DICTIONARY LEARNING FOR POISSON COMPRESSED SENSING

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Introduction

Compressed Sensing: Widely used paradigm to sense sparse signals using few incoherent observations

- Imaging involves Counting of Photons Poisson Distributed
- Poisson Noise Signal Dependent Non-Additive
- Reconstruction of data from Poisson corrupted compressed measurements is challenging

→ Dictionary Learning based reconstruction algorithm to handle Poisson statistics

Proposed simple and efficient algorithm to infer a sparsifying dictionary and the dictionary coefficients **in situ** from **Poisson-corrupted compressive measurements** in imaging instead of using standard bases like wavelets or DCT.

Problem Formulation

Noise free image: $\mathbf{X} \in \mathbb{Z}^{N_1 \times N_2}$

Division of image into patches of size n_1 x n_2.

Vectorise i[th] patch into $\mathbf{x}_i \in \mathbb{Z}^{n \times 1}$ where $n \triangleq n_1 n_2$.

Compressive sensor acquires m≪n measurements of each patch to produce $\mathbf{y}_i \in \mathbb{Z}^m$. Each measurement is Poisson corrupted $\forall i, 1 \le i \le N_p, \mathbf{y}_i \sim \text{Poisson}(\mathbf{\Phi}_i \mathbf{x}_i)$ where $\mathbf{\Phi}_i$ is forward model matrix of measuring device for i[th] patch.

Assume $\mathbf{x_i}$ to be sparse in some dictionary D.
Task: To infer dictionary D and sparse coefficients $\mathbf{s_i}$ from $\mathbf{y_i}$ with non negativity constraint on both D and $\mathbf{s_i}$ as the data itself is non-negative.

$$\mathbf{y}_i \sim \text{Poisson}(\mathbf{\Phi}_i \mathbf{D} \mathbf{s}_i) \text{ s.t. } \mathbf{D} \succeq \mathbf{0}^{n \times K}; \forall i, \mathbf{s}_i \succeq \mathbf{0}^{K \times 1}$$

We seek to minimise the negative log-likelihood function with sparsity promoting term and the objective function is as follows:

$$\mathcal{J}(\mathbf{D}, \mathbf{s}) = \min_{\mathbf{D}, \mathbf{s}} \left(-\mathbf{y}_{ij} \log((\mathbf{\Phi}_i \mathbf{D} \mathbf{s}_i)_j) + (\mathbf{\Phi}_i \mathbf{D} \mathbf{s}_i)_j \right) + \lambda \sum_{i=1}^{N_p} \|\mathbf{s_i}\|_1$$

subject to $\mathbf{D} \succeq \mathbf{0}^{n \times K}$ and $\forall i, \mathbf{s}_i \succeq \mathbf{0}^{K \times 1}$

Algorithm

Algorithm 1 Algorithm for reconstruction in Poisson CS

Input: Poisson corrupted data, $\mathbf{y}_i \sim \text{Poisson}(\mathbf{\Phi}_i \mathbf{D} \mathbf{s}_i), 1 \le i \le N_p$

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Set k = 0. Randomly initialize D^(0) and s^(0) with appropriate sizes and non-negative entries. Set the ℓ2 norm of all the columns of D to 1.
repeat
    while J(D^(k+1), s^(k)) > J(D^(k), s^(k)) do
        D̃ = D^(k) − α1 ∂J/∂D |_{D=D^(k)}
        Set all negative entries in D̃ equal to zero
        Rescale each column of D̃ to unit norm.
        Set D^(k+1) = D̃
        α1 ← ηα1
    end while
    while J(D^(k+1), s^(k+1)) > J(D^(k+1), s^(k)) do
        s̃ = s^(k) − α2 ∂J/∂s |_{s=s^(k)}
        Set all negative entries in s̃ equal to zero
        Set s^(k+1) = s̃
        α2 ← ηα2
    end while
    k = k + 1
until stopping criteria is met
```

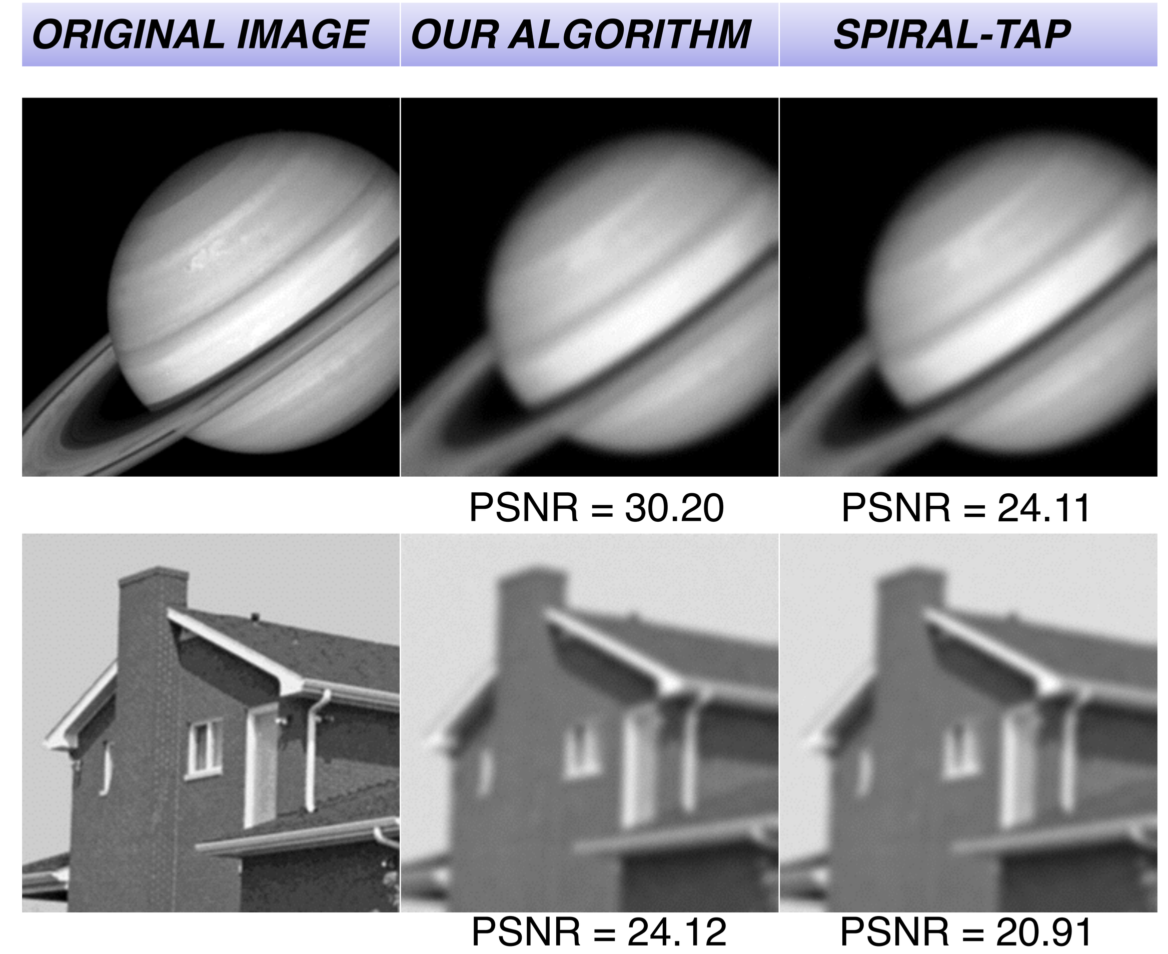
Experiments

- Patch based compressed sensing, size 7 x 7
- Different peak intensities to simulate low light scenarios, {3, 10}
- No of dictionary columns: 100
- Lambda in {0.1, 1, 5, 10, 20}
- Compression ratio m/n in {0.2, 0.5, 0.8}
- No of iterations: 100

- Comparison with SPIRAL-TAP using fixed bases such as 2-D DCT or Haar Wavelet for grayscale image reconstruction.

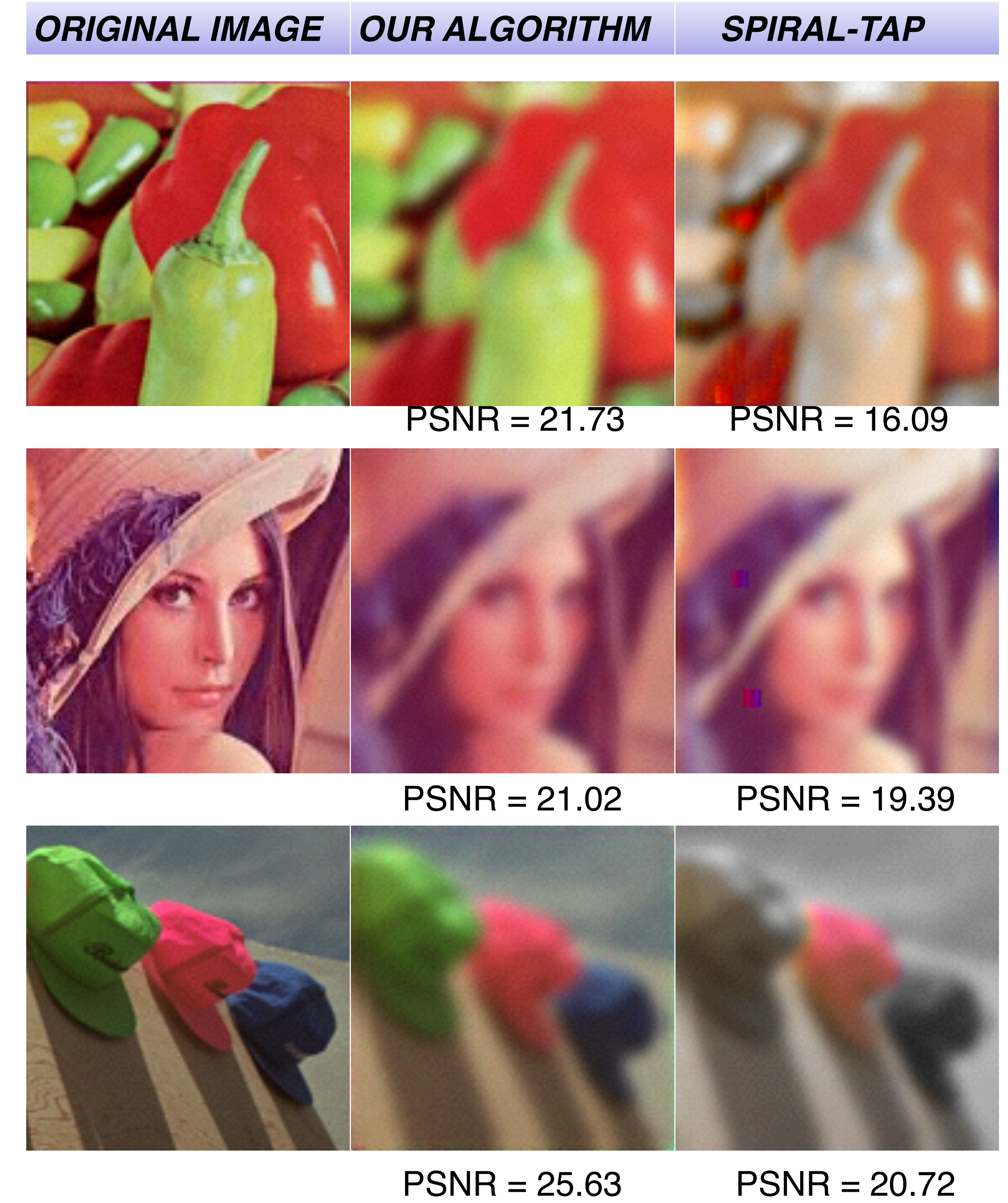
- Both the algorithms were iteratively run for 100 iterations and the lambda which gave highest PSNR was chosen as optimum reconstruction

Results



Peak 10 and Compression Ratio = 0.5

- Our method produced higher PSNR than SPIRAL-TAP but results are not significantly different in visual sense. SPIRAL-TAP results have DC-bias which results in lower PSNR than our algorithm.



Peak 10 and Compression Ratio = 0.5

Color CS

- Compressed measurements computed independently across each R, G, B channel
- Learn a single dictionary over R, G, B channels
- Use 3-D DCT as sparsifying basis for SPIRAL-TAP

- From result images, we see that SPIRAL-TAP fails to reconstruct color pattern and produces color artifacts
- 3-D DCT unable to compactly represent color image patches that contain pixels with significantly different R, G, B values

- Motivates the idea of dictionary learning for multidimensional signals such as color or hyperspectral images, or videos, instead of relying on prior knowledge of a sparsifying basis

Conclusions

- Proposed dictionary learning algorithm for reconstruction from Poisson-corrupted compressive measurements, for both gray-scale and color images

- Results illustrate the benefits of learning the dictionary in situ from the compressed measurements over fixed basis

- Recovers global pattern, blurs out fine texture
- Choice of lambda unclear

- Further applications in color image demosaicing, video and hyperspectral image reconstruction, and tomographic reconstruction