Estimating Structural Missing Values via Low-tubal-rank Tensor Completion

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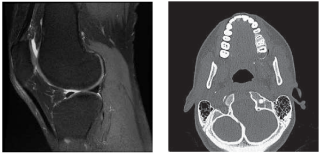
Tensor Based Signal Processing

Tensors in real world

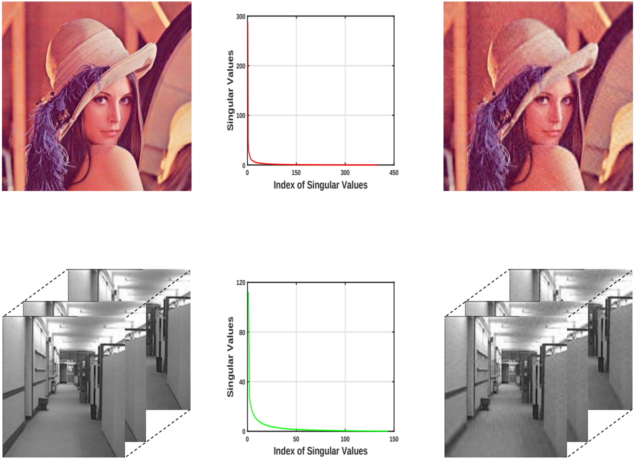
(a) Hyperspectral Image



(b) MRI

(c) RGB Image

Low-tubal-rank property



Low-tubal-rank tensor completion

The TNN minimization model [Zhang et al, 2016; Lu et al, 2018]:

$$\min_{\mathcal{X}} \|\mathcal{X}\|_{\star} \text{ s.t. } \mathcal{P}_{\Omega}(\mathcal{X}) = \mathcal{P}_{\Omega}(\mathcal{T})$$

Notice: TNN minimization method considers the low-tubal-rankness of the original tensor only, some other structural information are not be used.

Motivation: sparsity-based structure in the missing entries. For examples, chemical measurements, movie rating, (medical) survey data, sensor network, etc.

Our model:

$$\min_{\mathcal{X}} \|\mathcal{X}\|_\star + \lambda \|\mathcal{P}_{\Omega^c}(\mathcal{X})\|_1 \ \text{ s.t. } \ \mathcal{P}_\Omega(\mathcal{X}) = \mathcal{P}_\Omega(\mathcal{T})$$

Notice: it degenerates to the original tensor completion model when $\lambda = 0$, which means there is no structural difference between the observed and missing values.

Theorem

Let $\mathcal{T}_0$ be the ground truth tensor and Ω be the support set of the observed entries. Assume that structured observations satisfy $\mathcal{P}_{\Omega^c}(\mathcal{T}) = \mathbf{0}$. Then, for any tensor norm $\|\cdot\|$, we have

$$\|\mathcal{T}_2 - \mathcal{T}_0\| \leq \|\mathcal{T}_1 - \mathcal{T}_0\|,$$

where

$$\mathcal{T}_1 = \arg\min_{\mathcal{X}} \|\mathcal{X}\|_\star \ \ s.t.\ \ \mathcal{P}_\Omega(\mathcal{X}) = \mathcal{P}_\Omega(\mathcal{T}),$$

$$\mathcal{T}_2 = \arg\min_{\mathcal{X}} \|\mathcal{X}\|_\star + \lambda\|\mathcal{P}_{\Omega^c}(\mathcal{X})\|_1 \ \ s.t.\ \ \mathcal{P}_\Omega(\mathcal{X}) = \mathcal{P}_\Omega(\mathcal{T}).$$

Theorem

Suppose that $\mathcal{T}_0$ satisfies the tensor incoherence conditions as defined in [Lu et al, 2019] and Ω is uniformly distributed among all sets of cardinality m and the support set of sparse component $\mathcal{S}_0$ of non-zero unobserved entries is uniformly distributed among all sets of cardinality s contained in Ω^c. Then, there exist numerical constants $c_1, c_2 > 0$ such that with propability as least $1-c_1(n_{(1)}n_3)^{-c_2}$, the objective minimization problem with $\lambda = 1/\sqrt{n_{(1)}n_3}$ achieves exact recovery at $(\mathcal{X}_0, \mathcal{S}_0)$ provided that

$$\text{rank}_t\left(\mathcal{X}_0\right) \leq \frac{\rho_r n_{(2)} n_3}{\mu\left(\log\left(n_{(1)}n_3\right)\right)^2}, \textit{ and } s \leq \rho_s n_1 n_2 n_3,$$

where $\rho_r, \rho_s > 0$ are numerical constants.

Optimization-ADMM-based Algorithm

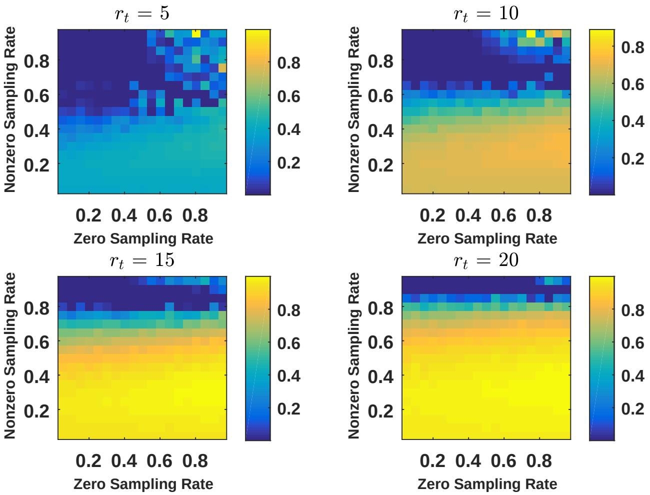
$$L(\mathcal{X},\mathcal{Y},\mathcal{Z},\mu)=\|\mathcal{X}\|_{\star}+\lambda\|\mathcal{Y}\|_1+\frac{\mu}{2}\|\mathcal{P}_{\Omega}(\mathcal{T})-\mathcal{X}+\mathcal{Y}+\frac{\mathcal{Z}}{\mu}\|_F^2$$

The update process:

$$\begin{cases}\mathcal{X}_{k+1}=\arg\min\|\mathcal{X}\|_{\star}+\frac{\mu_k}{2}\|\mathcal{P}_{\Omega}(\mathcal{T})-\mathcal{X}+\mathcal{Y}_k+\frac{\mathcal{Z}_k}{\mu_k}\|_F^2\\ \mathcal{Y}_{k+1}=\arg\min\lambda\|\mathcal{P}_{\Omega^c}(\mathcal{Y})\|_1+\frac{\mu_k}{2}\|\mathcal{P}_{\Omega^c}(-\mathcal{X}_{k+1}+\mathcal{Y}+\frac{\mathcal{Z}_k}{\mu_k})\|_F^2\\ \mathcal{Z}_{k+1}=\mathcal{Z}_k+\mu_k(\mathcal{P}_{\Omega}(\mathcal{T}-\mathcal{X}_{k+1})+\mathcal{Y}_{k+1})\end{cases}$$

Enhanced Performance: $\|\mathcal{T}_2 - \mathcal{T}_0\|_F / \|\mathcal{T}_1 - \mathcal{T}_0\|_F$ (conduct $\mathcal{T}_0 = \mathcal{T}_L * \mathcal{T}_R$ with

tubal rank r_t, where $\mathcal{T}_L \in \mathbb{R}^{n \times r_t \times n}$ and $\mathcal{T}_R \in \mathbb{R}^{r_t \times n \times n}$ are sparse tensors with density d. Set $r_t = 5, 10, 15, 20$, $n = 100$, and $d = 0.05$. The observations are subsampled from the zero and nonzero entries at various rates from 0 to 1 with interval 0.05)



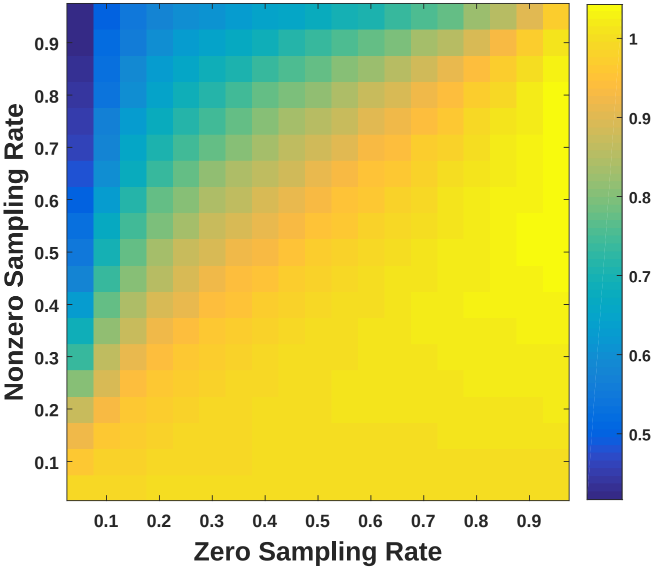
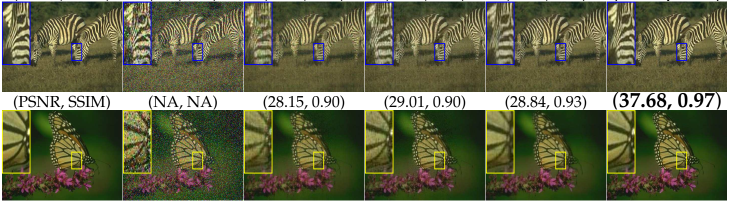

Nonzero Sampling Rate
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Zero Sampling Rate
0.5
0.6
0.7
0.8
0.9
1

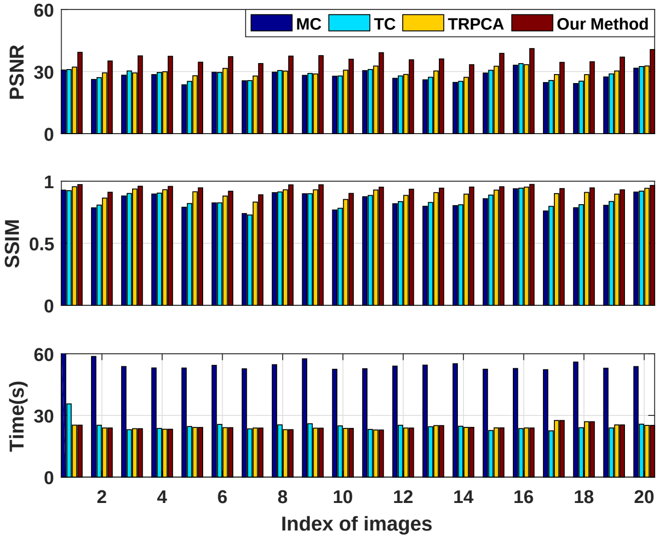
Image denoising: Salt and pepper noise

(PSNR, SSIM) (NA, NA) (23.59, 0.79) (25.25, 0.82) (27.96, 0.91) **(34.51, 0.95)**

(PSNR, SSIM) (NA, NA) (28.15, 0.90) (29.01, 0.90) (28.84, 0.93) **(37.68, 0.97)**



Original Noisy MC TC TRPCA Our Method


MC
TC
TRPCA
Our Method
PSNR
SSIM
Time(s)
Index of images
60
30
0
1
0.5
0
60
30
0
2
4
6
8
10
12
14
16
18
20

Summary:

1. The structural information on missing values is useful for tensor completion;
2. The proposed method has the theoretical recovery guarantee and better performance than the classical TNN minimization method;
3. Sufficient experiments verify the superiority of our work.

References:

Zhang et al, 2016: Z. Zhang and S. Aeron, "Exact tensor completion using t-SVD," IEEE Transactions on Signal Processing, vol.65, no. 6, pp. 1511-1526, 2016.

Lu et al, 2018: C. Lu, J. Feng, Z. Lin, and S. Yan. "Exact low tubal rank tensor recovery from Gaussian measurements," In Proceedings of the 27th International Joint Conference on Artificial Intelligence (IJCAI), AAAI Press, pp. 2504-2510, 2018.

Lu et al, 2019: C. Lu, J. Feng, Y. Chen, W. Liu, Z. Lin and S. Yan, "Tensor robust principal component analysis with a new tensor nuclear norm," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 42, no. 4, pp. 925-938, 2019.

THANKS!