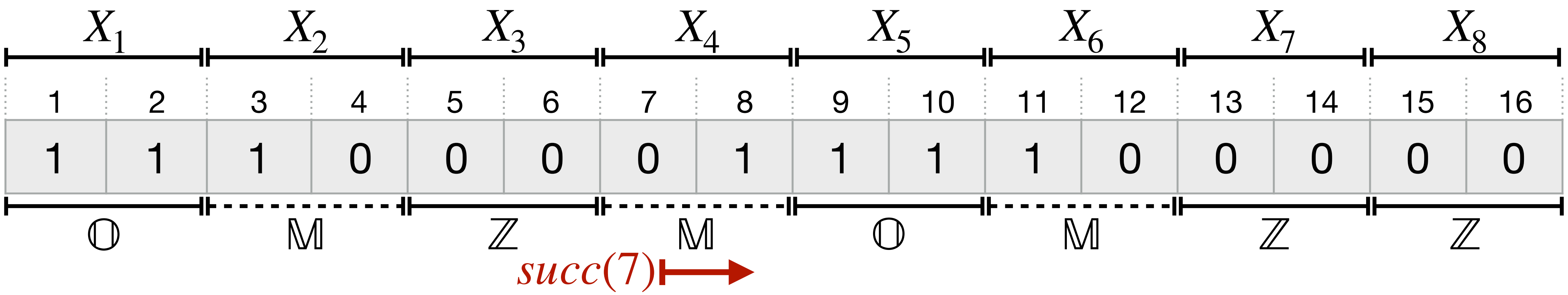
zombit-vector

	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

	1	2	3	4	5	6
M:	**1**	**0**	**0**	**1**	**1**	**0**

zombit-vector

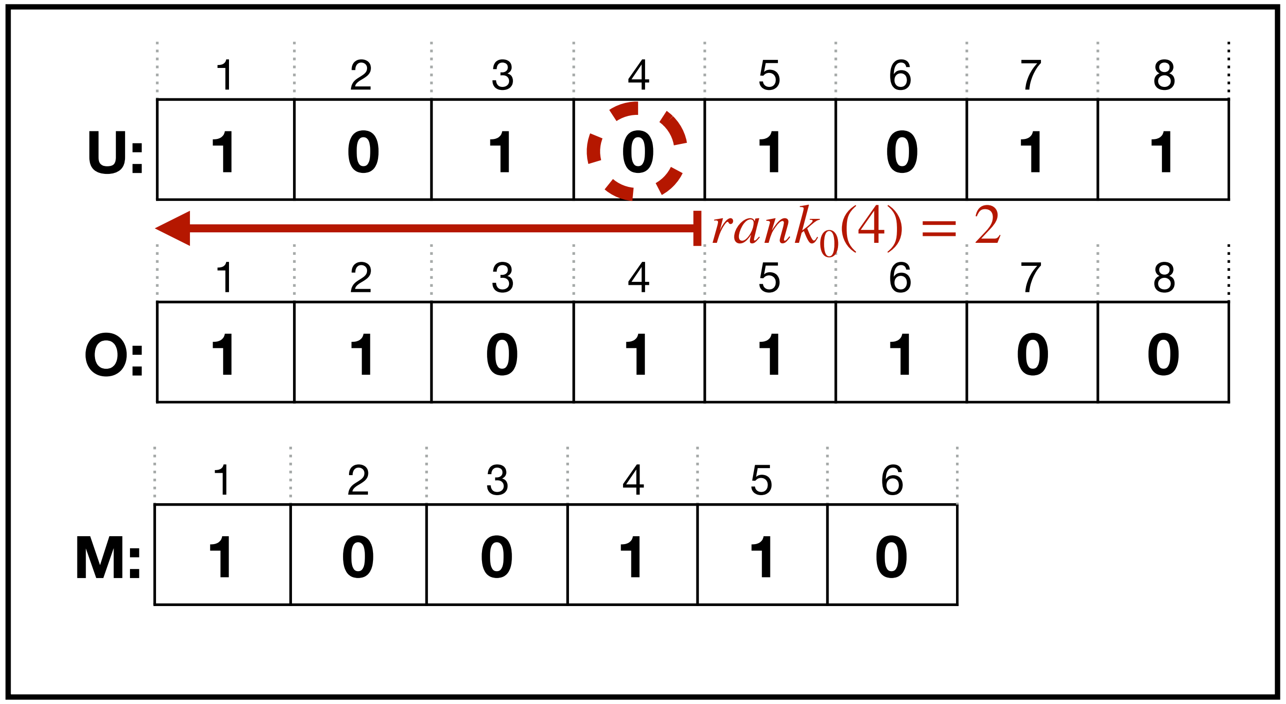


	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
	$\mathbb{O}$	$\mathbb{M}$	$\mathbb{Z}$	$\mathbb{M}$	$\mathbb{O}$	$\mathbb{M}$	$\mathbb{Z}$	$\mathbb{Z}$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

$succ(7) \mapsto$

zombit-vector



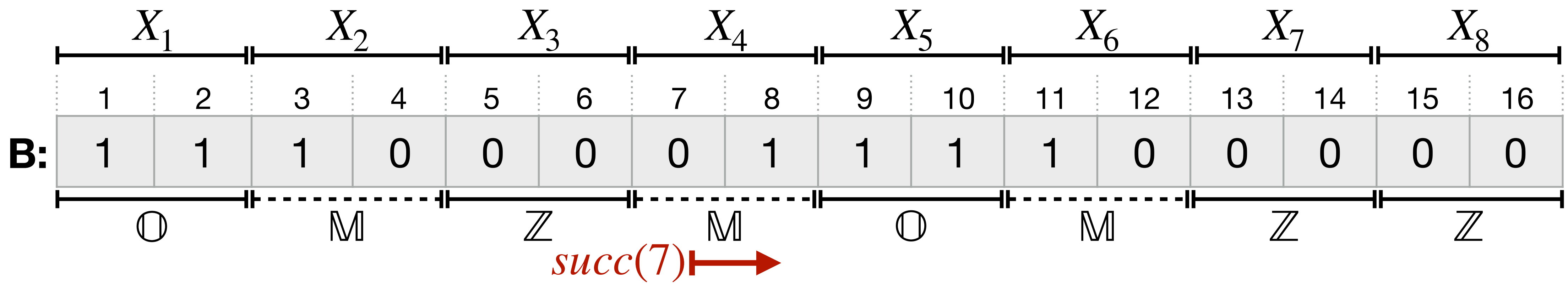
	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

$rank_0(4) = 2$

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

	1	2	3	4	5	6
M:	1	0	0	1	1	0

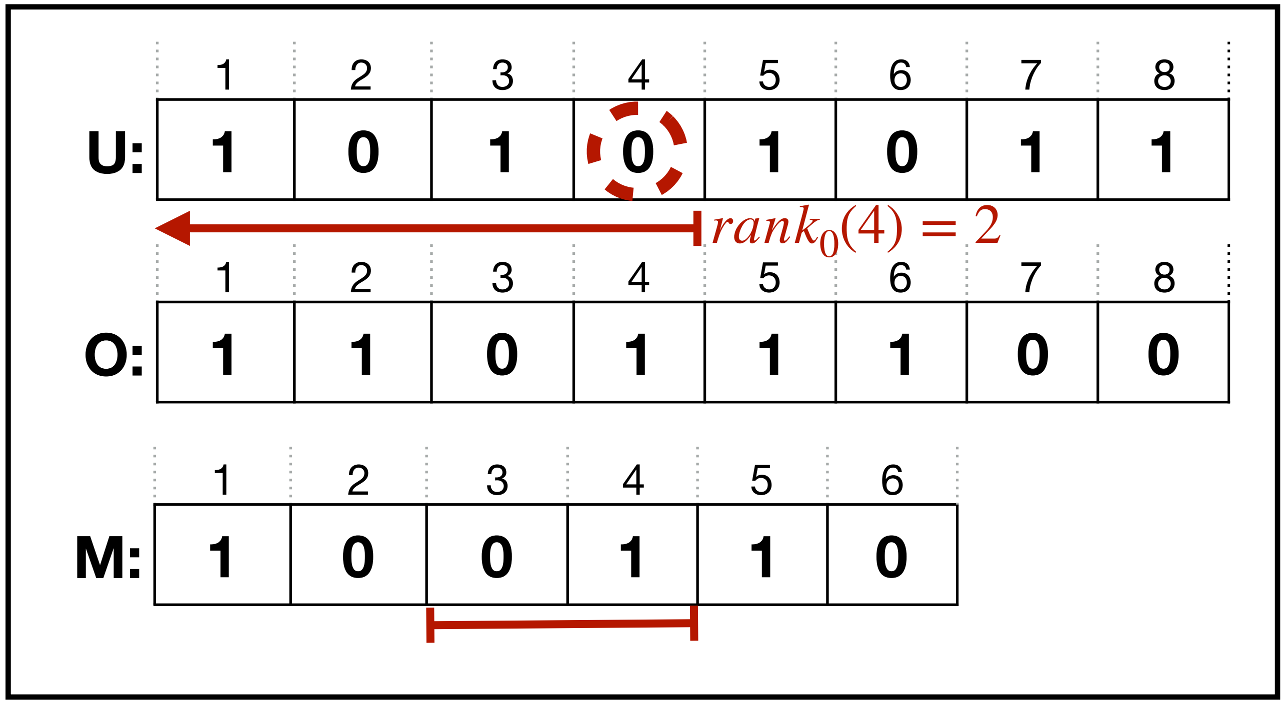
zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(7)$

zombit-vector



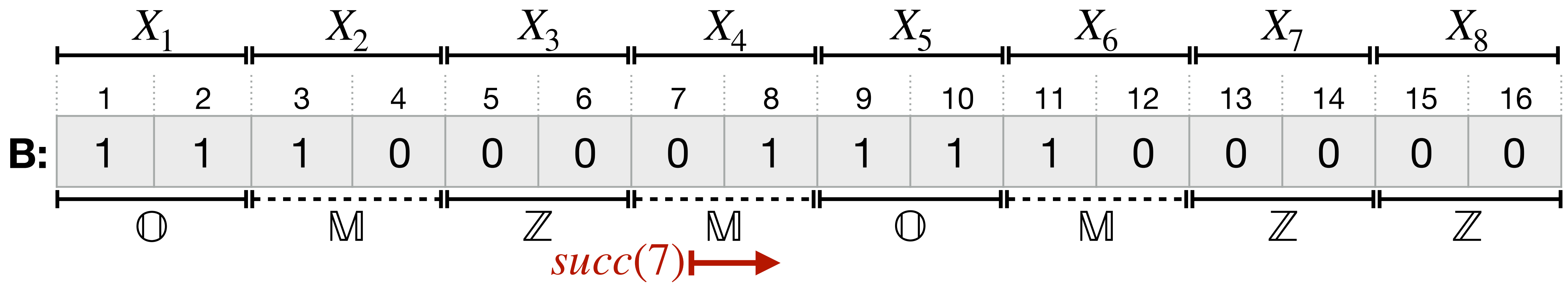
	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

$rank_0(4) = 2$

	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

	1	2	3	4	5	6
M:	**1**	**0**	**0**	**1**	**1**	**0**

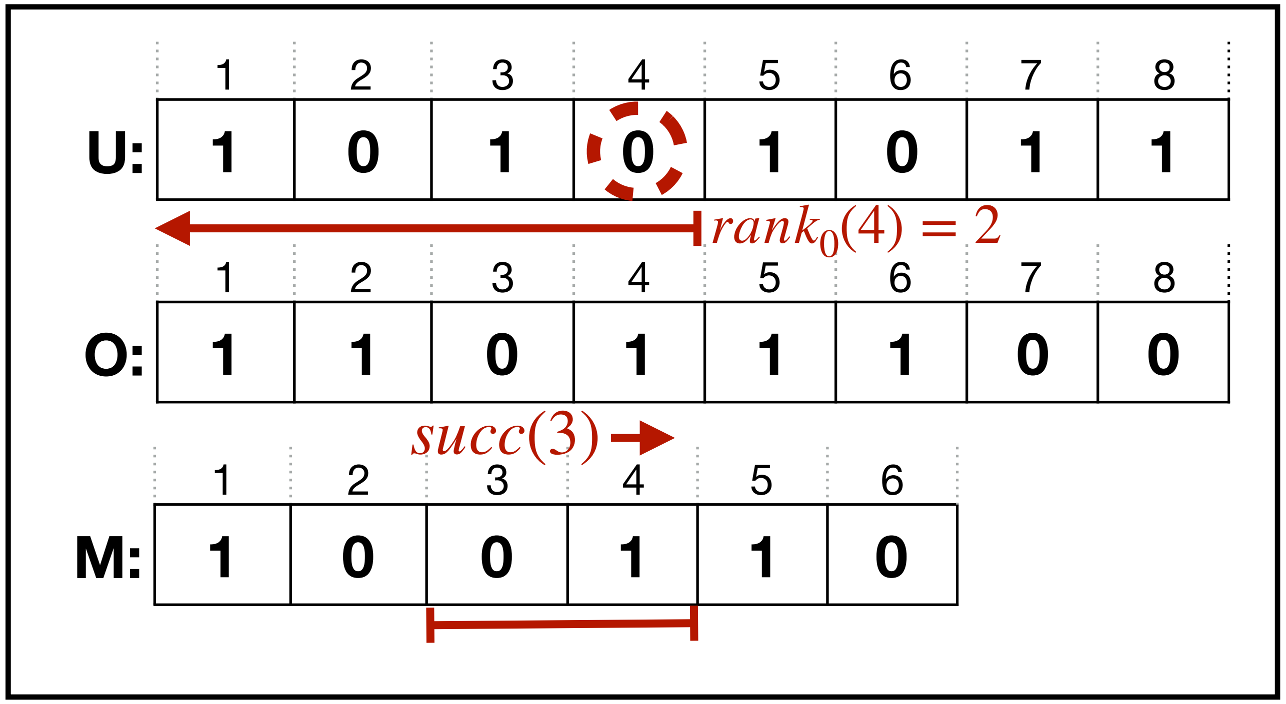
zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(7)$

zombit-vector



	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

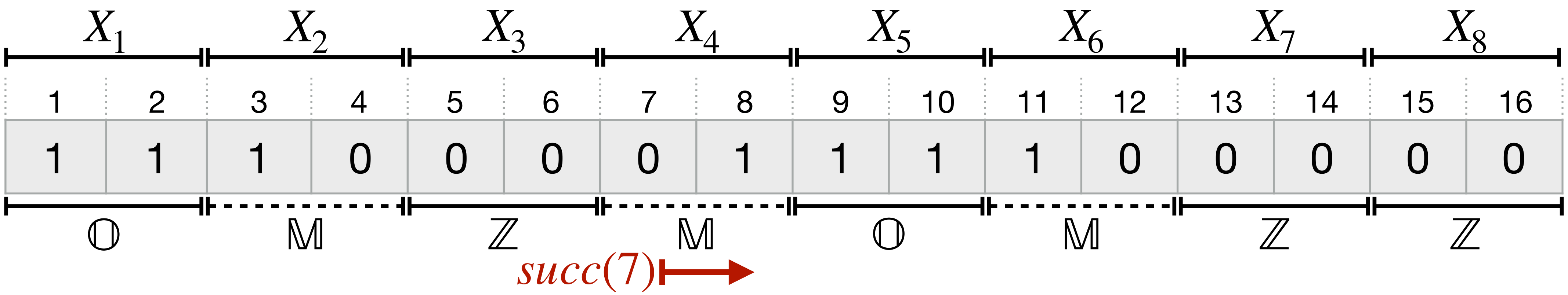
$rank_0(4) = 2$

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

$succ(3)$

	1	2	3	4	5	6
M:	1	0	0	1	1	0

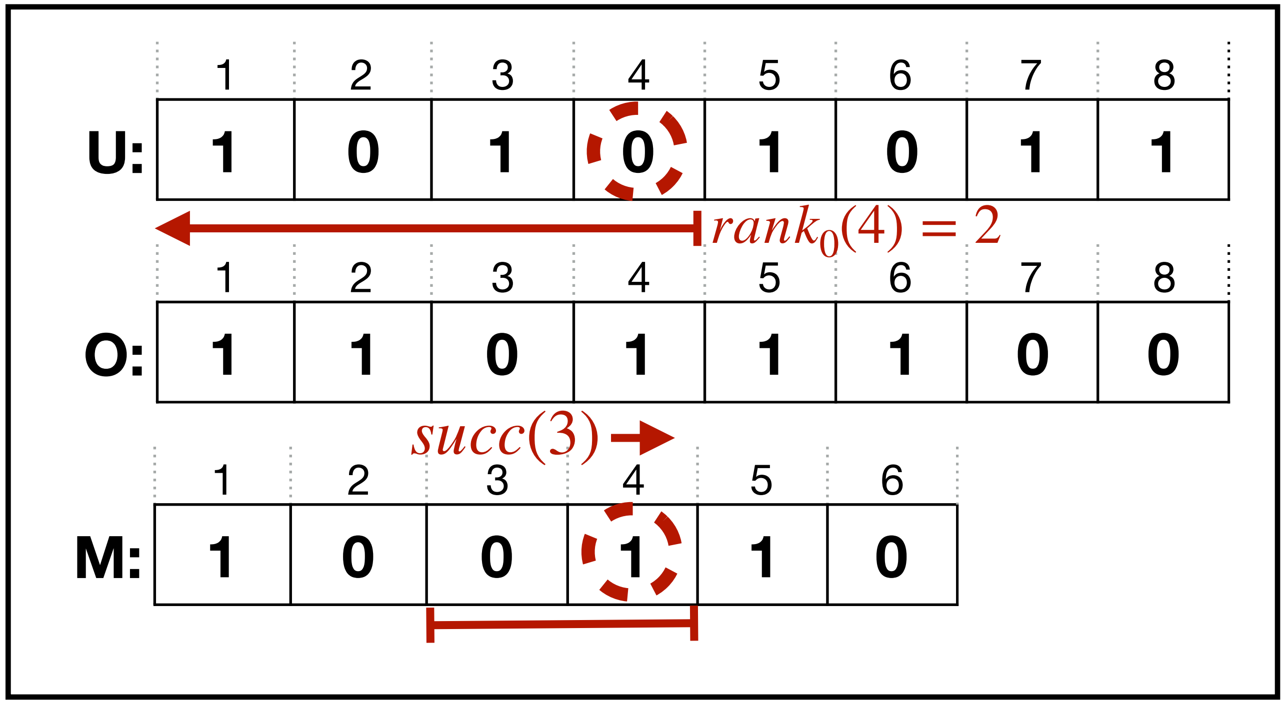
zombit-vector



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

$succ(7) \mapsto$

zombit-vector



	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

$rank_0(4) = 2$

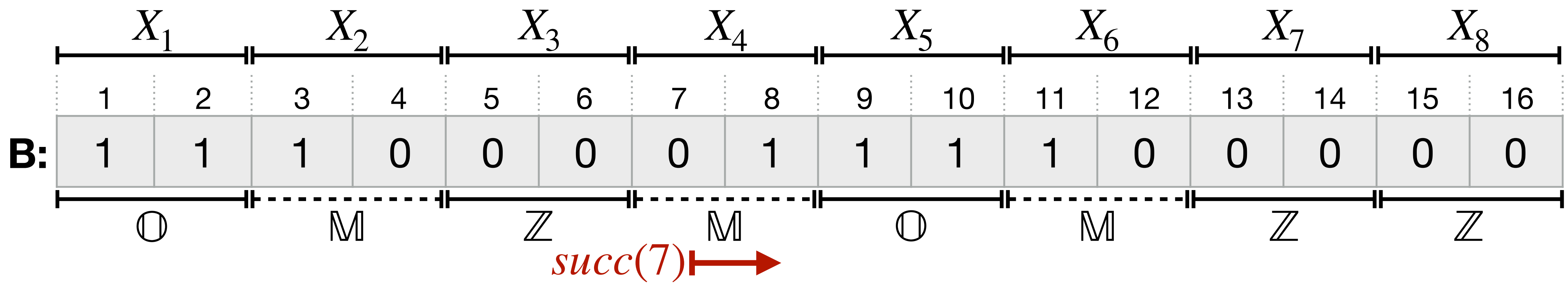
	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

$succ(3) \rightarrow$

	1	2	3	4	5	6
M:	1	0	0	1	1	0

Data
Compression
Conference

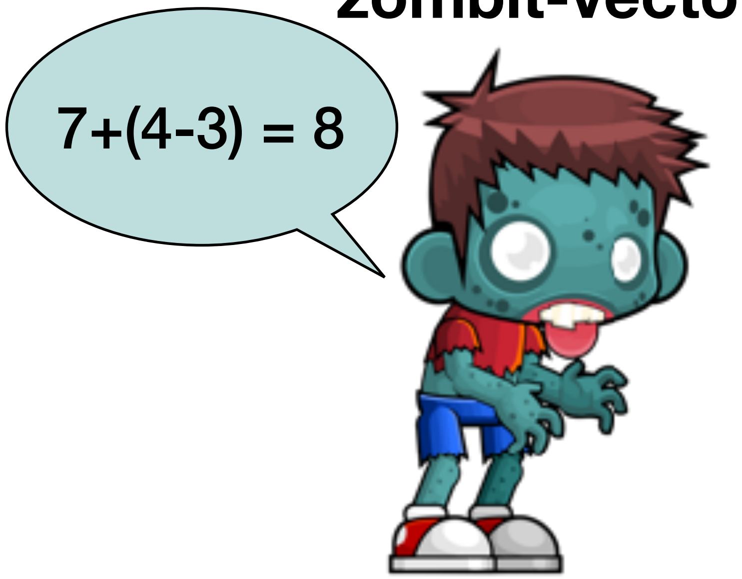
zombit-vector

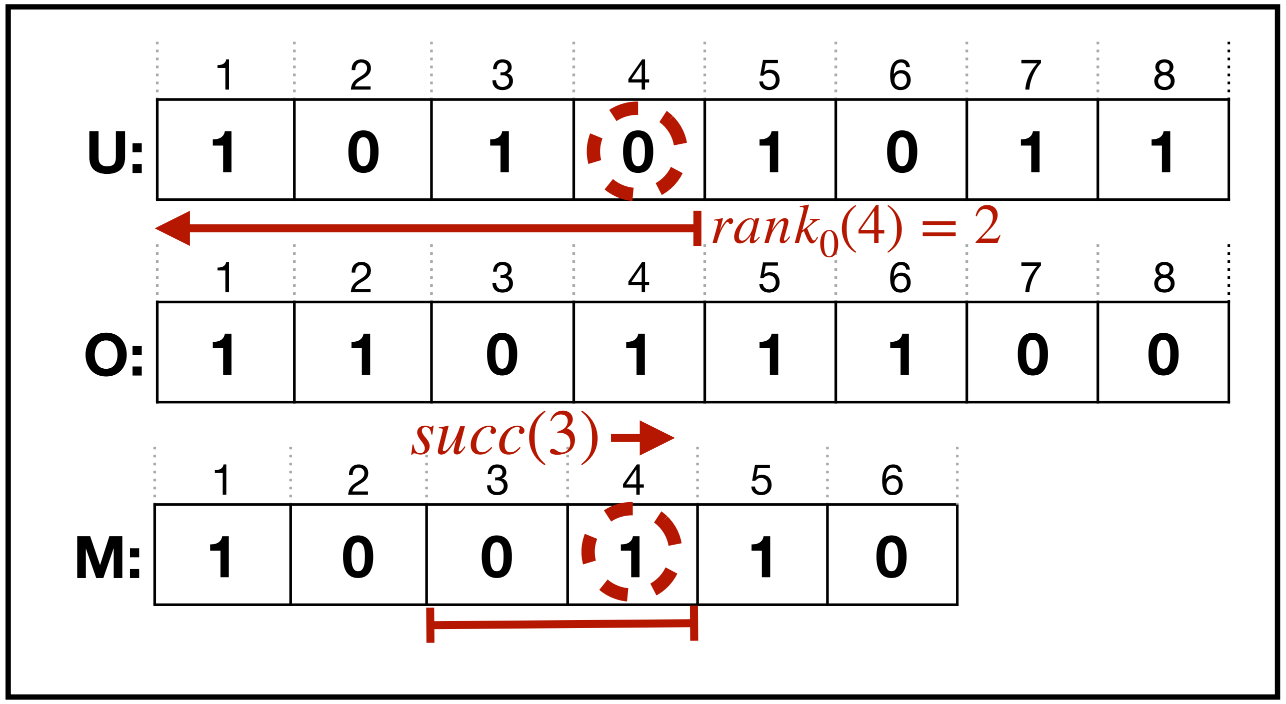


	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(7)$

zombit-vector





	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

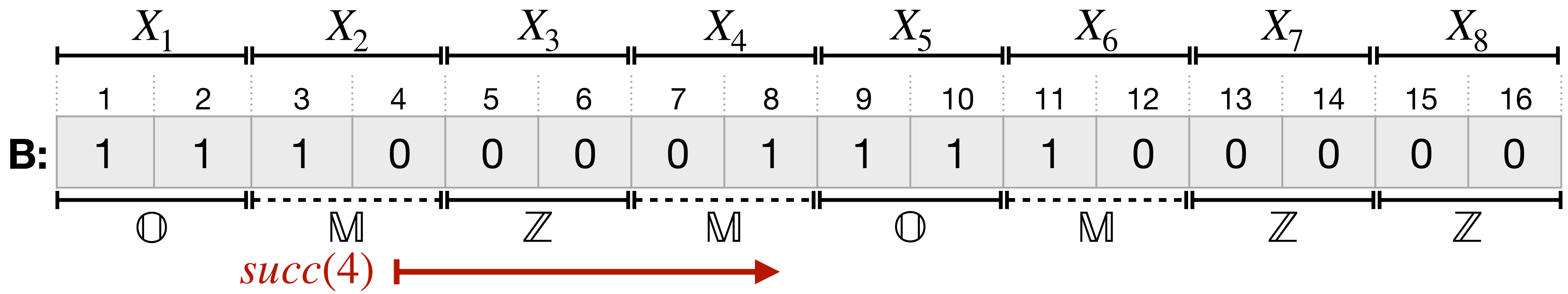
$rank_0(4) = 2$

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

$succ(3)$

	1	2	3	4	5	6
M:	1	0	0	1	1	0

zombit-vector



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

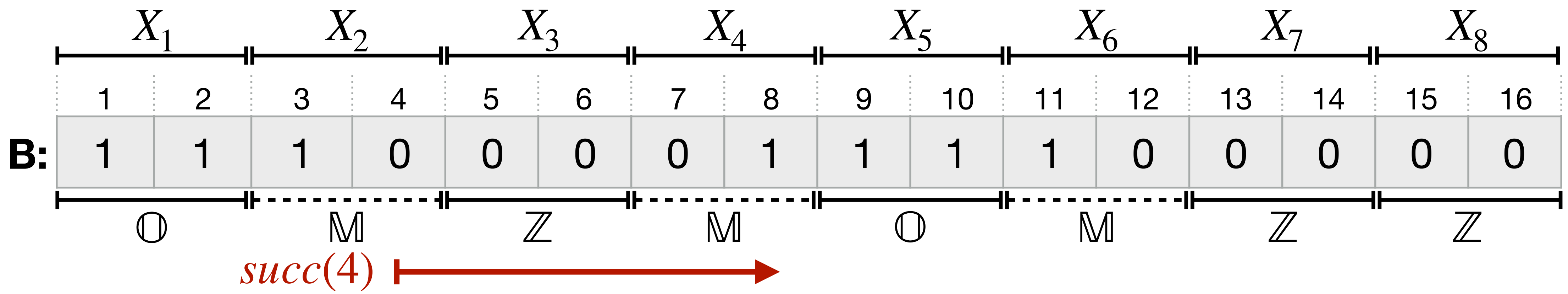
zombit-vector

	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

	1	2	3	4	5	6
M:	**1**	**0**	**0**	**1**	**1**	**0**

zombit-vector

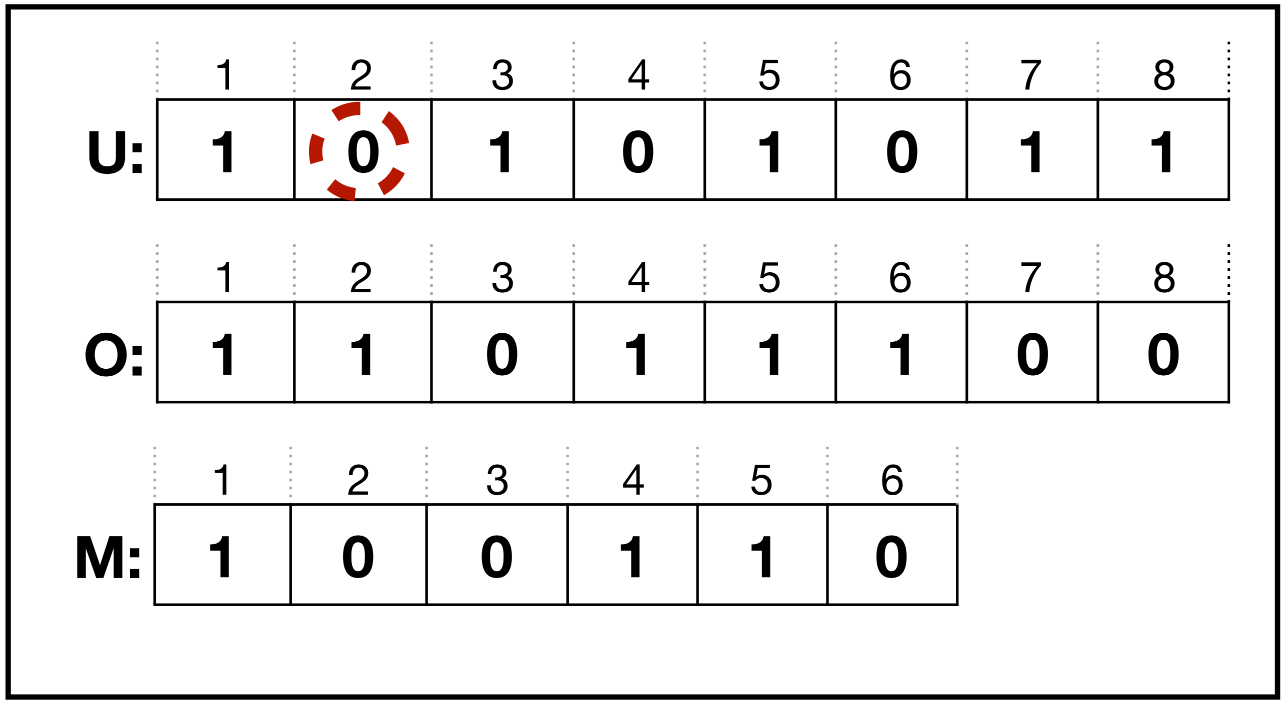


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
$\mathbb{O}$	$\mathbb{M}$	$\mathbb{Z}$	$\mathbb{M}$	$\mathbb{O}$	$\mathbb{M}$	$\mathbb{Z}$	$\mathbb{Z}$

$succ(4)$

zombit-vector

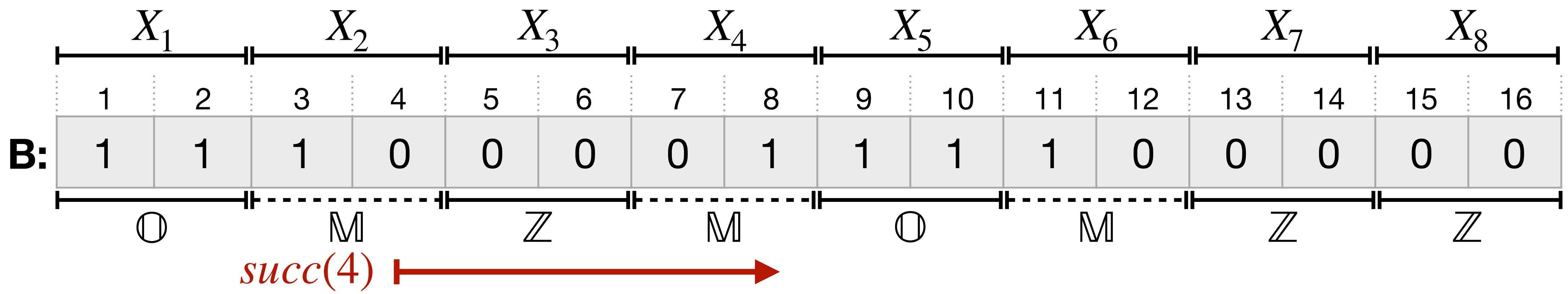


	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

	1	2	3	4	5	6
M:	**1**	**0**	**0**	**1**	**1**	**0**

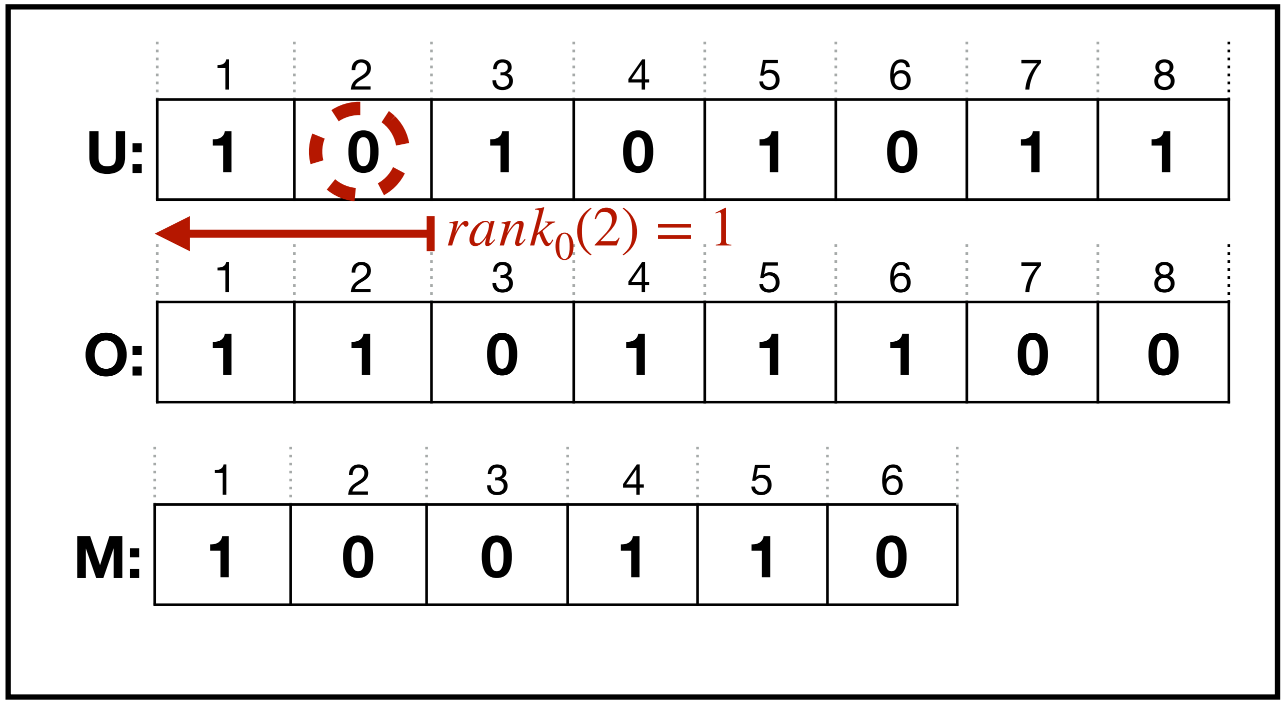
zombit-vector



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

$succ(4)$

zombit-vector



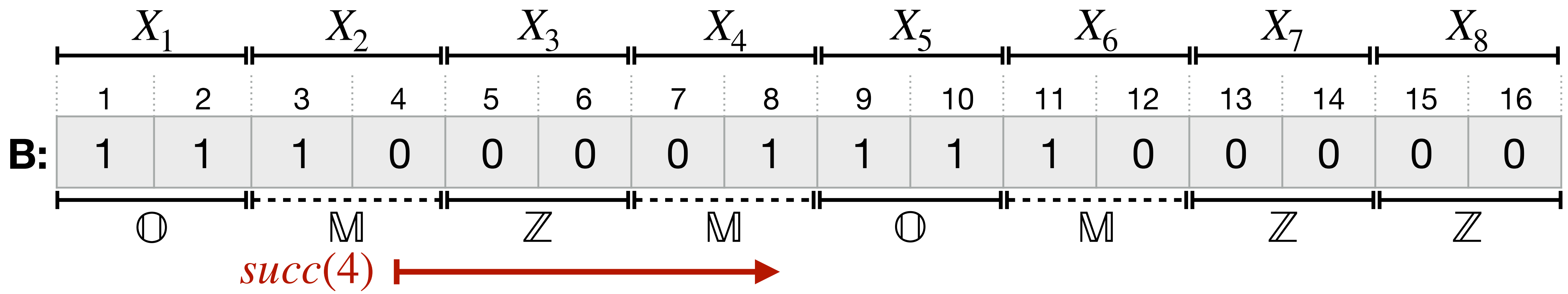
	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

$rank_0(2) = 1$

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

	1	2	3	4	5	6
M:	1	0	0	1	1	0

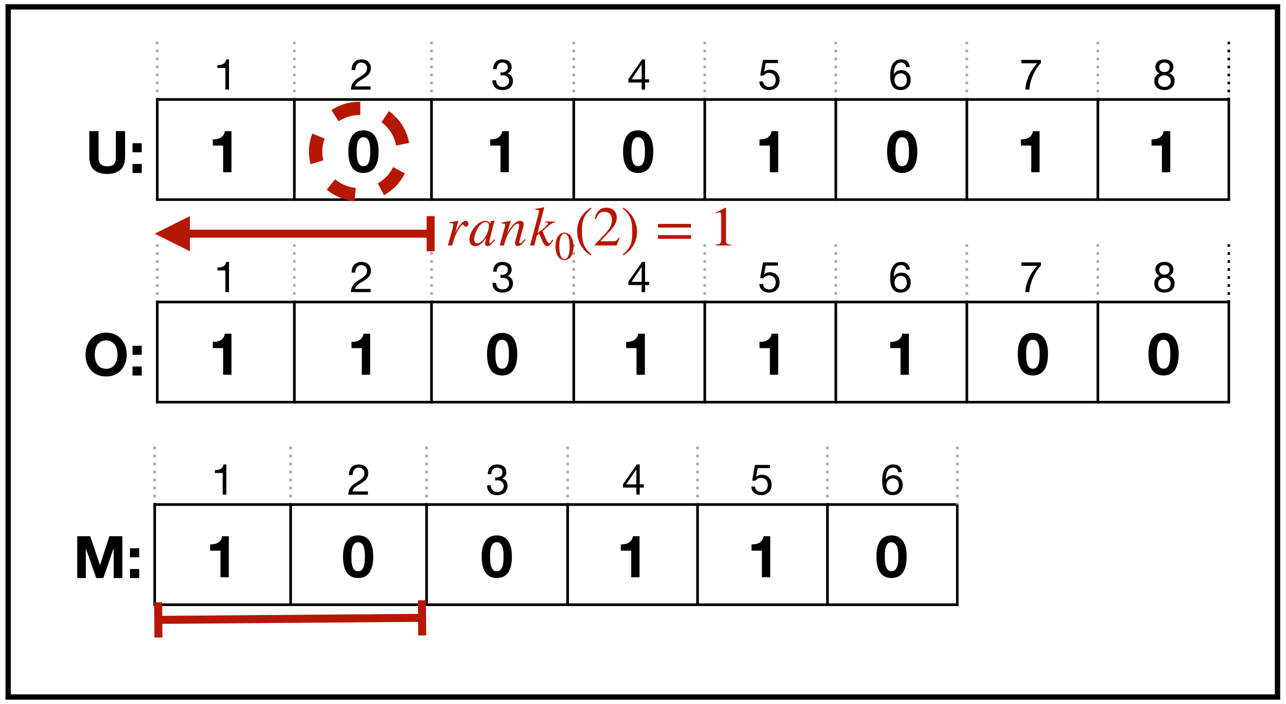
zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(4)$

zombit-vector



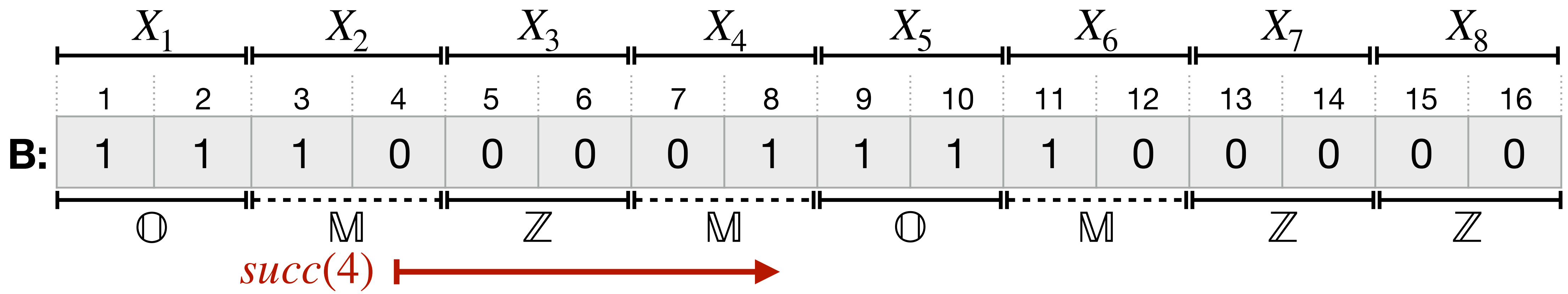
	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

$rank_0(2) = 1$

	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

	1	2	3	4	5	6
M:	**1**	**0**	**0**	**1**	**1**	**0**

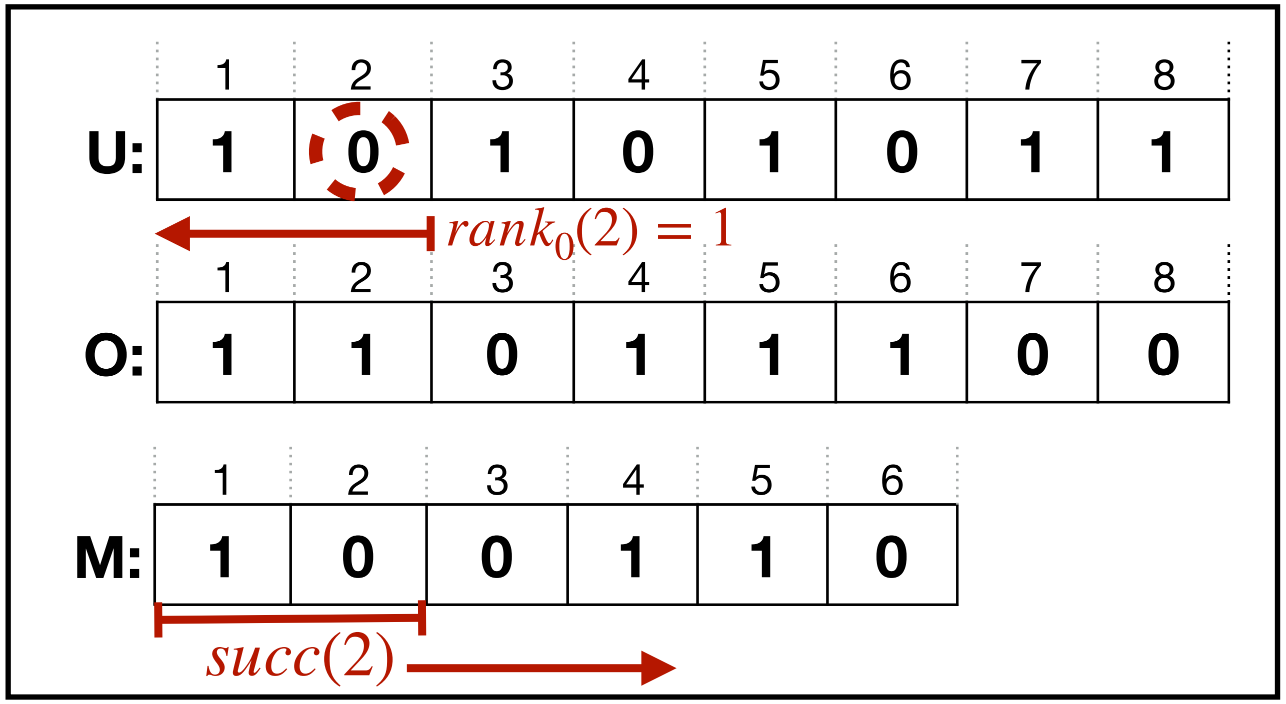
zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(4)$

zombit-vector



	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

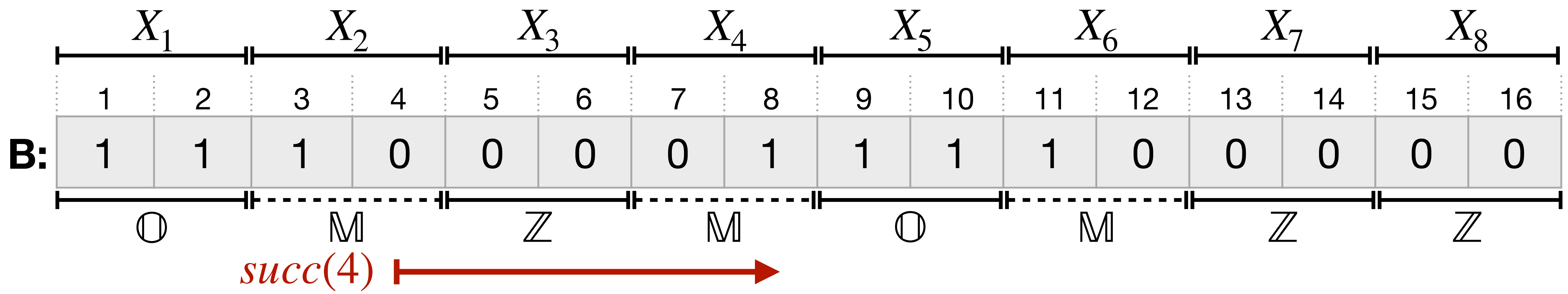
$rank_0(2) = 1$

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

	1	2	3	4	5	6
M:	1	0	0	1	1	0

$succ(2)$

zombit-vector

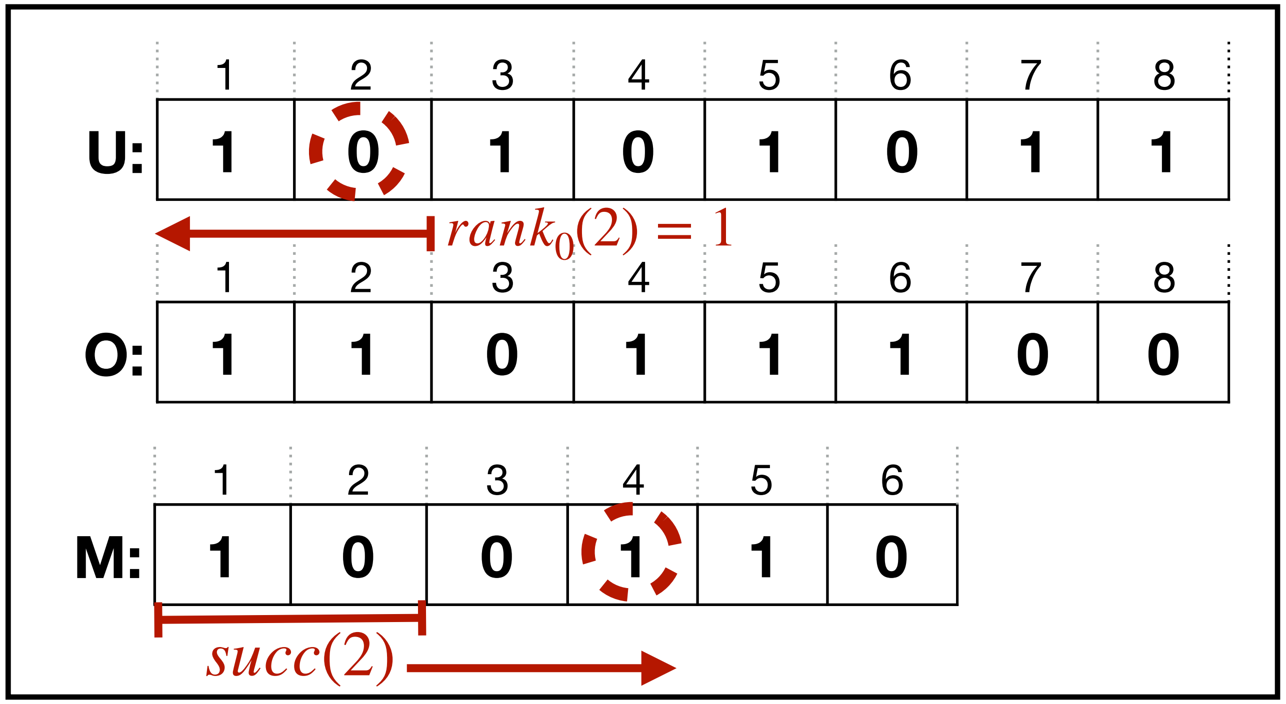


X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
1 2	3 4	5 6	7 8	9 10	11 12	13 14	15 16
1 1	1 0	0 0	0 1	1 1	1 0	0 0	0 0
𝕆	𝕄	ℤ	𝕄	𝕆	𝕄	ℤ	ℤ

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

$succ(4)$

zombit-vector



	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

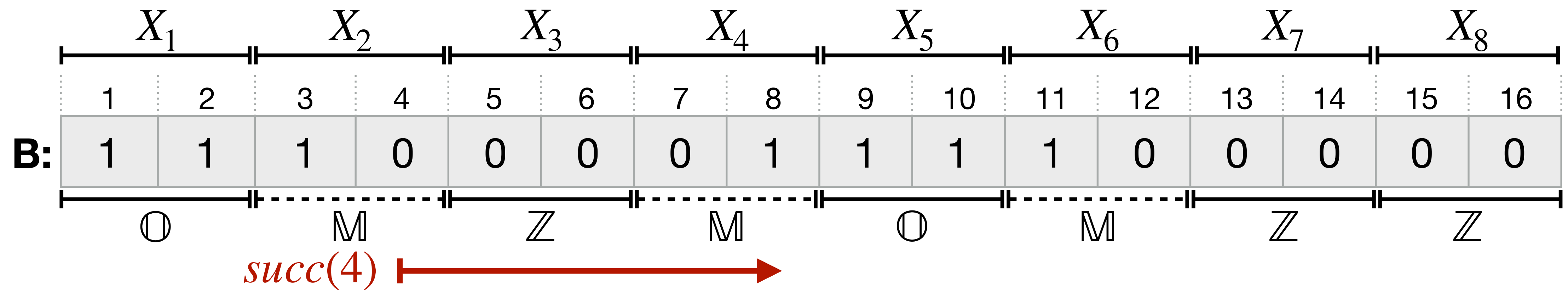
$rank_0(2) = 1$

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

	1	2	3	4	5	6
M:	1	0	0	1	1	0

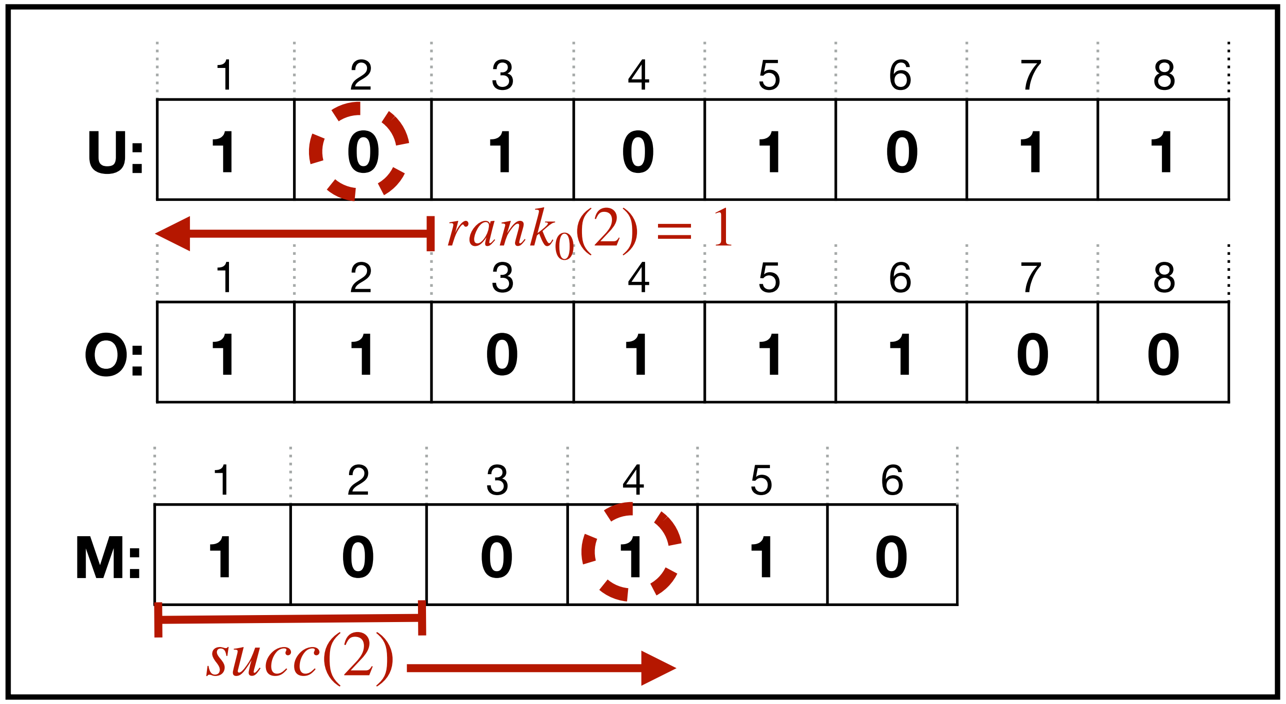
$succ(2)$

zombit-vector



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

zombit-vector

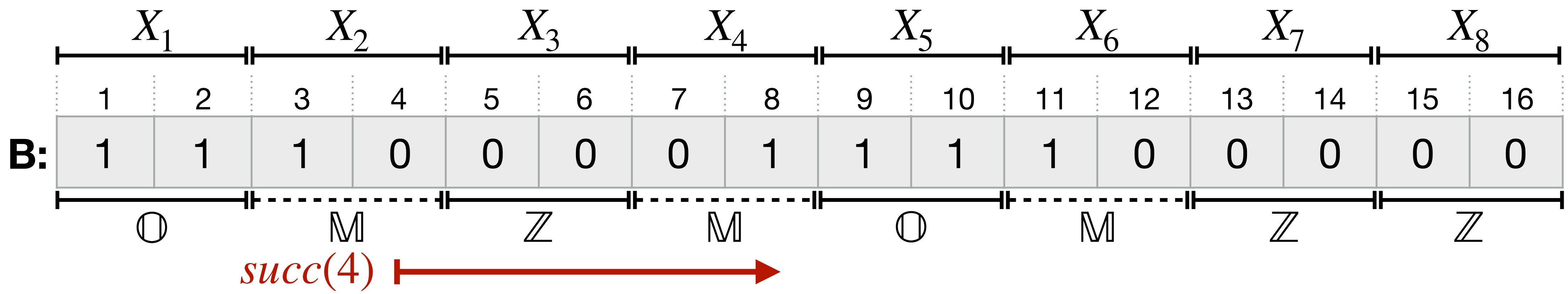


	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

	1	2	3	4	5	6
M:	**1**	**0**	**0**	**1**	**1**	**0**

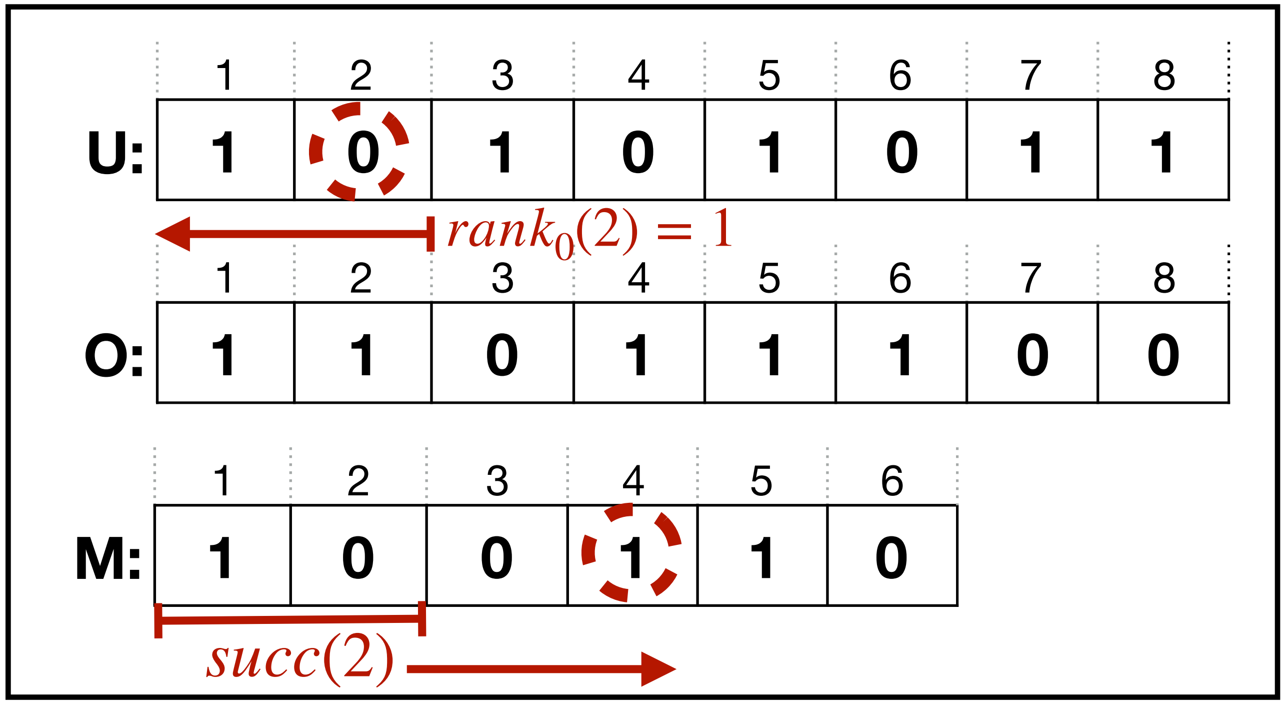
zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(4)$

zombit-vector



	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

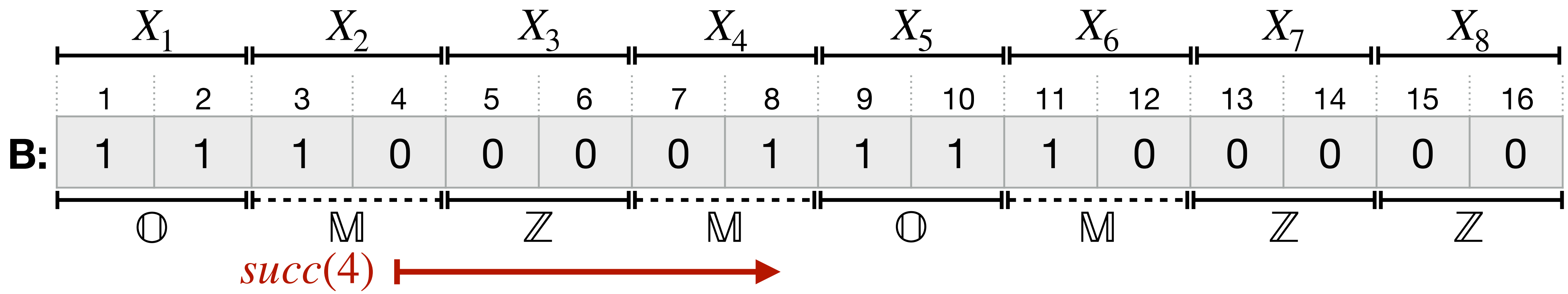
$rank_0(2) = 1$

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

	1	2	3	4	5	6
M:	1	0	0	1	1	0

$succ(2)$

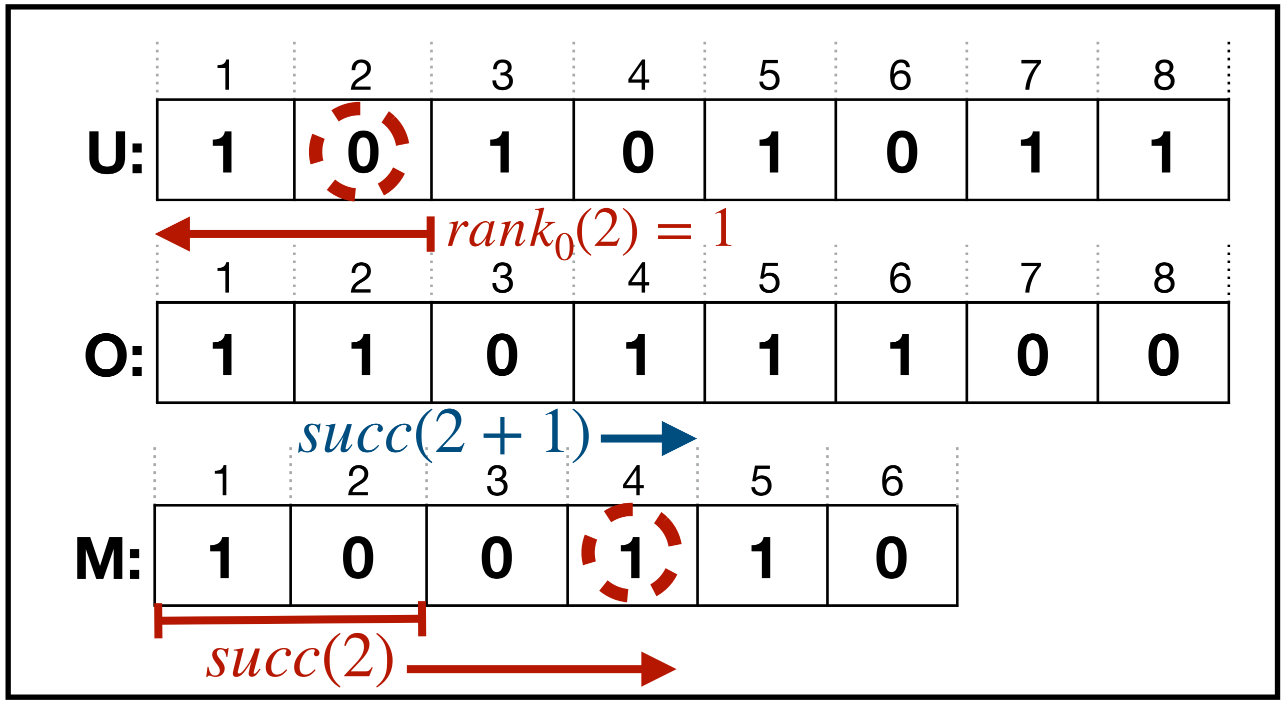
zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(4)$

zombit-vector



	1	2	3	4	5	6	7	8
U:	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**1**

$rank_0(2) = 1$

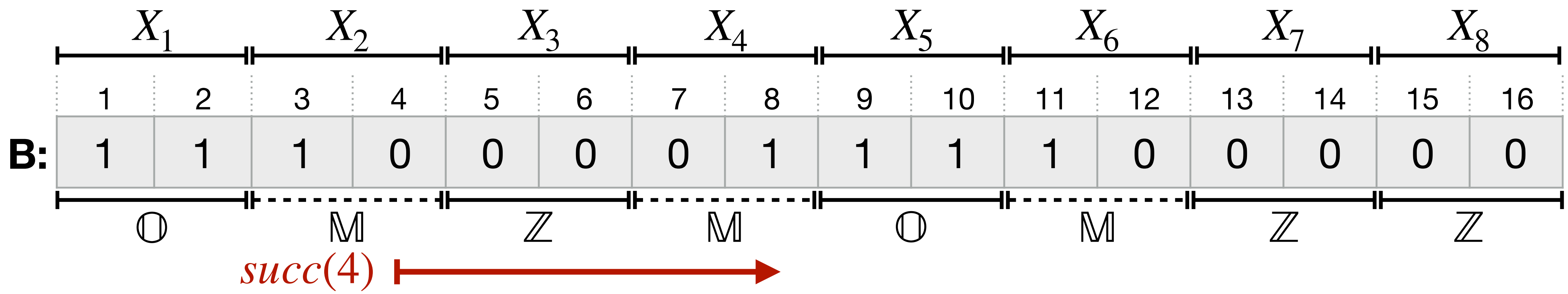
	1	2	3	4	5	6	7	8
O:	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**

$succ(2 + 1)$

	1	2	3	4	5	6
M:	**1**	**0**	**0**	**1**	**1**	**0**

$succ(2)$

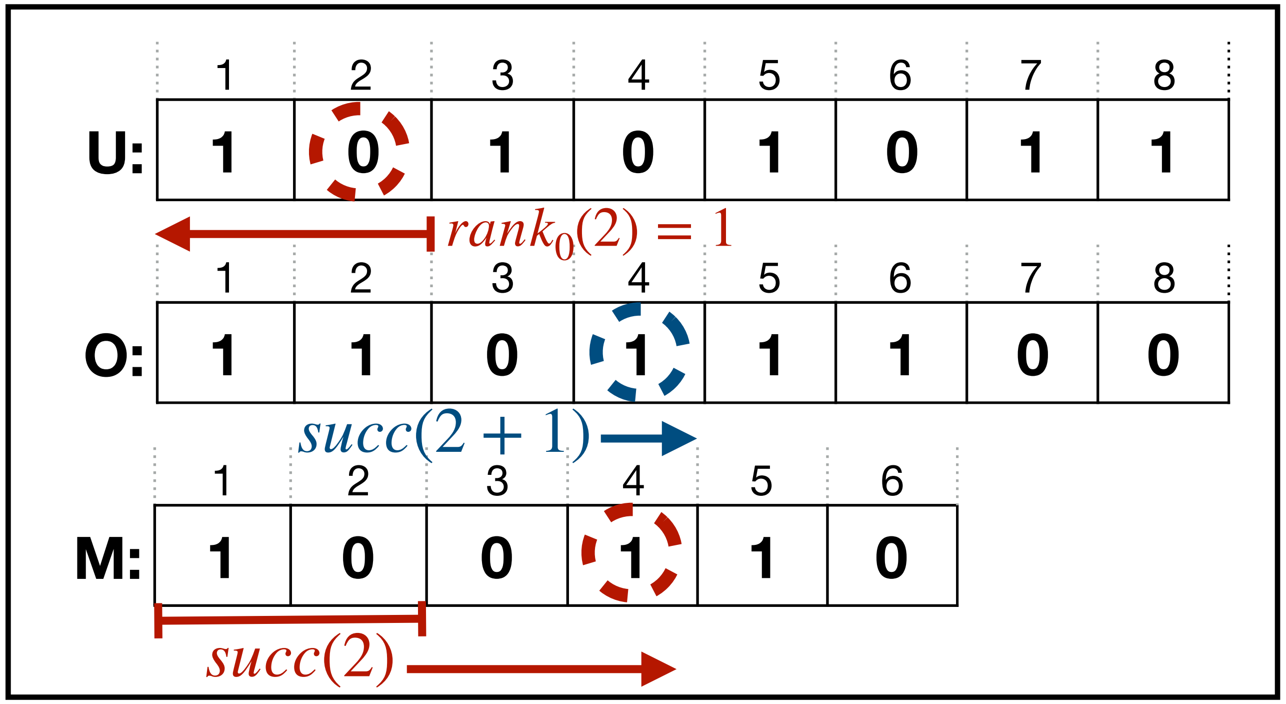
zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(4)$

zombit-vector



	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

$rank_0(2) = 1$

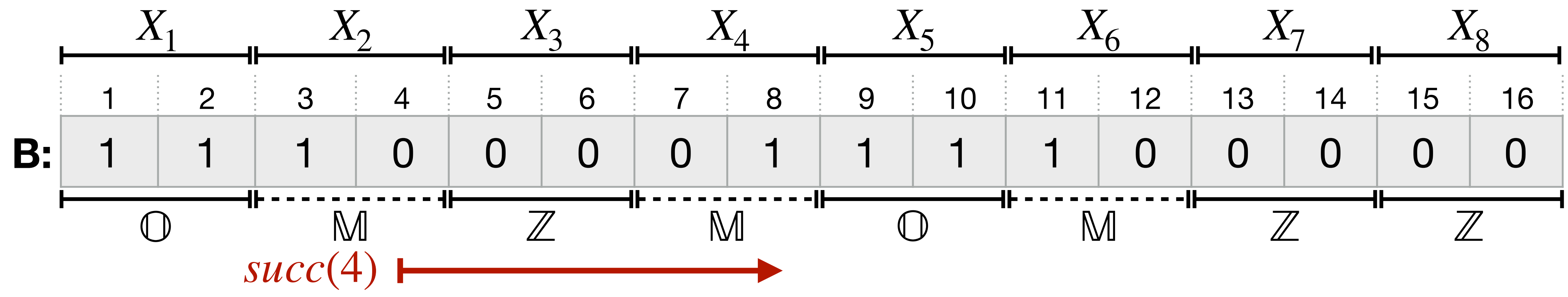
	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

$succ(2 + 1)$

	1	2	3	4	5	6
M:	1	0	0	1	1	0

$succ(2)$

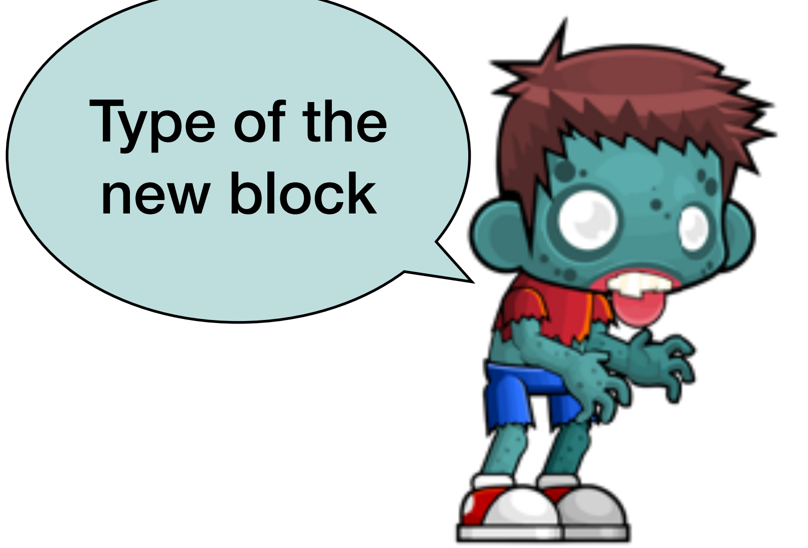
zombit-vector

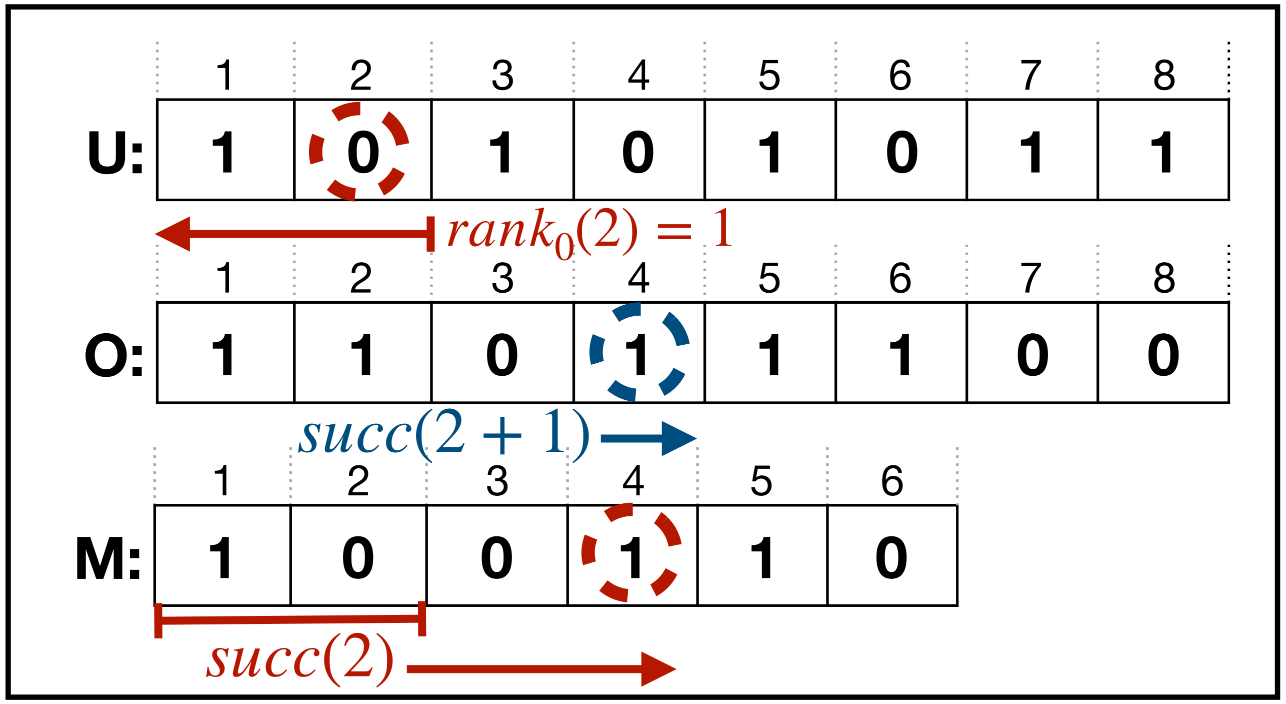


	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(4)$

zombit-vector





	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

$rank_0(2) = 1$

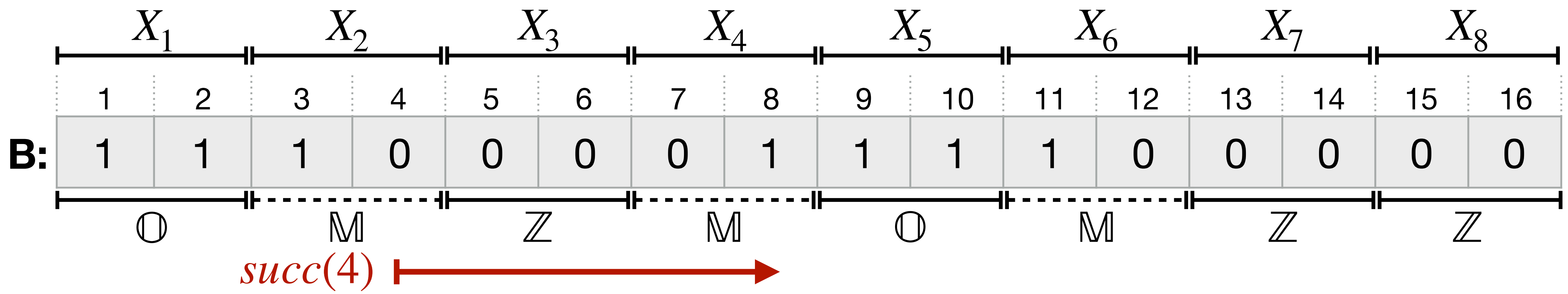
	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

$succ(2+1)$

	1	2	3	4	5	6
M:	1	0	0	1	1	0

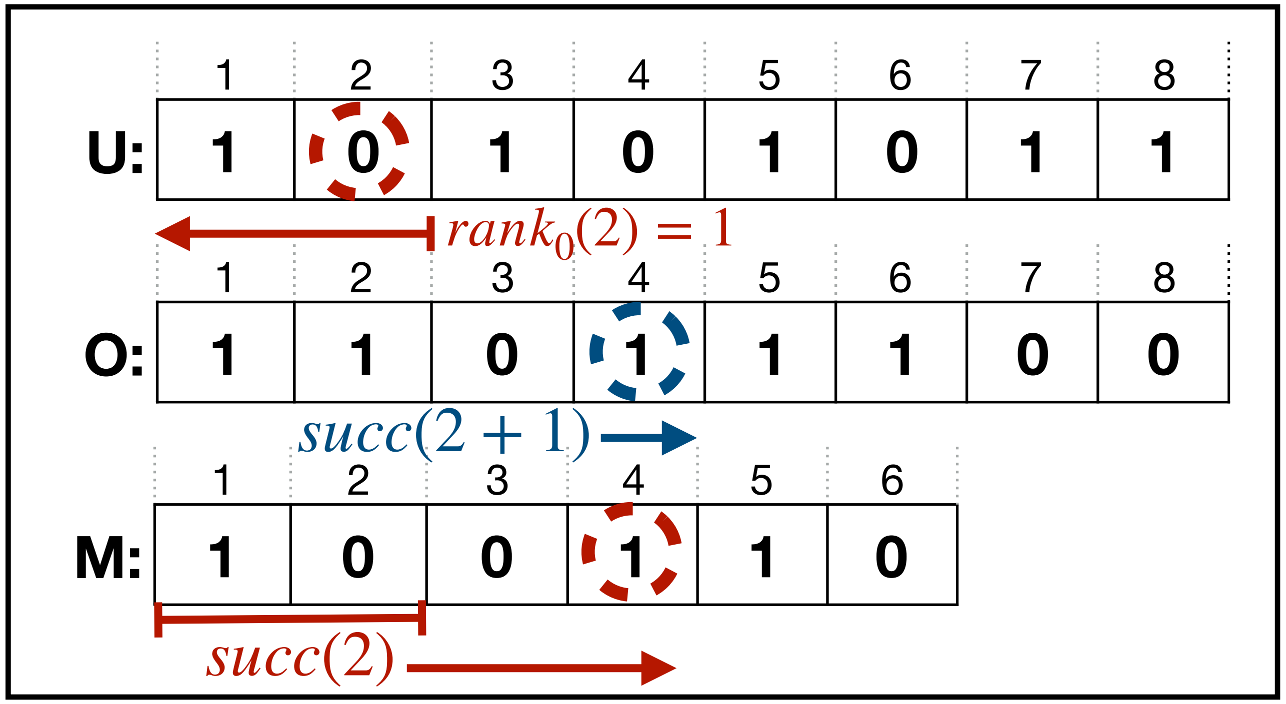
$succ(2)$

zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

zombit-vector

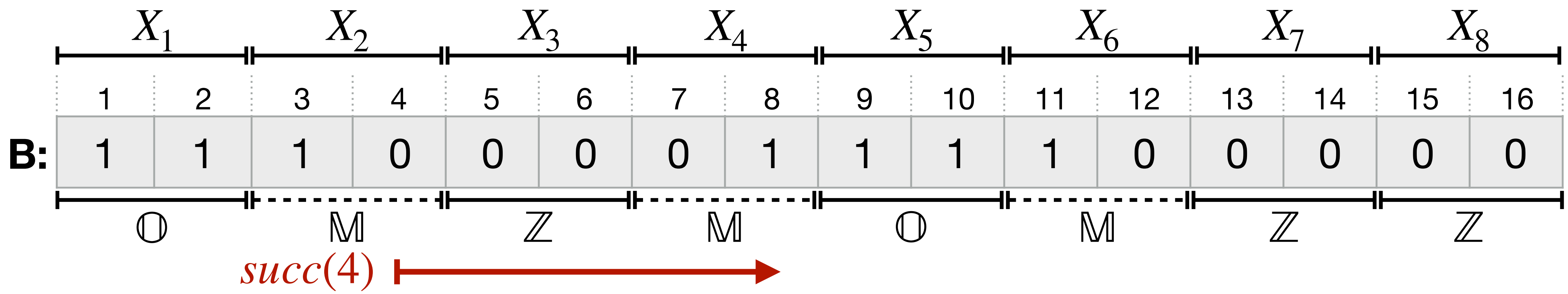


	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

	1	2	3	4	5	6
M:	1	0	0	1	1	0

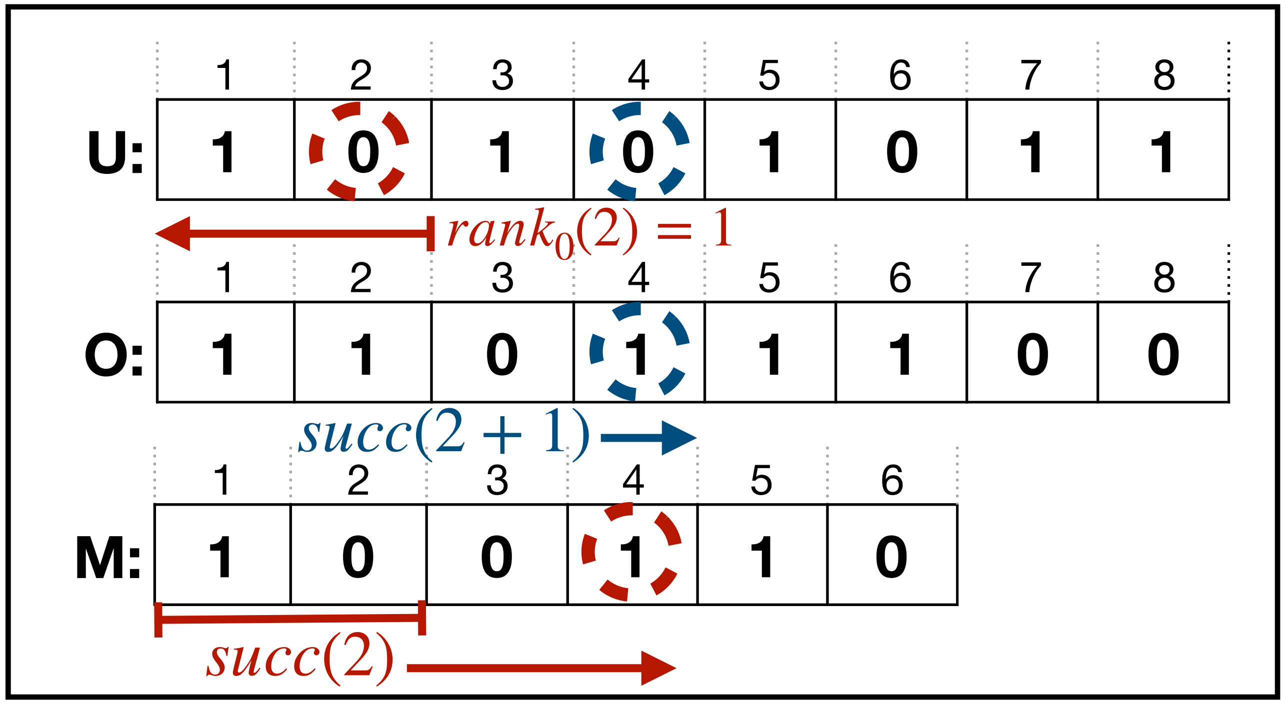
zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(4)$

zombit-vector



	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

$rank_0(2) = 1$

	1	2	3	4	5	6	7	8
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$succ(2 + 1)$

	1	2	3	4	5	6
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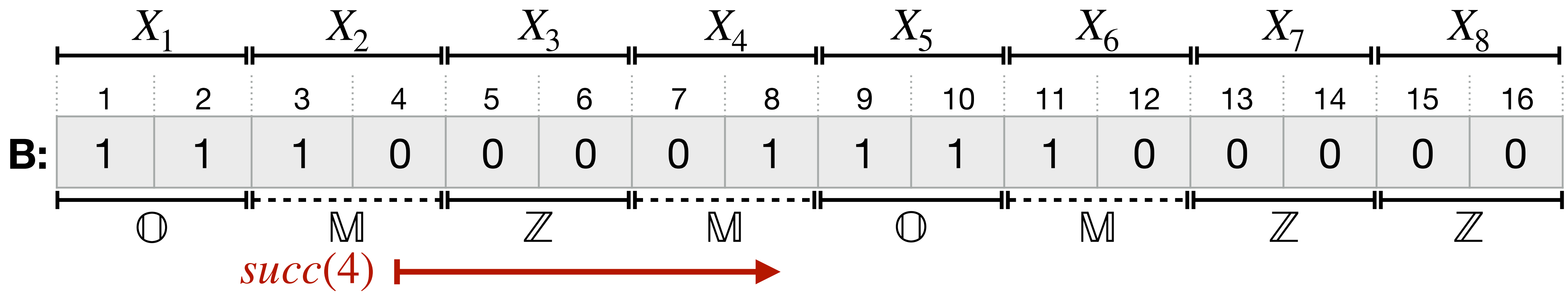
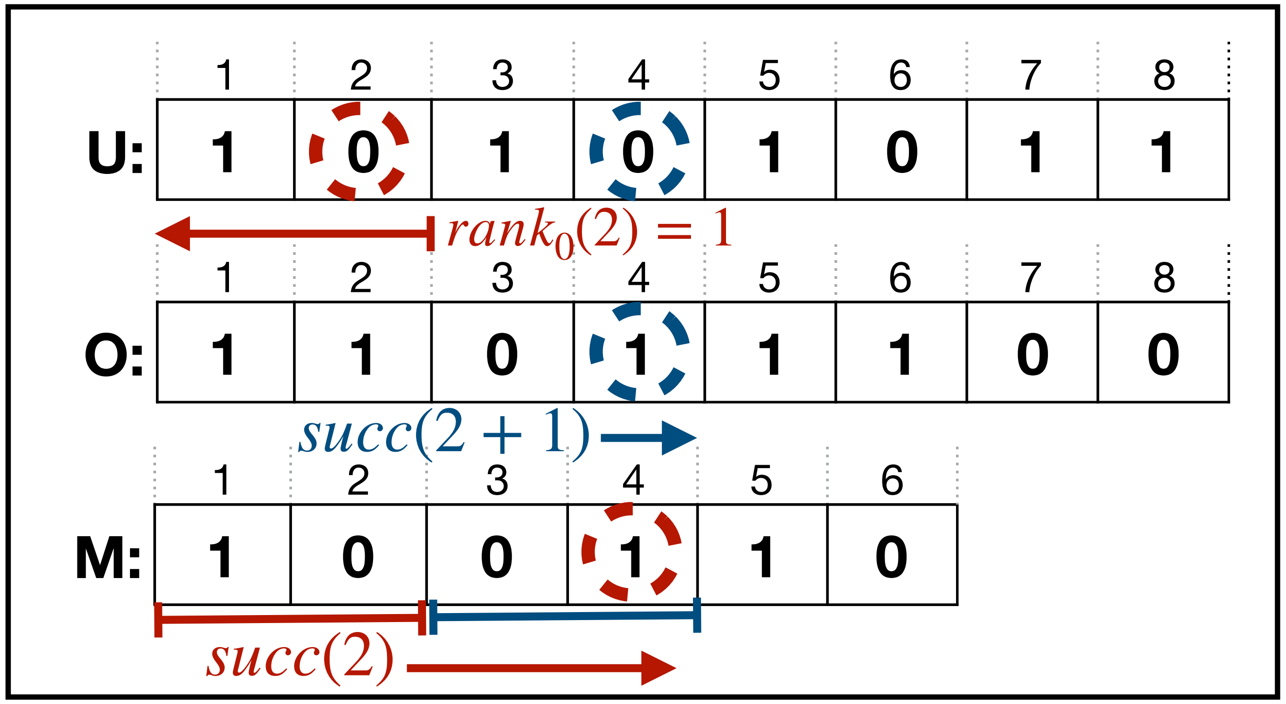
$succ(2)$

zombit-vector



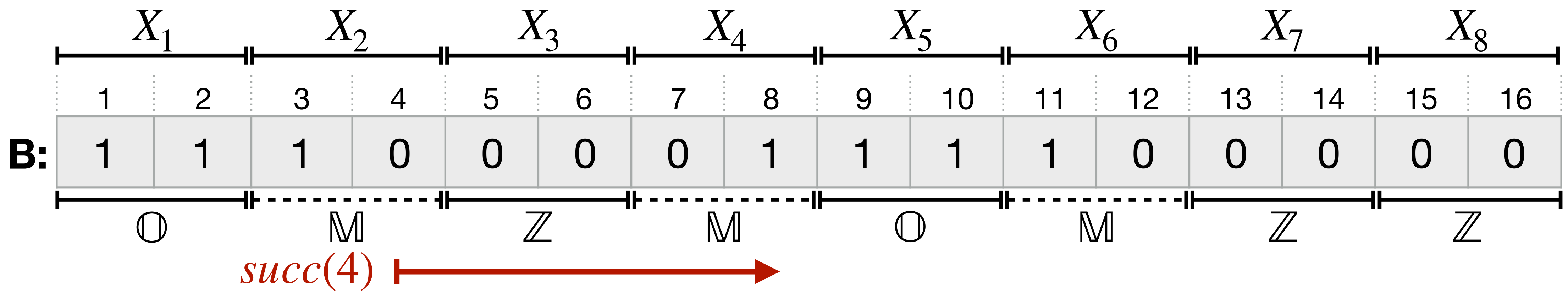
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

zombit-vector



	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1
O:	1	1	0	1	1	1	0	0
M:	1	0	0	1	1	0		

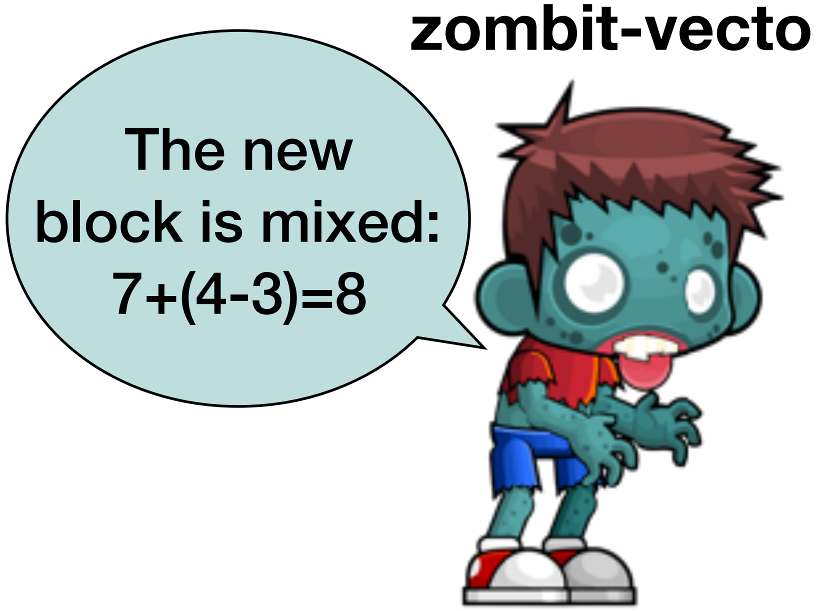
zombit-vector

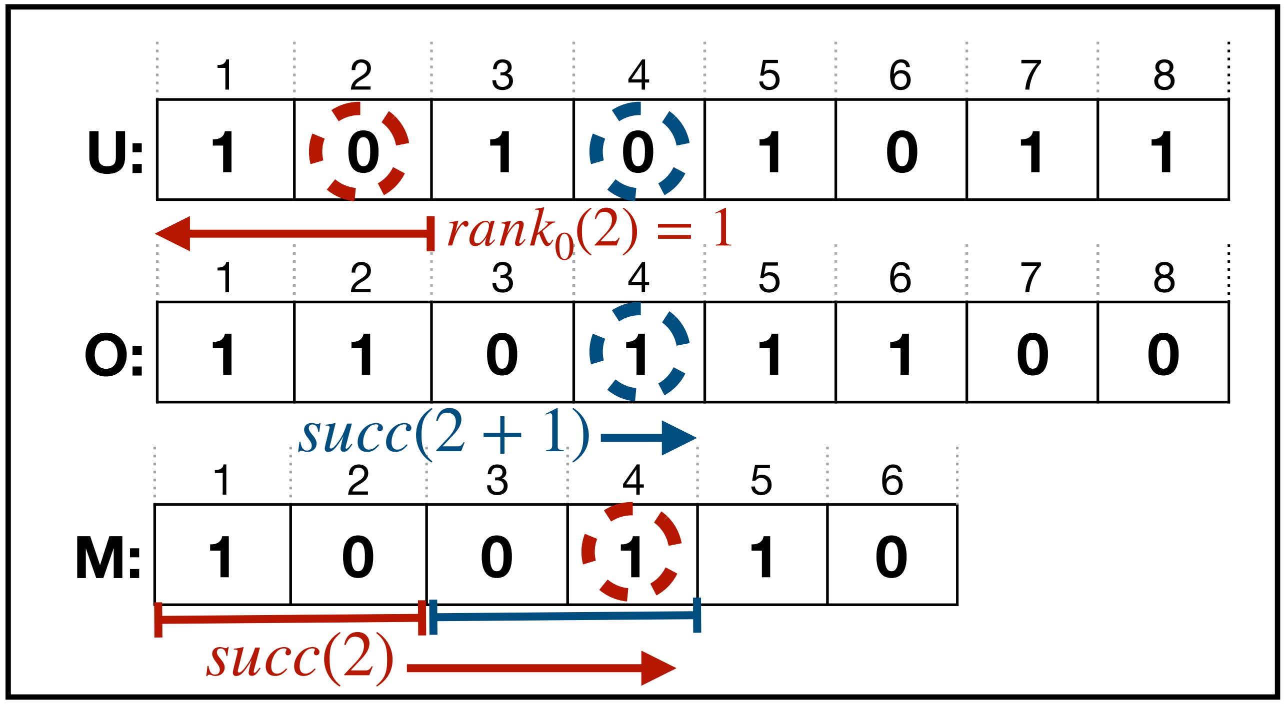


	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(4)$ ⟶

zombit-vector





	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

$rank_0(2) = 1$

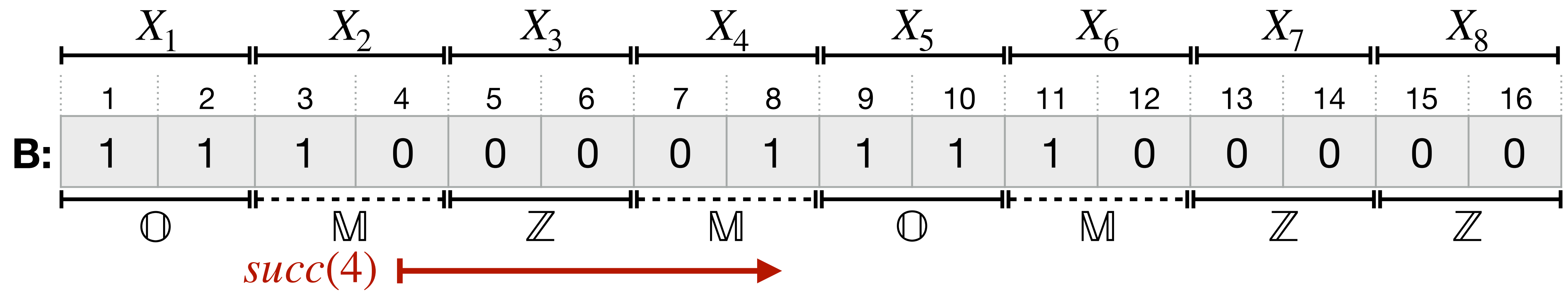
	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

$succ(2+1)$ ⟶

	1	2	3	4	5	6
M:	1	0	0	1	1	0

$succ(2)$ ⟶

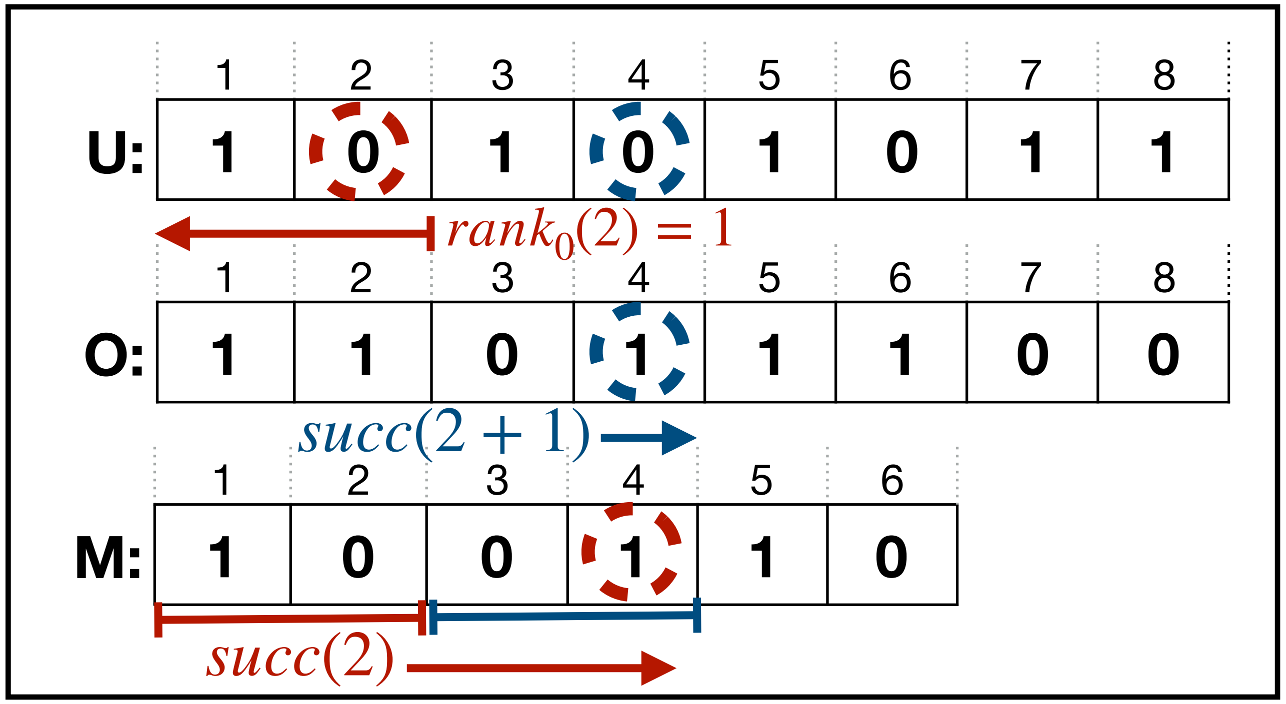
zombit-vector



	X_1		X_2		X_3		X_4		X_5		X_6		X_7		X_8	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B:	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
	$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{M}$		$\mathbb{O}$		$\mathbb{M}$		$\mathbb{Z}$		$\mathbb{Z}$	

$succ(4)$

zombit-vector



	1	2	3	4	5	6	7	8
U:	1	0	1	0	1	0	1	1

$rank_0(2) = 1$

	1	2	3	4	5	6	7	8
O:	1	1	0	1	1	1	0	0

$succ(2 + 1)$

	1	2	3	4	5	6
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$succ(2)$

zombit-vector

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zombit-vector

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- *zombit-rec* converges to **O(k)** bits and solves access, rank, succ and pred in **O(log(n/k))** time.

zombit-vector

Bitvector	Time	Space
plain	$O(1)$	$n + o(n)$
rrr	$O(1)$	$nH_0 + o(n)$
rec-rank	$O(\log \frac{n}{m})$	$\log \frac{n}{m} + m + o(n)$
sd-array	$O(\log \frac{n}{m})$	$m \log \frac{n}{m} + O(m)$
oz-vector	$O(\log k)$	$k \log \frac{2n}{k} + O(k)$
hybrid	$O(\log n)$	$min(k, m)\lceil \log b \rceil + o(n)$
zombit	$\mathbf{O(1)}$	$\mathbf{O(\sqrt{kn})}$
zombit-rec	$\mathbf{O(\log \frac{n}{k})}$	$\mathbf{O(k)}$

Data
Compression
Conference

Outline

- ☑ Introduction
- ☑ Background
- ☑ zombit-vector
- ☐ **Experimental evaluation**
- ☐ Conclusions
- ☐ Future work

Experimental evaluation

- We evaluate the behavior of *zombit* and *zombit-rec* in three queries: **successor**, **access**, and **rank**.

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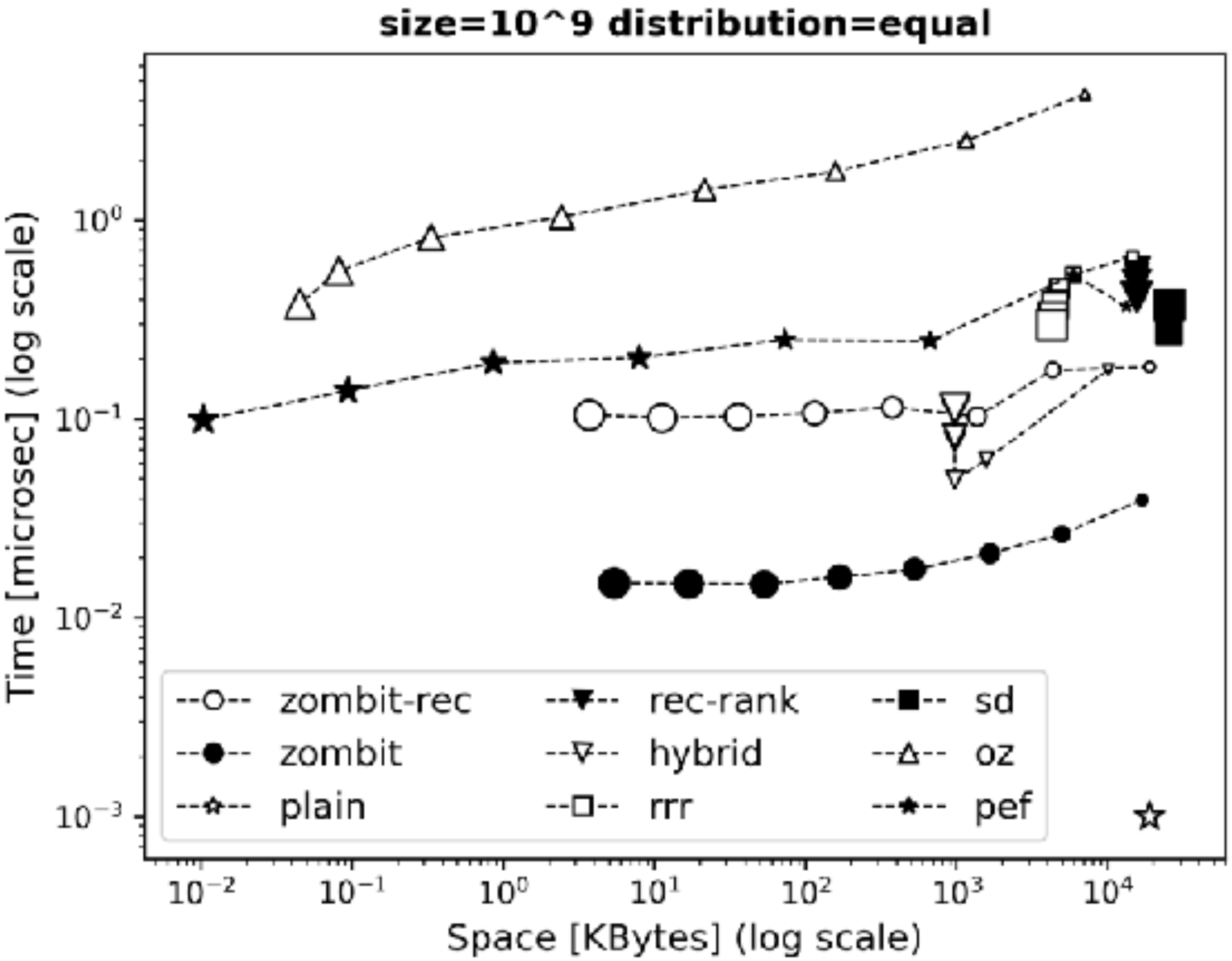
- Synthetic bitvectors with different sizes **10^9**, **10^8** and **10^7** with different **lengths of runs**.

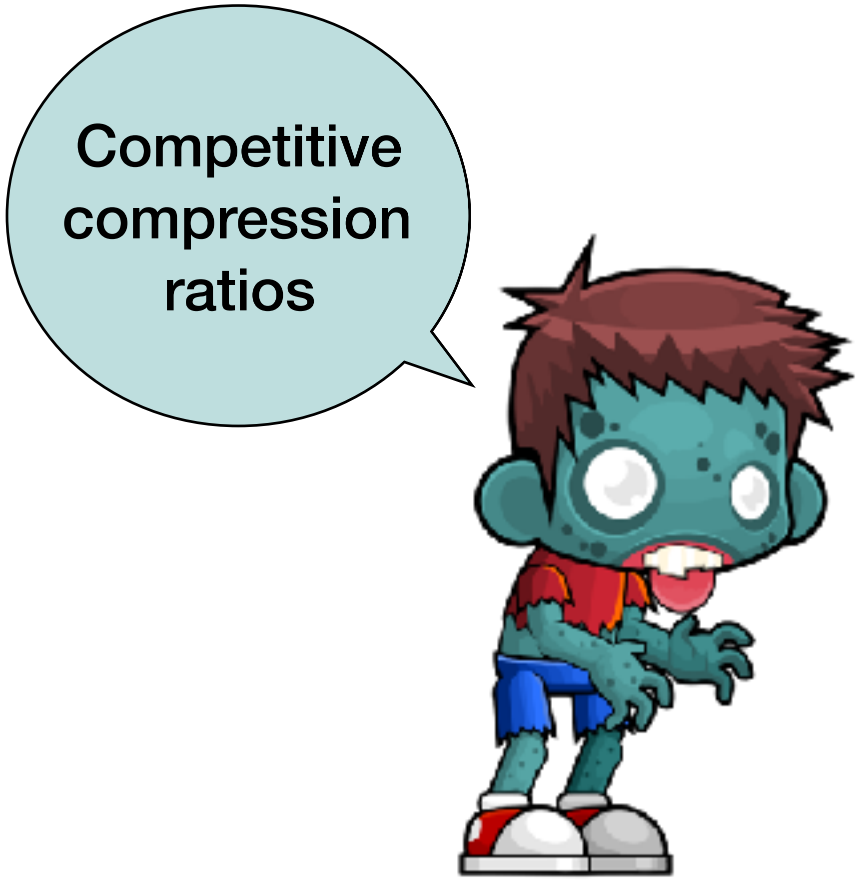
Experimental evaluation

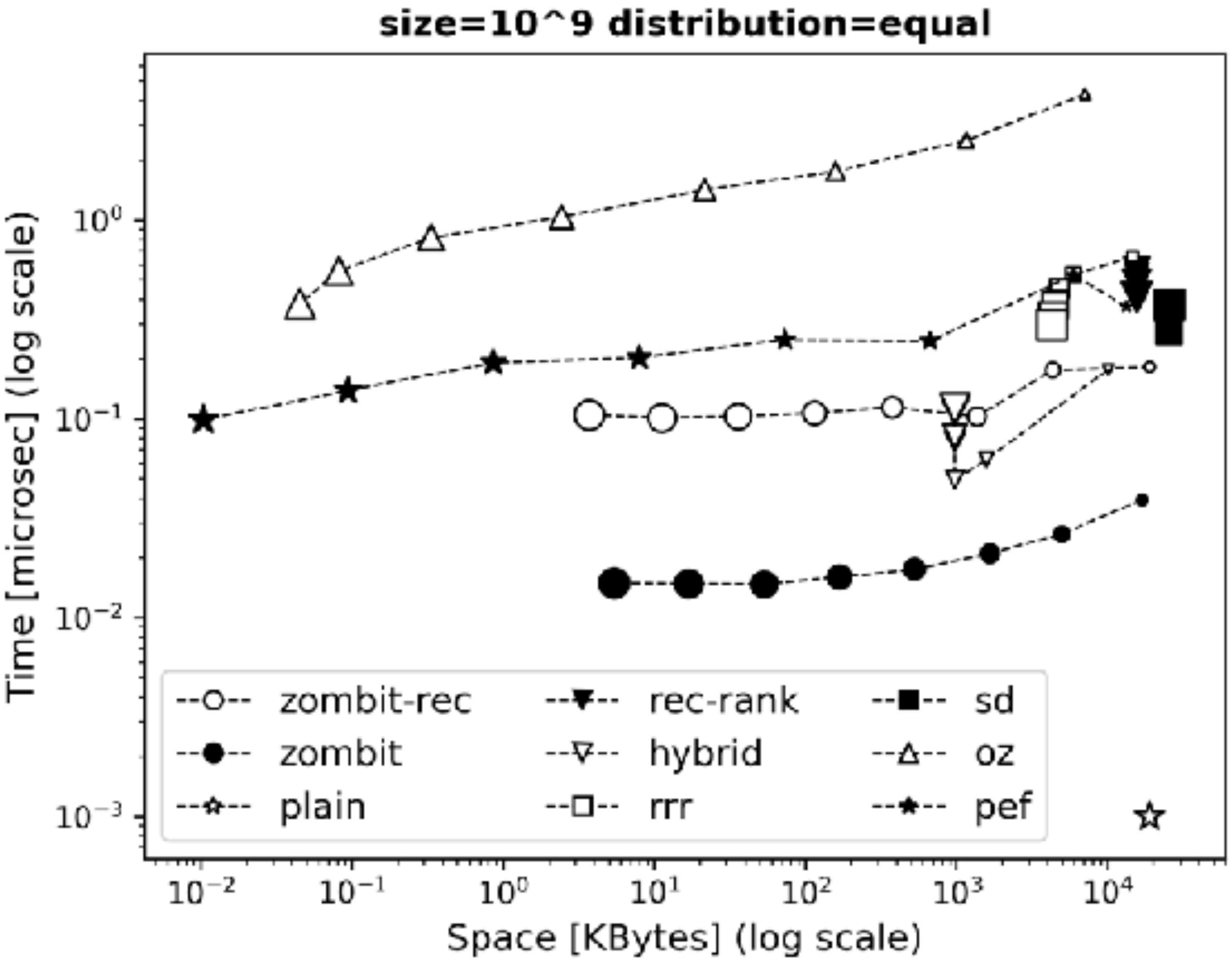
- **Successor operation**



Experimental evaluation

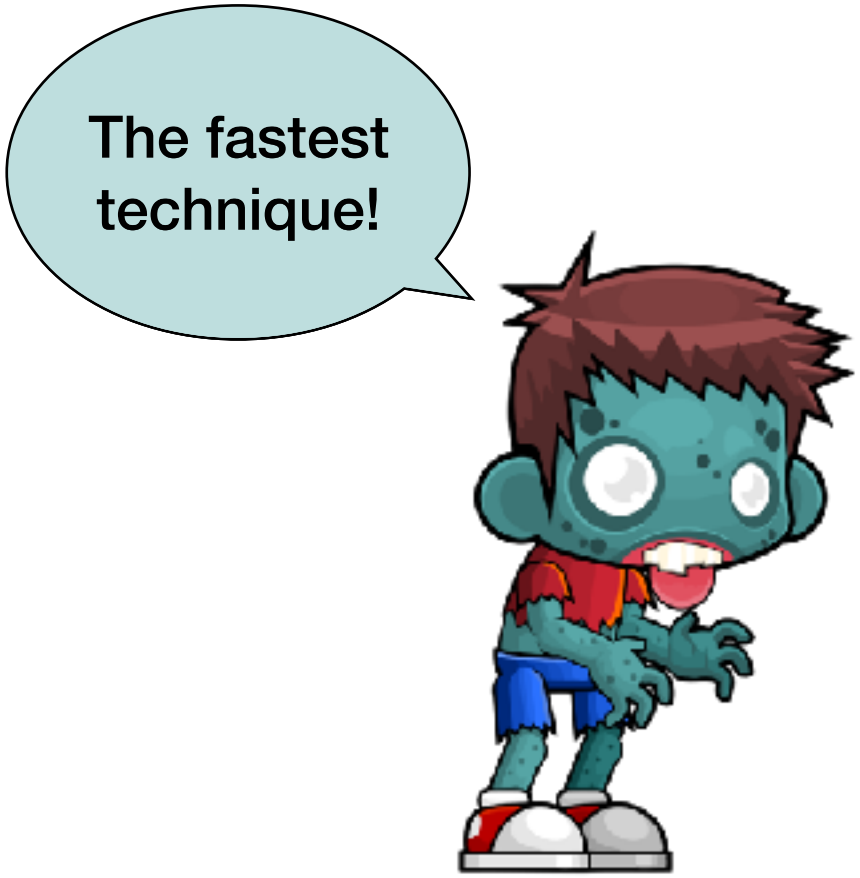
- **Successor operation**

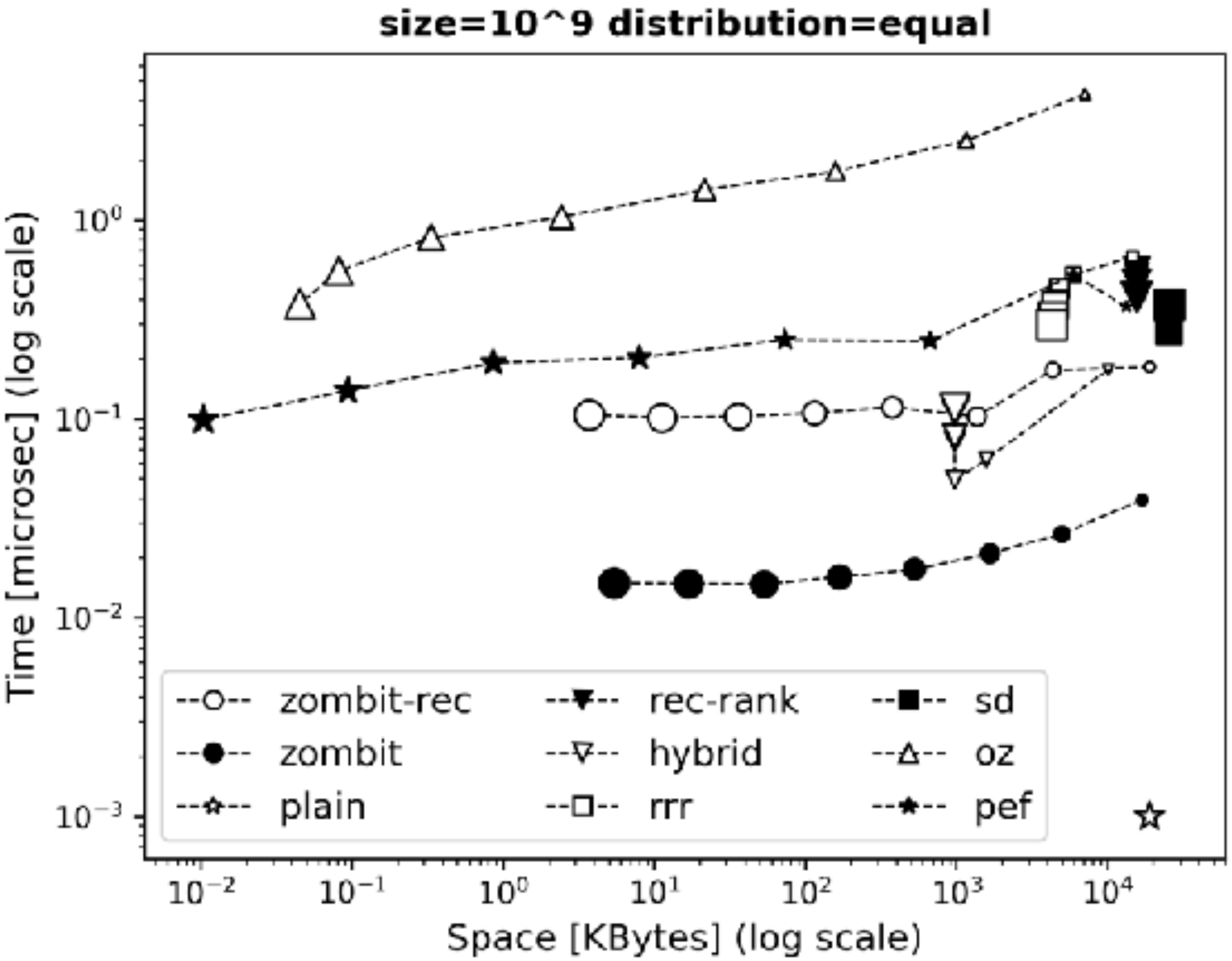

Competitive compression ratios



Experimental evaluation

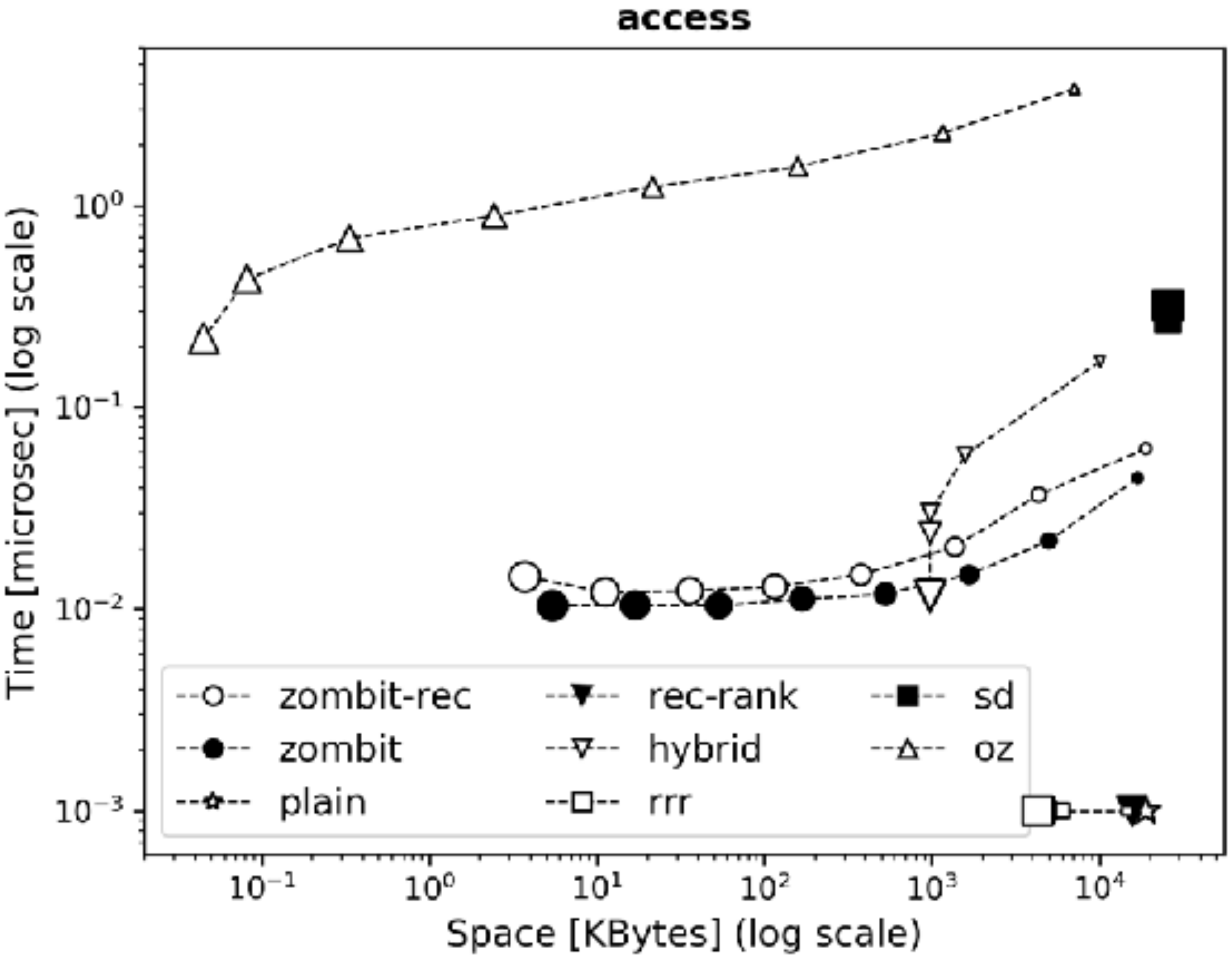
- **Successor operation**





Experimental evaluation

- **Access operation**



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Experimental evaluation

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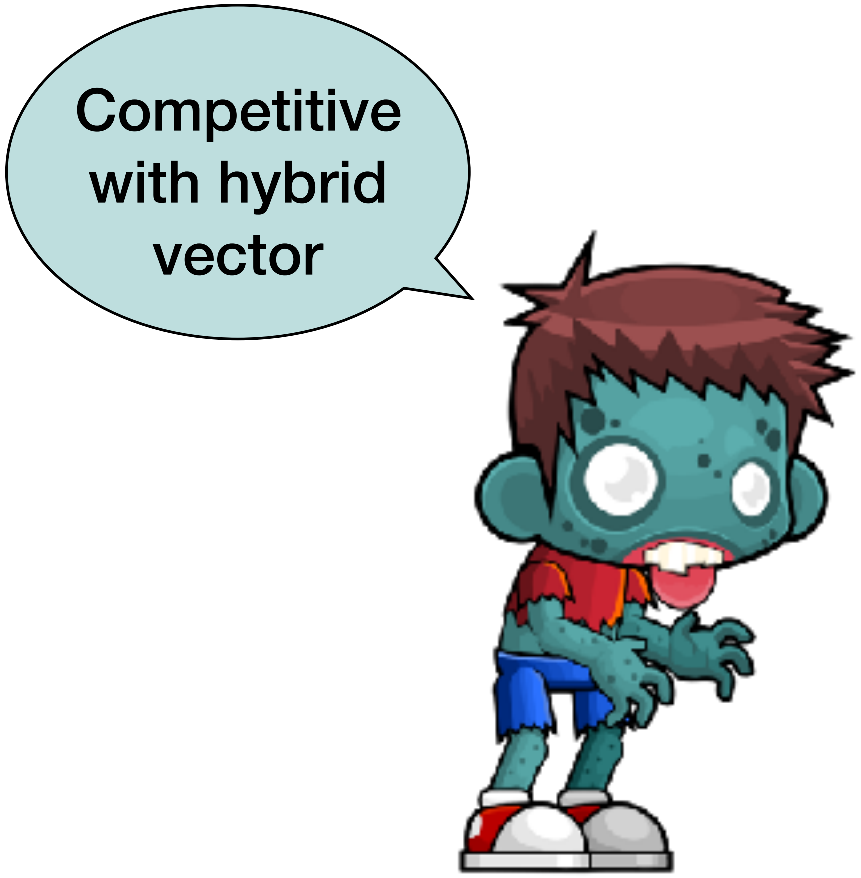

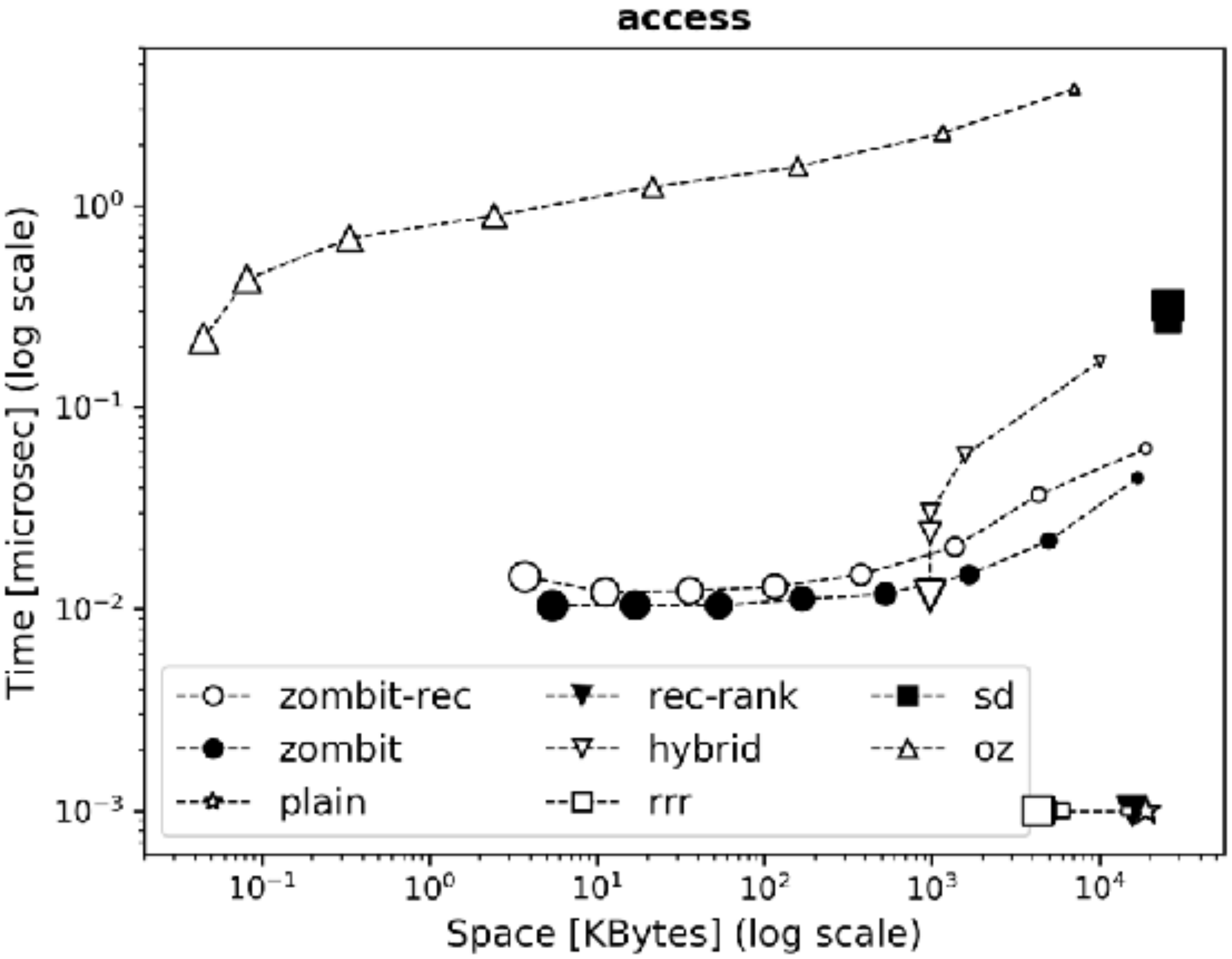

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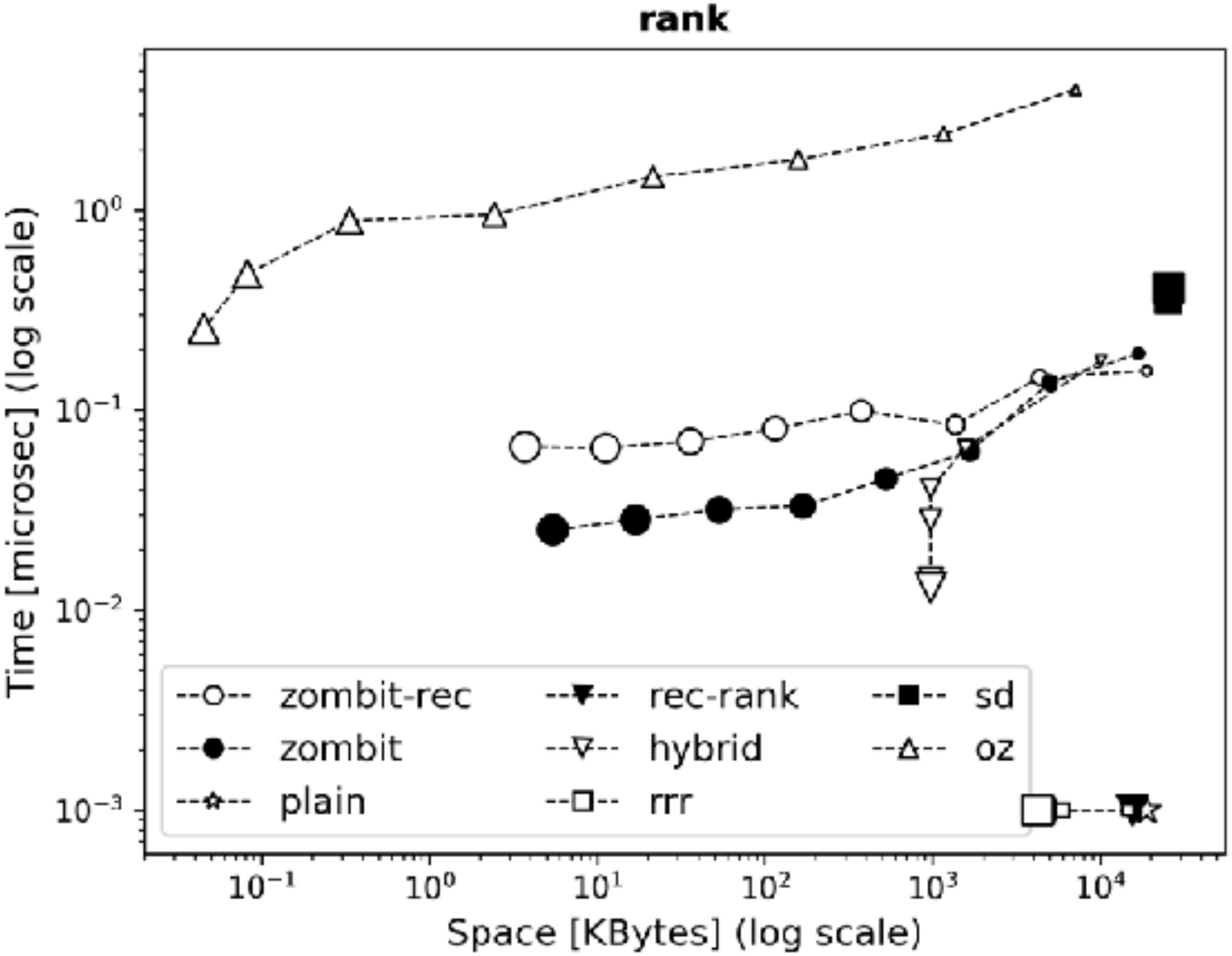
Experimental evaluation

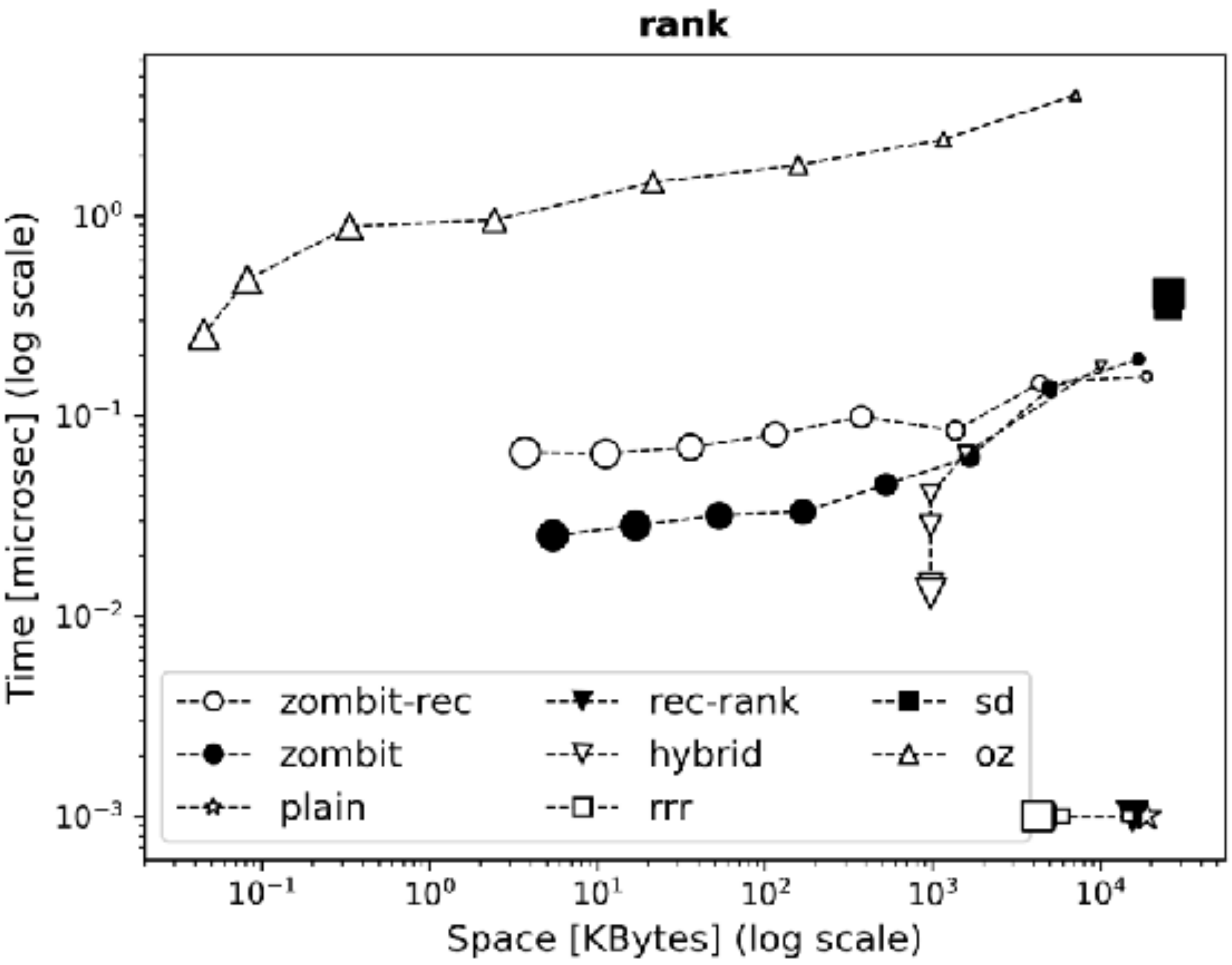
- **Rank operation**



Experimental evaluation

- **Rank operation**

Slower than hybrid vector



Outline

- ☑ Introduction
- ☑ Background
- ☑ zombit-vector
- ☑ Experimental evaluation
- ☐ **Conclusions**
- ☐ Future work

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times faster

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- Solving **select** operation with *o(n)* bits of extra-space.

Questions

Bitvectors with runs and the successor/predecessor problem

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