Robust Self-Calibration of Constant Offset Time-Difference-of-Arrival

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Constant Offset Time-difference-of-arrival

Given a set of microphones and sound events at a constant interval, we present a method to find the locations of the microphones and locations of the sound events.

Problem 1. *(Constant Offset Time-Difference-of-Arrival Self-Calibration) Given measurements* $\tilde{z}_{ij}$

$$\tilde{z}_{ij} = \|\mathbf{r}_i - \mathbf{s}_j\|_2 + o + \epsilon_{ij}, \quad (1)$$

Here there are m *receiver positions* $\mathbf{r}_i \in \mathbb{R}^3$, $i = 1, \ldots, m$, *and* n *sender positions* $\mathbf{s}_j \in \mathbb{R}^3$, $j = 1, \ldots, n$, *with a constant offset* o *and errors* ϵ_{ij}.

Minimal problems and solvers

Given time-difference-of-arrival measurements from five receivers to five senders, there are four possible offsets o, given as the roots to the fourth degree polynomial $f(o)$, counting complex roots and multiplicity of roots.

$$f(o) = \det(C^T (Z - o)^{\circ 2} C) = 0 \quad (2)$$

for a compaction matrix C and a 5 x 5 matrix block Z with elements $\tilde{z}_{ij}$.

Local optimization and the low rank relaxation

If an initial estimate of the parameters $\theta_1 = \{R, S, o\}$ is given and if the set of inliers is known, then refinement of the estimate can be found by optimization methods, *e.g.* Levenberg-Marquardt (LM) [1],

$$\min_{\theta_1} f(\theta_1) = \sum_{(i,j) \in W_{\text{in}}} (z_{ij} - (\|\mathbf{r}_i - \mathbf{s}_j\|_2 + o))^2. \quad (3)$$

There is an interesting relaxation to the problem, that exploits the fact that the matrix with elements $(z_{ij} - o)^2$ is rank 5, [2]. The relaxed problem involves a set of parameters $\theta_2 = \{U, V, b, a, o\}$. Here the constraints can be written as

$$z_{ij} = \sqrt{u_i^T v_j + a_i + b_j} + o, \quad (4)$$

where u_i denotes column i of U and v_j denotes column j of V. Refinement of parameters can be done by performing local optimization on

$$\min_{\theta_2} f(\theta_2) = \sum_{(i,j) \in W_{\text{in}}} \left(z_{ij} - (\sqrt{u_i^T v_j + a_i + b_j} + o) \right)^2. \quad (5)$$

Computational Times for Each Solver

Implementation	Matlab	C++
Calculation of o	$38\,\mu s$	$3.7\,\mu s$
Calculation of $\theta_2 = \{U, V, a, b, o\}$	$100\,\mu s$	N/A
Calculation of $\theta_1 = \{R, S, o\}$	$600\,ms$	$22\,ms$

Table 1: Execution times for 5×5 minimal solvers steps. Notice that the steps of calculating o and the relaxed solution is significantly faster than upgrading to the full solution

Evaluation of Minimal Solver

- 10,000 instance problems with known offsets
- Ran our solvers and compared the returned solutions with the ground truth solution
- From the 4 returned offsets, the closest solution to the ground truth was compared

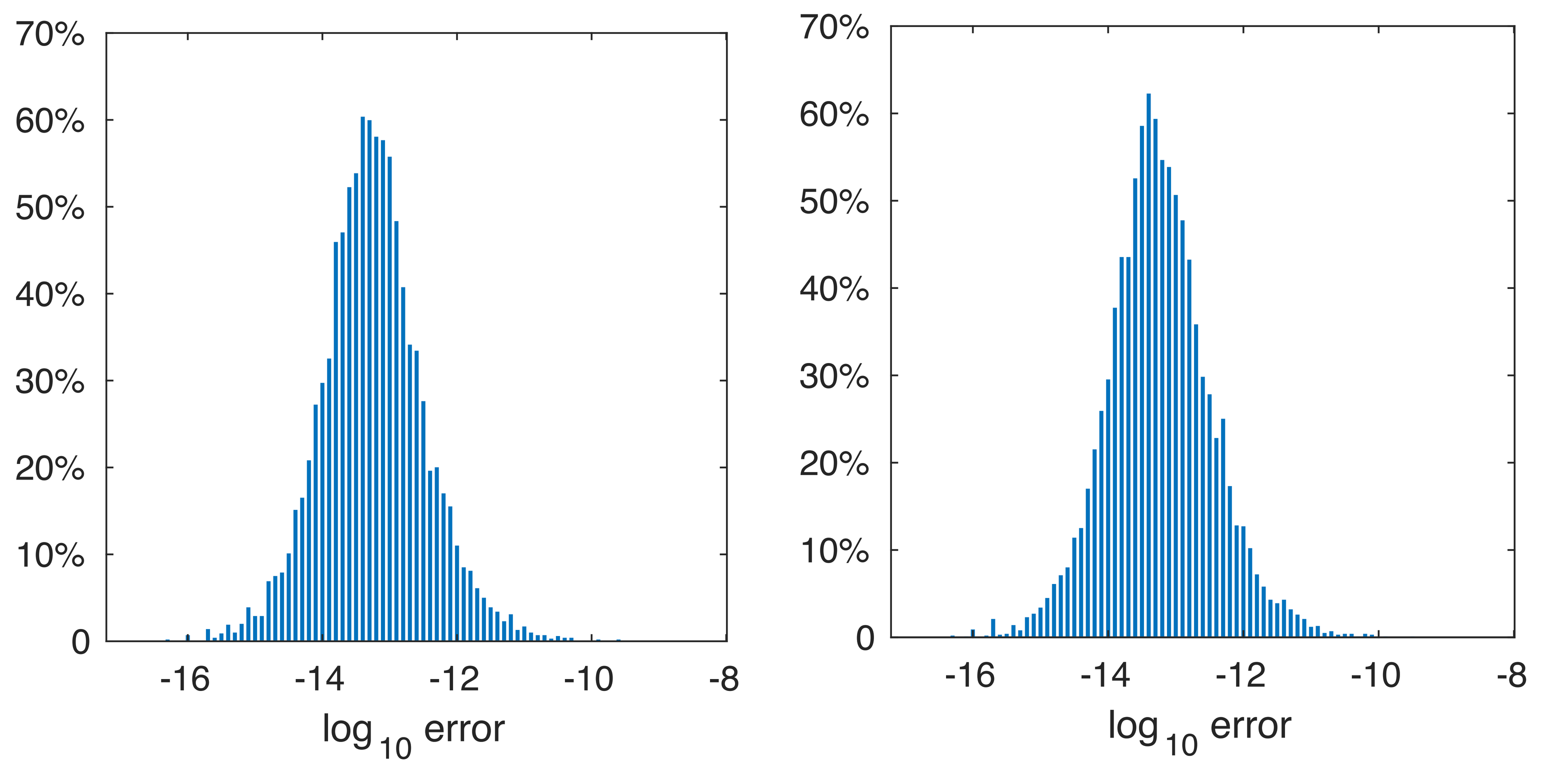

Figure 1: Left shows the histogram of the logarithm of the absolute errors, for the Matlab implementation of our minimal solver. To the right the corresponding histogram for the C++ implementation.

Evaluation of Office Experiment

- 12 microphones (8x t.bone MM-1, 4x Shure SV100)
- Roland UA-1610 Sound Capture audio interface
- Audacity 2.3.0 with a sampling frequency of 96 kHz on a laptop
- Synthetically generated chirp played on a loudspeaker every half second

12 microphones were positioned around an open room ($\sim 3 \times 5\ m^2$) and measured using a laser to obtain ground truth positions of the microphones with an error of $\pm 2\ mm$.

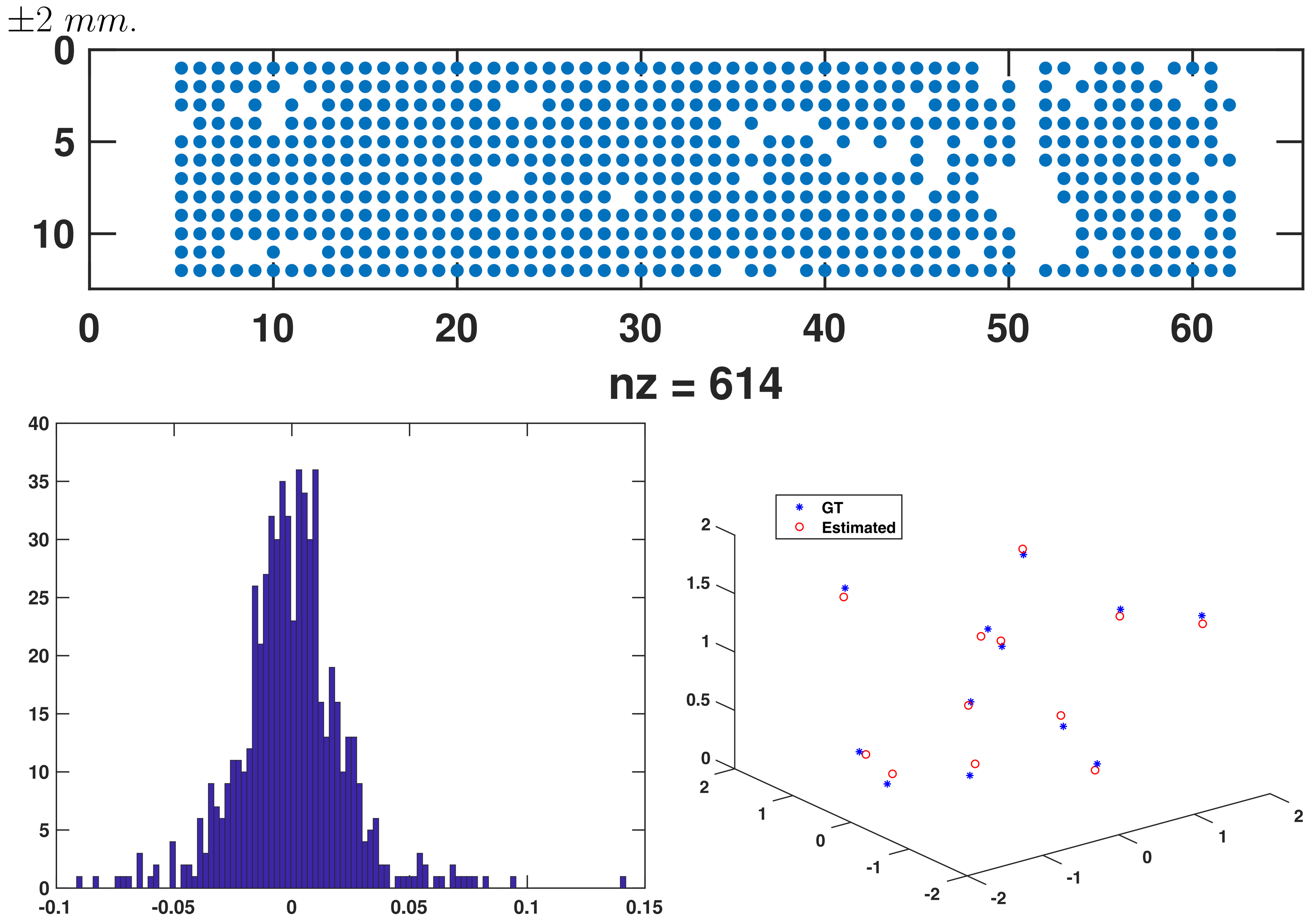

Figure 2: For the office experiment the figure shows detected inliers W_{in} (top), inlier residual histogram (bottom left), and estimated and ground truth microphone positions (bottom right).

Evaluation of Bat Cave Experiment

- 12 microphones (4x Sanken CO-100K, 8x Knowles SPU0410)
- The sound recordings were captured using pre-amplifiers (Quadmic, RME)
- Two synchronised Fireface 800 (RME) audio interfaces running at a sampling frequency of 192 kHz
- Ultrasonic chirps (8 ms, $16-96\ kHz$ upward hyperbolic sweep) played on a lPeerless XT25SC90-04 loudspeaker

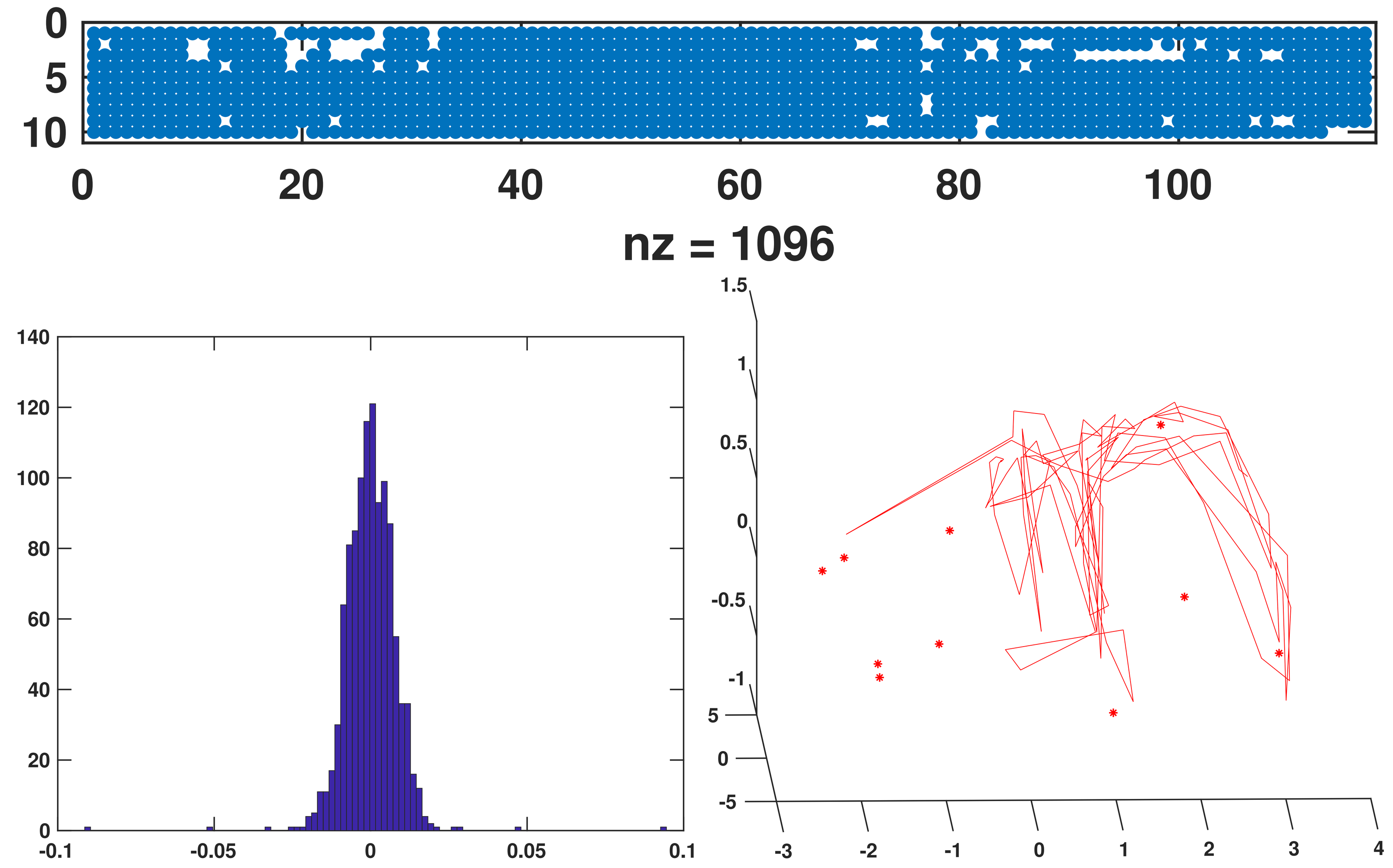

Figure 3: For the cave experiment the figure shows detected inliers W_{in} (top), inlier residual histogram (bottom left) and estimated microphone and sound source positions, red dots and line respectively (bottom right).

Conclusions

- A novel method has been constructed to efficiently solve a TDOA problem with a constant offset.
- The calculation of the offsets and the calculation of the relaxed form θ_2 are very fast solvers without loss in numerical accuracy.
- From the office experiment, we can see that the calculated microphone positions are accurate and the residuals are small, mostly in the range $\pm 0.04\ m$.
- A comparison of the calculated microphone positions were made to a solution from a Full TDOA system, [3], which produced similar results.
- From the Bat Cave experiment, we can see that the calculated microphone positions are accurate since the residuals are small.

References

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