Tensor Dictionary Learning with representation quantization for Remote Sensing Observation Compression

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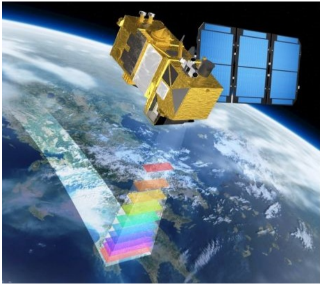
Motivation

Remotely sensed images are used for:

- forest monitoring
- disaster evaluation
- land cover estimation



Challenges:

- Increasing spatial, spectral and temporal resolutions of the images
- Increasing storage and transmission requirements
- High dimensional observations modeled as **tensors**
- High spacial, spectral and temporal redundancies

Problem

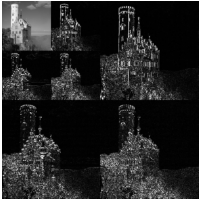
Compression of High-Dimensional Remote Sensing Observations that

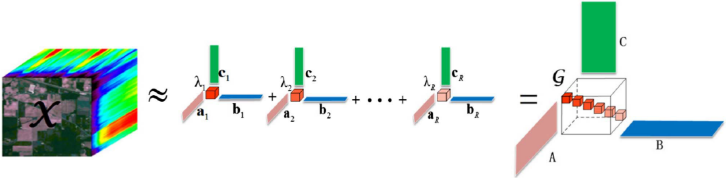
- Includes quantization and coding
- Achieves high compression ratio
- Retains the structure of the data
- Can handle arbitrary high dimensional data structures

Proposed solution:
A novel *tensor dictionary learning* method is used to compress every new sample as a vector of sparse coefficients corresponding to the elements of the learned tensor dictionary, given a set of previous samples.

Related Work

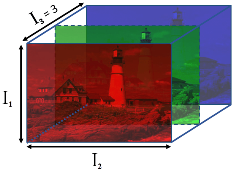
- 3D Wavelet transform on the full data cube.
- A combination of JPEG2000 with Discrete Wavelet Transform or Principal Components Analysis for spectral decorrelation.
- Tensor-based approaches using tensor decompositions by transmitting all the factors of the decomposition.

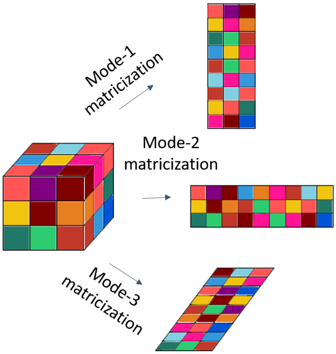




Tensor Based Observation Modeling

A *tensor* $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is a N-way array, a higher-order generalization of vectors and matrices.

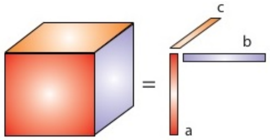




The *mode-n unfolded matrix* $\mathbf{X}_{(n)} \in \mathbb{R}^{I_n \times \prod_{i \neq n} I_i}$ corresponds to a matrix with columns being the vectors obtained by fixing all indices of $\mathcal{X}$ except the n-th index.

Tensor Rank

The outer product of N vectors yields a *rank-1* N-way tensor.



Every tensor can be written as a sum of rank-1 tensors

$$\mathscr{X} \approx \sum_{r=1}^{R} a_r^{(1)} \circ a_r^{(2)} \circ \ldots \circ a_r^{(N)}$$

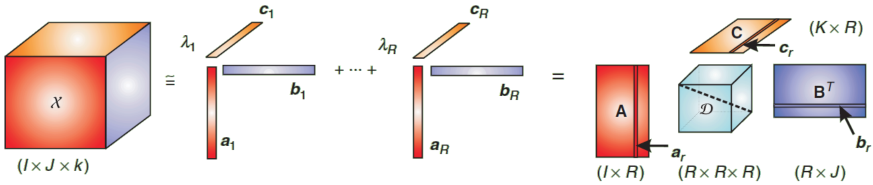
- The *rank* of a N-way tensor $\mathscr{X}$ is the smallest number R of rank-1 tensors needed to synthesize $\mathscr{X}$.
- No straightforward algorithm to determine the rank of a specific given tensor (NP-hard problem).

CP Decomposition

CANDECOMP/PARAFAC (CP) decomposition represents a N-order tensor $\mathscr{X} \in \mathbb{R}^{I_1 \times .. \times I_N}$ as a linear combination of rank-1 tensors in the form

$$\mathscr{X} = \sum_{r=1}^{R} \lambda_r a_r^{(1)} \circ a_r^{(2)} \circ ... \circ a_r^{(N)} = \mathscr{D} \times_1 A^{(1)} \times_2 A^{(2)} \times_3 \cdots \times_N A^{(N)}$$

where $A^{(n)} = [a_1^{(n)} \; a_2^{(n)} \; \cdots \; a_R^{(n)}],\; n = 1, .., N$ are the factor matrices and $\mathscr{D} = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_R) \in \mathbb{R}^{R \times \cdots \times R}$ is a diagonal core tensor.



Training Process

1. Learn a *tensor dictionary* $\mathscr{D} \in \mathbb{R}^{I_1 \times \cdots \times I_N \times K}$ of K rank-1 tensors $\mathscr{D}^{(k)} \in \mathbb{R}^{I_1 \times \cdots \times I_N}$, $k = 1, .., K$, by minimizing

$$\begin{array}{c} \min_{\mathscr{D}, \mathbf{A}} \frac{1}{2} \|\mathscr{X} - \mathscr{D} \times_{N+1} \mathbf{A}\|_F^2 \\ \text{subject to } \|\mathbf{A}(t,:)\|_0 \leq \lambda, \ \forall t = 1, .., T \end{array},$$

   where $\mathscr{X} = (\mathscr{X}^1, \mathscr{X}^2, ..., \mathscr{X}^T) \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N \times T}$ is a set of T training samples and $\mathbf{A} \in \mathbb{R}^{T \times K}$ contains the corresponding sparse coefficients.

2. Learn a *coding dictionary* which maps a binary number to a set of 2^{bit} symbols by equally partitioning the range of values of the coefficient matrix $\mathbf{A}$.

Proposed Tensor Dictionary Learning Method

Introducing the auxiliary variable $\mathbf{G} \in \mathbb{R}^{T \times K}$, we apply the Alternating Direction Method of Multipliers to solve the reformulated problem

$$\min_{\mathscr{D},\mathbf{A},\mathbf{G}} \frac{1}{2}\|\mathscr{X} - \mathscr{D} \times_{N+1} \mathbf{A}\|_F^2 + \lambda \sum_{t=1}^{T} \|\mathbf{G}(t,:)\|_0$$
$$\text{subject to } \mathbf{G} = \mathbf{A}$$

The augmented Lagrangian function is given by

$$\mathscr{L}(\mathbf{A},\mathbf{G},\mathscr{D},\mathbf{Y}) = \frac{1}{2}\|\mathscr{X} - \mathscr{D} \times_{N+1} \mathbf{A}\|_F^2 + \lambda \sum_{t=1}^{T} \|\mathbf{G}(t,:)\|_0 + < \mathbf{Y}, \mathbf{G} - \mathbf{A} > + \frac{p}{2}\|\mathbf{G} - \mathbf{A}\|_F^2,$$

where $\mathbf{Y} \in \mathbb{R}^{T \times K}$ is the Lagrange multiplier matrix and $p > 0$ denotes the step size parameter.

Tensor Dictionary Learning Algorithm

We solve the problem iteratively by minimizing $\mathscr{L}$ with respect to each variable while keeping the others fixed. At each iteration l we update:

- Sparse coefficient matrix $\mathbf{A}$:

$$\nabla_{\mathbf{A}}\mathscr{L} = 0 \quad \Rightarrow \quad \mathbf{A} \leftarrow (\mathbf{X}_{(N+1)} \cdot \mathbf{D}_{(N+1)}^{T} + \mathbf{Y} + p \cdot \mathbf{G}) \cdot (\mathbf{D}_{(N+1)} \cdot \mathbf{D}_{(N+1)}^{T} + p \cdot \mathbf{I})^{-1}$$

- Auxiliary variable $\mathbf{G}$:

$$\nabla_{\mathbf{G}}\mathscr{L} = 0 \quad \Rightarrow \quad \mathbf{G} \leftarrow H_{\lambda}(\mathbf{A} - \frac{\mathbf{Y}}{p}), \text{ where } H_{\lambda}(x) = \begin{cases} x, & |x| > \lambda \\ 0, & \text{otherwise} \end{cases}$$

- Tensor dictionary $\mathscr{D}$:

$$\nabla_{\mathscr{D}}\mathscr{L} = 0 \quad \Rightarrow \quad \mathscr{D}^{(l)} \leftarrow \mathscr{D}^{(l-1)} + \mathscr{X} \times_{N+1} \mathbf{A}^{-1} \text{ and normalization}$$

- Lagrangian multiplier matrix $\mathbf{Y}$:

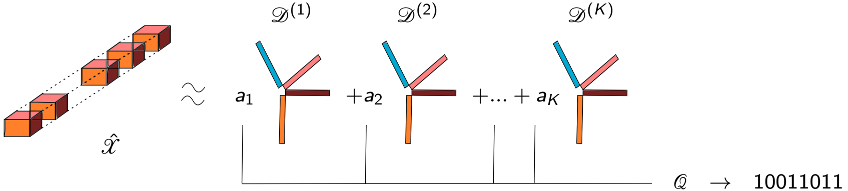
$$\mathbf{Y}^{(l)} \leftarrow \mathbf{Y}^{(l-1)} + p \cdot (\mathbf{G} - \mathbf{A}) \text{ where } p = 0.6 \text{ in our setup}$$

Compression

1. Compress each new sample $\hat{\mathscr{X}} \in \mathbb{R}^{I_1 \times \cdots \times I_N}$ as a sparse vector of coefficients $\mathbf{a} \in \mathbb{R}^K$ such that

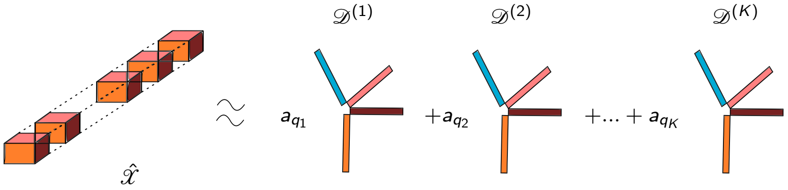
$$\hat{\mathscr{X}} = \mathscr{D} \times_{N+1} \mathbf{a}, \text{ where } \|\mathbf{a}\|_0 \leq \lambda,$$

2. Quantize $\mathbf{a}$ to b bits using a uniform quantizer $\mathscr{Q}$.
3. Encode $\mathscr{Q}(\mathbf{a})$ using Huffman coding and the learned encoding dictionary.



Decompression

1. Decode the transmitted coefficients $\mathbf{a}_q = \mathcal{Q}(\mathbf{a})$ using the learned Huffman dictionary.
2. Decompress the sample as $\hat{\mathcal{X}} \approx \mathcal{D} \times_{N+1} \mathbf{a}_q$.



Experiments

- Data: Time series of satellite derived observations of normalized difference vegetation index (NDVI).
- Sample size: $200 \times 200 \times 7$
  The last dimension indicates the number of days.
- Training samples: 50
- The recovery performance is measured in terms of the Normalized Mean Square Error (NMSE) which is defined as $\mathrm{NMSE} = \frac{\|\mathscr{Y} - \hat{\mathscr{Y}}\|_2^2}{\|\mathscr{Y}\|_2^2}$.

Number of Atoms of the Dictionary

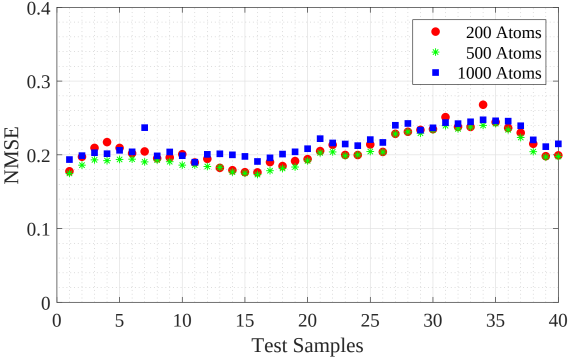


Figure: Reconstruction quality for each test sample and different number of atoms of the dictionary, using 80% sparsity level and 8 bits of quantization.

Sparsity Level

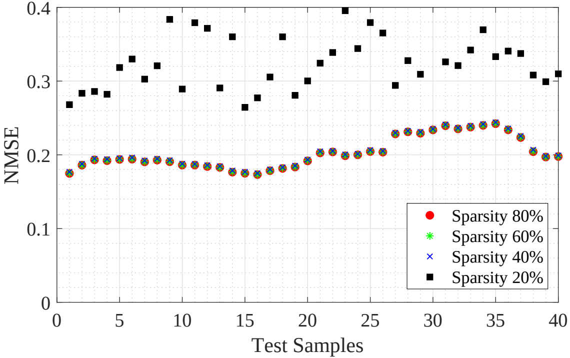


Figure: Reconstruction quality for each test sample and different sparsity levels, using a dictionary with 500 atoms and 8 bits of quantization.

Number of Quantization Bits

Table: Reconstruction error for different samples as a function of quantization bit number.

Number of Bits	NMSE			
	1st Sample	10th Sample	25th Sample	35th Sample
4	0.2915	0.3226	0.3510	0.3936
6	0.1816	0.1941	0.2134	0.2510
8	0.1748	0.1860	0.2044	0.2425

Comparison with State-of-the-art Compression Algorithm

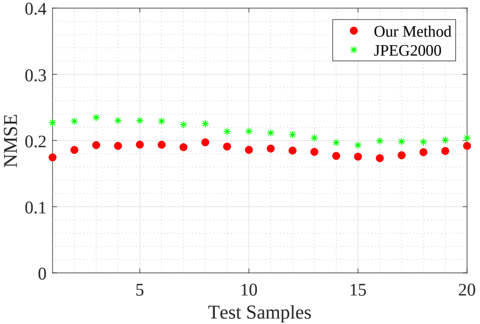


Figure: Reconstruction quality for several test samples, using 0.06 bpppb.

Table: Reconstruction quality for different number of bpppb.

NMSE	bpppb			
	0.20	0.16	0.08	0.03
Our Method	0.1754	0.1751	0.1748	0.1777
JPEG2000	0.1838	0.1929	0.2169	0.2493

Conclusion

- An end-to-end compression algorithm is proposed that includes quantization and coding.
- A novel tensor dictionary learning method based on CP decomposition is presented for compression purposes.
- The proposed scheme can handle arbitrary high dimensions.
- Our method is evaluated on 3D remote sensing observations.

Acknowledgments

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