audio

Decompressing Lempel-Ziv Compressed Text

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background

motivation

Right now, when *everything* is being done online, *compression* is important — but since there's even more downloading than uploading, *decompression* is even more important!

LZ77 is one of the most elegant, powerful and — thanks to its inclusion in several standards and tools such as `gzip` — popular compression schemes.

There have been several recent papers on small-space LZ77 compression but almost nothing is known about small-space LZ77 decompression.

setting

Suppose we're given the z-phrase parse Z of a string $S[1..n]$ over $\{0, \ldots, \sigma - 1\}$.

In this talk we consider the version of LZ77 in which a phrase can be

- a single character $S[i] \neq S[i']$ for all $i' < i$, encoded as $(S[i], 0)$;
- a substring $S[i..j] = S[i'..j']$ for some $j' < i$, encoded as $(i, j - i + 1)$.

Our results can be generalized to other versions as well.

setting

For example, if

$$S = 0100110001101001100011010011 00$$

then $z = 8$ and

$$Z = (0,0),(1,0),(1,1),(1,2),(2,3),(4,4),(2,8),(10,10)\,.$$

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setting

For example, if

$$S = 0\textcolor{red}{1}00110001101001100011010011 00$$

then $z = 8$ and

$$Z = (0,0), \textcolor{red}{(1,0)}, (1,1), (1,2), (2,3), (4,4), (2,8), (10,10)\,.$$

setting

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setting

For example, if

$$S = 010\textcolor{blue}{0110}\textcolor{red}{0110}100110001101001100$$

then $z = 8$ and

$$Z = (0,0), (1,0), (1,1), (1,2), (2,3), \textcolor{red}{(4,4)}, (2,8), (10,10)\,.$$

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past solutions I

The standard solution is to decompress S from left to right, phrase by phrase, which takes $O\left(z + n \cdot \frac{\log \sigma}{\log n}\right)$ time and space.

Another solution is to turn Z into a context-free grammar for S and generate S from that, which takes $O\left(z \log \frac{n}{z} + n \cdot \frac{\log \sigma}{\log n}\right)$ time and $O\left(z \log \frac{n}{z}\right)$ space.

past solutions II

Puglisi and Rossi's DCC 2019 solution was a simplified implementation of our arXiv poster.

It doesn't have good worst-case time bounds but works well in practice.

(Rossi's slides: `https://tinyurl.com/ub6yyxd`.)

step 1

S^τ definition audio

We first want to extract and store the subsequence S^τ of S consisting of characters within distance τ of the nearest phrase boundary, where τ is a parameter.

For example, if $\tau = 2$ and

$$S = 0\,1\,0\,01\,100\,0110\,10011000\,1101001100$$

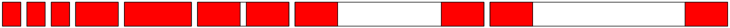
then

$$S^\tau = 0\,1\,0\,01\,100\,0110\,1000\,1100\,.$$

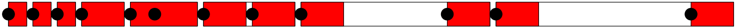
S^{τ} extraction audio

0	1	0	0 1	1 0 0	0 1 1 0	1 0 0 1 1 0 0 0	1 1 0 1 0 0 1 1 0 0

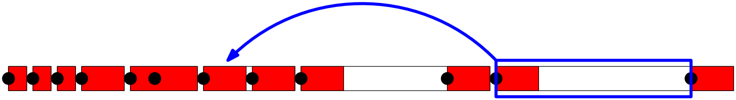
S^{τ} extraction audio



S^{τ} extraction audio

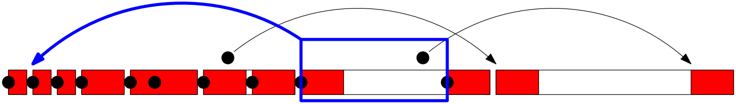


S^{τ} extraction audio

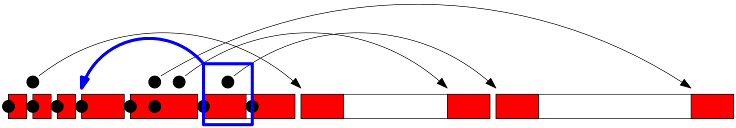


Decompressing LZ77

S^τ extraction audio



S^τ extraction
audio



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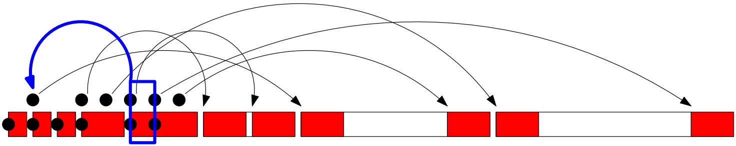

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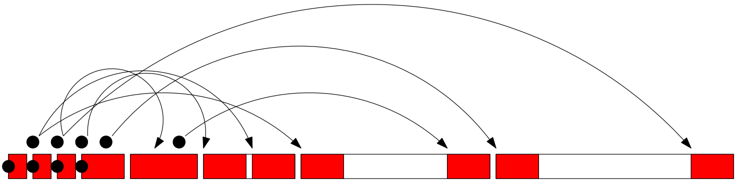
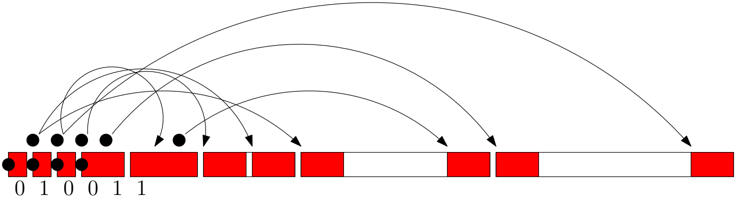
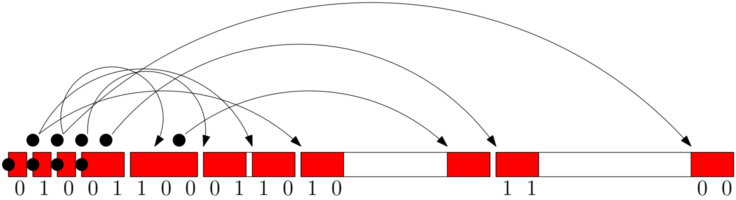


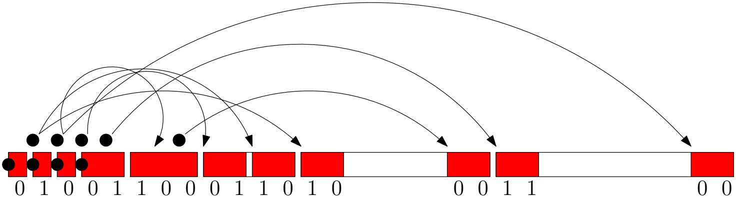
S^{τ} extraction
audio



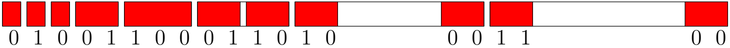
S^τ extraction audio



S^τ extraction audio



S^τ extraction
audio



mergeable dictionary

In my graphics editor, I can just select all the dots in a box that's τ shorter than a phrase and move them to the phrase's source.

Iacono and Özkan's *mergeable dictionary* has the same functionality now that Bille et al. have shown how to implement shifts.

If $\tau = O\left(\frac{\log n}{\log \sigma}\right)$ — so τ characters fit in $O(1)$ machine words — then extracting S^τ takes $O(z \log n)$ time and $O(z)$ space.

step 2

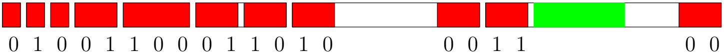
substring division

We extract substrings of length greater than $z\tau$ by breaking them into pieces of length at most $z\tau$ and extracting the pieces *consecutively*.

We extract substrings of length between $\tau + 1$ and $z\tau$ by breaking them into pieces of length of at most τ and extracting the pieces *simultaneously*.

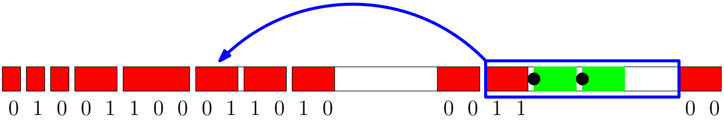
We extract substrings of length at most τ by treating those substrings as part of S^{τ} and repeating its extraction.

substring extraction
audio

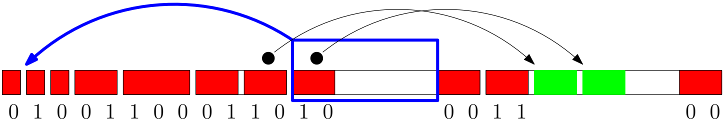


substring extraction
audio

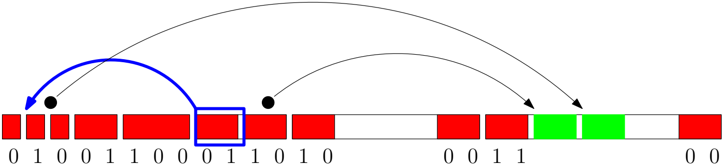
substring extraction audio



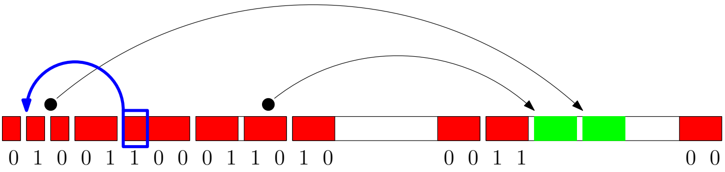
substring extraction
audio



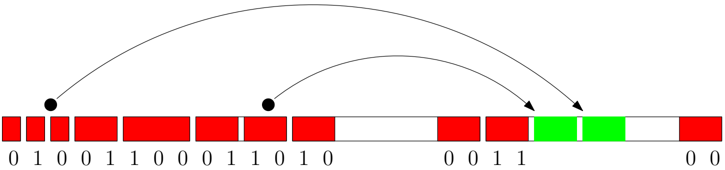
substring extraction
audio



substring extraction
audio



substring extraction audio



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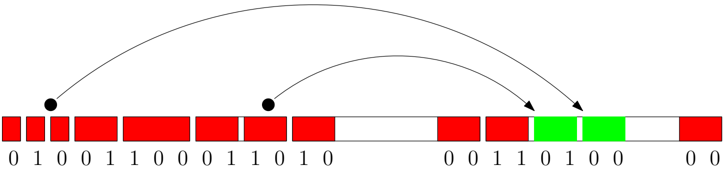

substring extraction

Extracting a substring of length ℓ takes

$$O\left(\left(\frac{\ell}{z\tau} + 1\right) \cdot z \log n\right)$$

time and

$$O\left(z\tau \cdot \frac{\log \sigma}{\log n}\right)$$

space.

If $\sigma = O(1)$ and $\tau = \Theta(\log n)$ then these bounds are $O(\ell + z \log n)$ and $O(z)$.

applications

decompression

The obvious application of our results is decompression LZ77-compressed texts in small space.

For example, if $\sigma = O(1)$ then we can choose $\tau = \log n$ so we decompress in

$$O\left(\frac{n}{z\tau} \cdot z \log n\right) = O(n)$$

time using $O(z)$ space.

pattern matching

If we are looking for approximate matches to a pattern of length m allowing k mismatches, then we need only check characters within distance $m + k$ of the nearest phrase boundary.

The best known algorithm uses $O(z \log(n/z) + z \min(mk, k^4 + m) + \text{occ})$ time and $O(z \log(n/z) + m + \text{occ})$ space.

With our results, we can keep the same time and reduce the space to $O(z \log\log(n/z) + m + \text{occ})$ in general or $O(z + m + \text{occ})$ when $\sigma = O(1)$.

questions?