# Simplified Dynamic SC-Flip Polar Decoding

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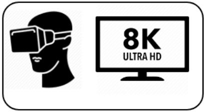
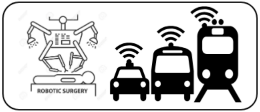
# 5G Use Cases

Enhanced Mobile Broadband (**eMBB**)




**High Throughput**

Ultra-Reliable Low-Latency Communications (**URLLC**)

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Massive Machine-Type Communications (**mMTC**)

**Massive Connectivity**
**Energy Efficiency**

- 5G prioritizes various targets based on the use case.

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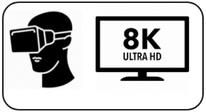


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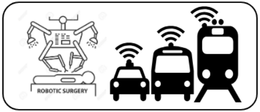


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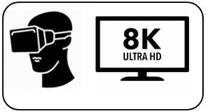


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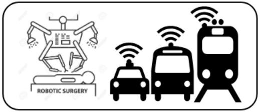


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- 5G prioritizes various targets based on the use case.
- Polar codes provably achieve channel capacity.
- They are involved in 5G eMBB control channel.
- Currently, polar codes are being evaluated for other use cases.

# An Overview of SC Algorithms

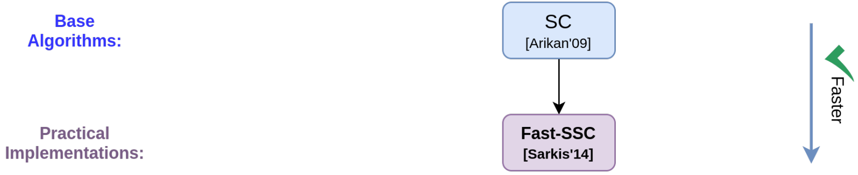
**Base Algorithms:**

**SC**
**[Arikan'09]**

Successive Cancellation (SC) Decoding

- ✓ Simple encoding/decoding
- ✗ Mediocre performance at practical lengths
- ✗ Sequential, long latency

# An Overview of SC Algorithms




Fast-SSC Decoding

✓ $\approx 10\times$ less latency

- No error correction performance degradation

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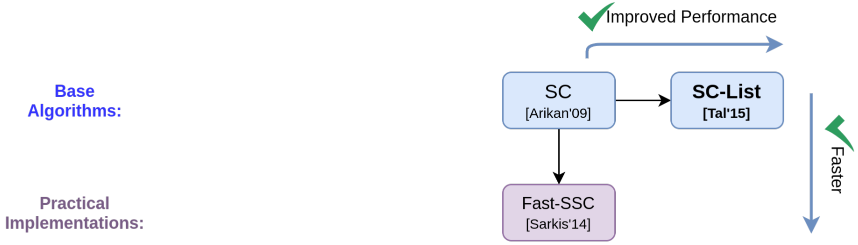


SC-List (SCL) Decoding

- ✓ Improved performance
- ✗ Increased complexity

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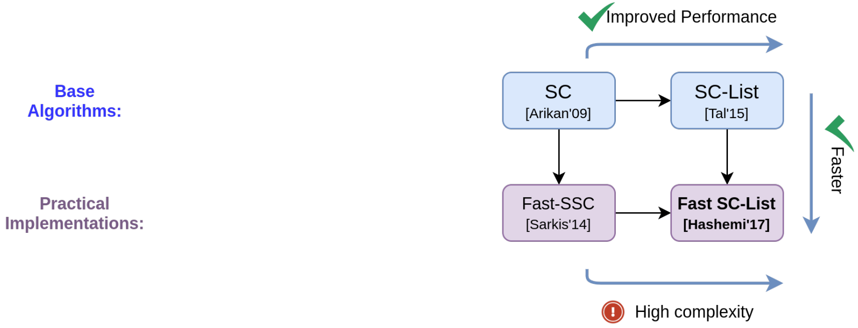


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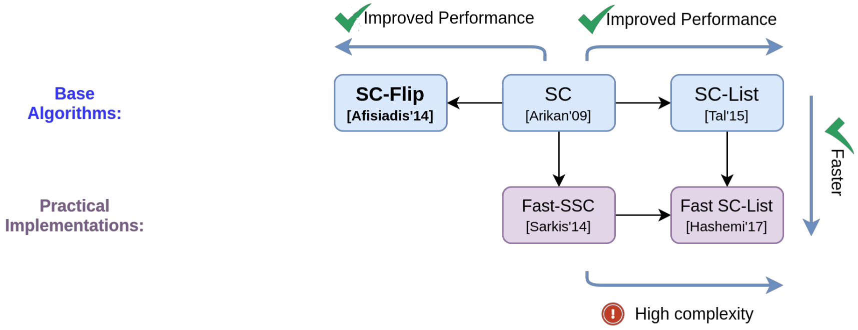


SC-Flip (SCF) Decoding

- ✓ Some improved performance
- ✓ Low complexity
- ✗ Variable latency

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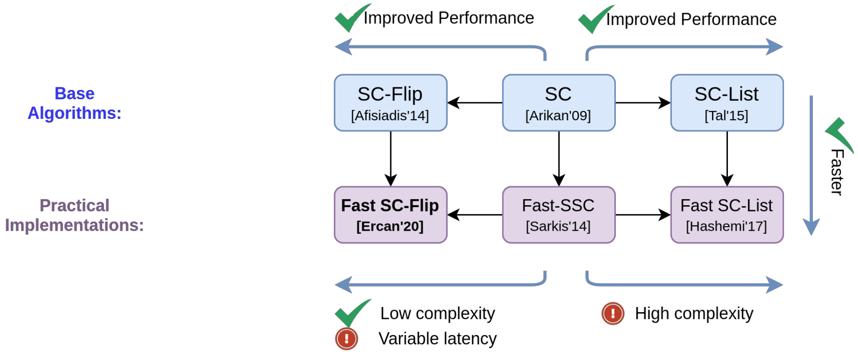


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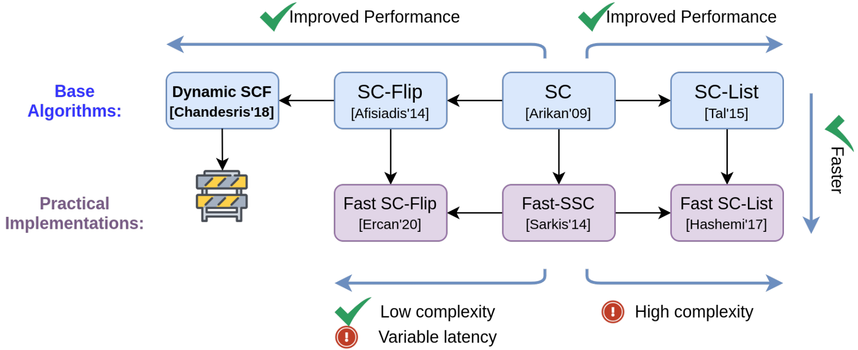
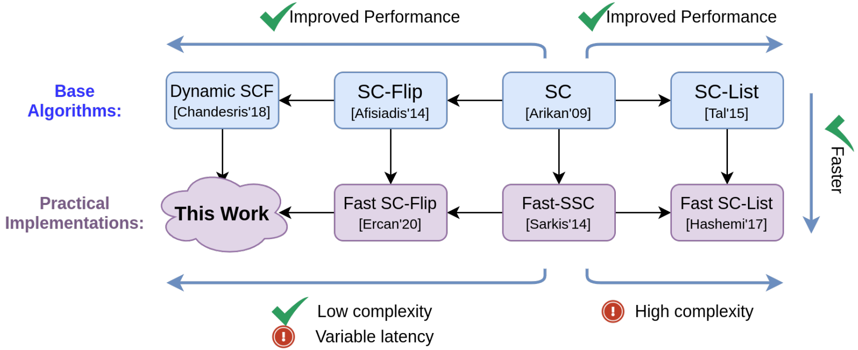



Dynamic SCF (DSCF) Decoding

- ✓ Better improved performance
- ✗ Expensive computations (log, exp, $\times$)
- ✗ No practical implementation

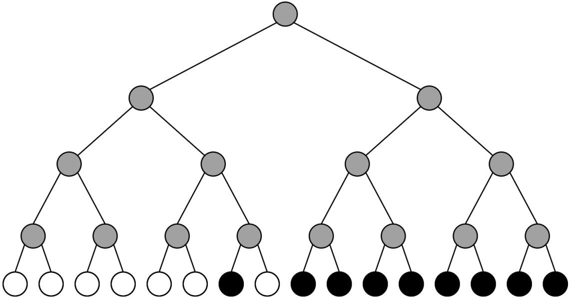
# An Overview of SC Algorithms



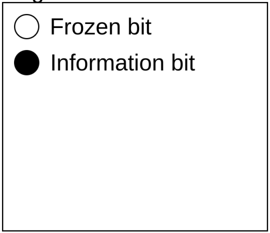

This Work

- ✓ No expensive computations
- ✓ Introduce fast decoding techniques
- ✓ First steps towards practical implementations

# SC-Flip (SCF) Decoding



Legend



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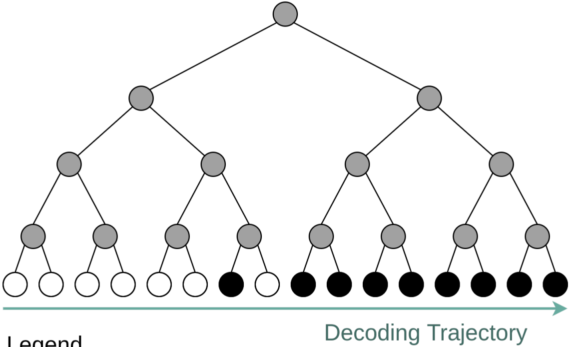


Legend

- ○ Frozen bit
- ● Information bit

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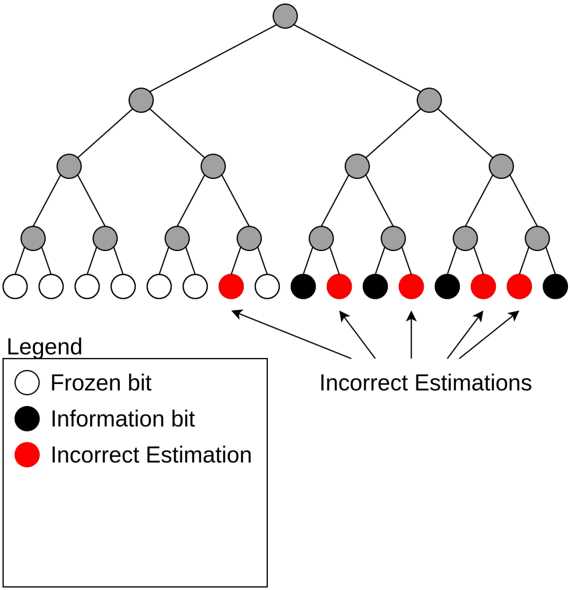

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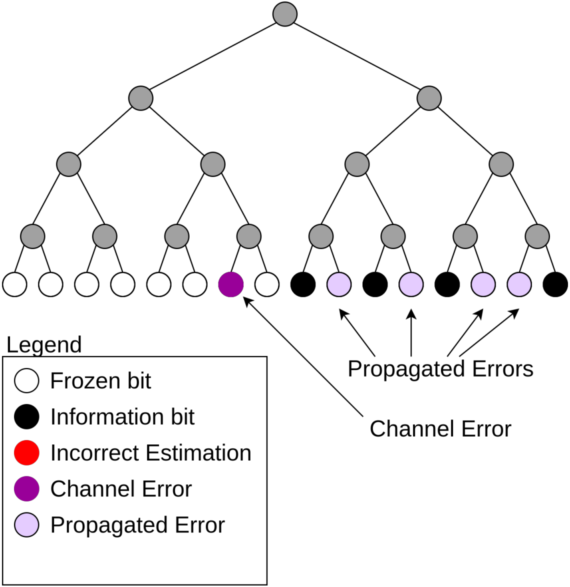

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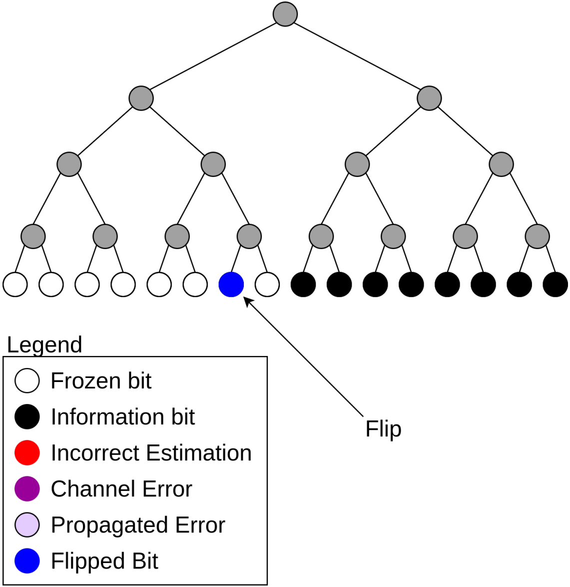

# Problems with SCF Algorithm

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- Metric for SCF for node index i: $|L_i|$ where $L$ is LLR.
- Performance improvement of SCF is limited:
  - Comparable to SCL with small list sizes.
- Two main problems of SCF:
  - Metric cannot distinguish channel errors from propagated errors.
  - Only one error can be corrected.

# Dynamic SC-Flip (DSCF) Decoding

Dynamic SC-Flip (DSCF) algorithm tackles both issues of SCF decoding:

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Dynamic SC-Flip (DSCF) algorithm tackles both issues of SCF decoding:

- A better metric that can distinguish channel errors from propagated errors.
- This gives an opportunity to tackle more than one channel error.
- Therefore, performance is improved greatly.

# Dynamic SC-Flip (DSCF) Decoding

- Let $\omega$ denote the *decoding order*.
- Let $\mathcal{E}_\omega = \{i_1, \ldots, i_\omega\}$ denote the set of bit-flipping indices.
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The metric for DSCF decoding, based on LLRs, is computed as

$$M_\alpha(\mathcal{E}_\omega) = \sum_{j \in \mathcal{E}_\omega} |L^0[\mathcal{E}_{\omega-1}]_j| + S_\alpha(\mathcal{E}_\omega)$$

where

$$S_\alpha(\mathcal{E}_\omega) = \frac{1}{\alpha} \sum_{\substack{j \leqslant i_\omega \\ \forall j \in \mathcal{A}}} \log(1 + \exp(-\alpha |L^0[\mathcal{E}_{\omega-1}]_j|))$$

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where

$$S_\alpha(\mathcal{E}_\omega) = \frac{1}{\underbrace{\alpha}_{\text{positive constant}}} \sum_{\substack{j \leqslant i_\omega \\ \forall j \in \mathcal{A}}} \log(1 + \exp(-\alpha |L^0[\mathcal{E}_{\omega-1}]_j|))$$

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LLR magnitudes at flipping indices

Part that distinguishes channel errors from others

So we can't ignore it

$$M_\alpha(\mathcal{E}_\omega) = \sum_{j\in\mathcal{E}_\omega} |L^0[\mathcal{E}_{\omega-1}]_j| + S_\alpha(\mathcal{E}_\omega)$$

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positive constant

Based only on LLRs

# Simplifications over the DSCF Metric

First, we reformulate $S_\alpha(\mathcal{E}_\omega)$ as

$$S_\alpha(\mathcal{E}_\omega) = \sum_{\substack{j \leqslant i_\omega \\ \forall j \in \mathcal{A}}} f_\alpha(|L^0[\mathcal{E}_{\omega-1}]_j|),$$

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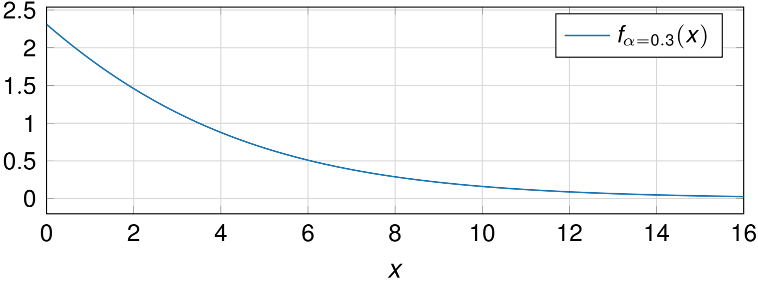
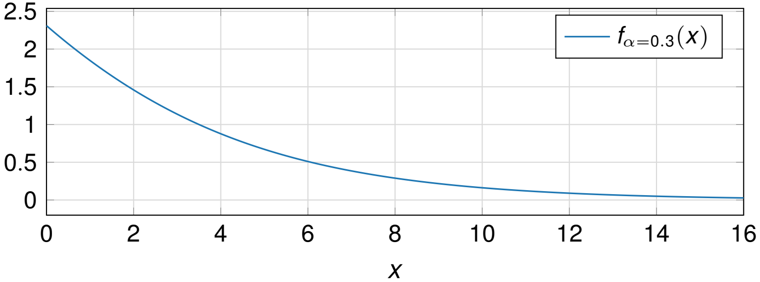
Then, we take a closer look at the behavior of $f_\alpha(x)$. Following the original DSCF algorithm, $\alpha = .3$.



# Simplifications over the DSCF Metric

- Interestingly, a similar problem was encountered for Turbo codes in 1998*.

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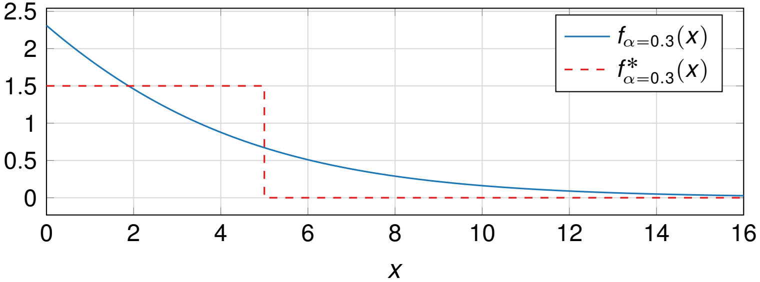


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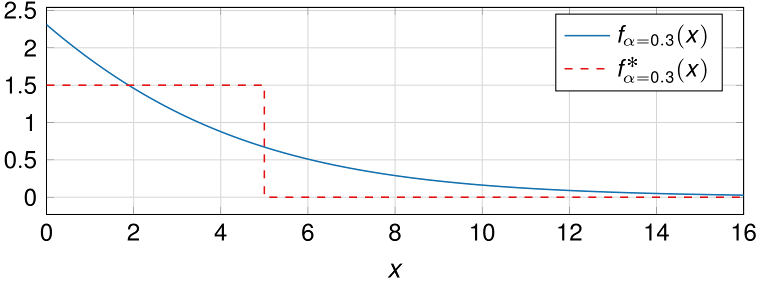


We replace $f_{\alpha=0.3}(x)$ with

$$f^*_{\alpha=0.3}(x) = \begin{cases} \frac{3}{2}, & \text{if } |x| \leqslant 5 \\ 0, & \text{otherwise.} \end{cases}$$

The values in $f^*_{\alpha=0.3}(x)$ are chosen in favor of quantization.

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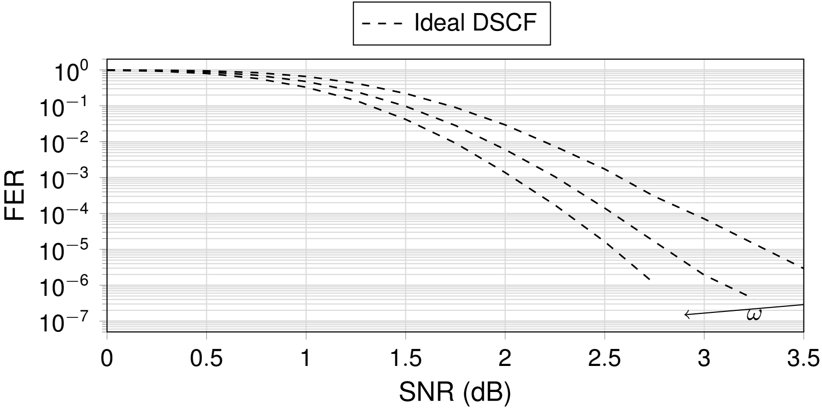
# Impact of the Simplification on Performance

- $PC(N, K) = PC(1024, 512)$, CRC length $C = 16$ bits.
- $T_{\max} \in \{10, 40, 200\}$ for $\omega \in \{1, 2, 3\}$.



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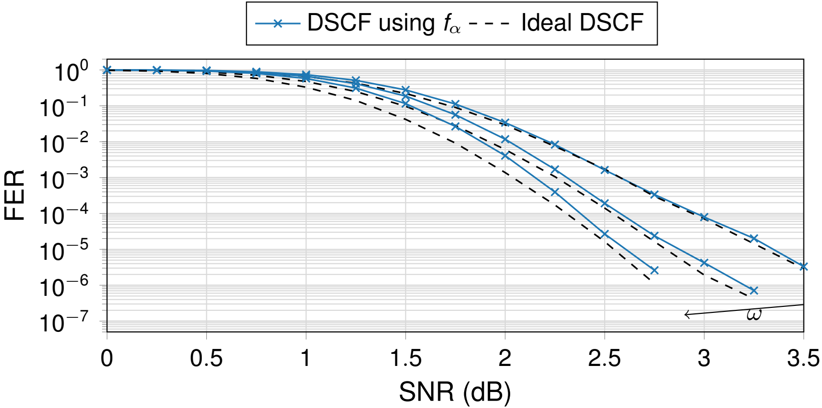

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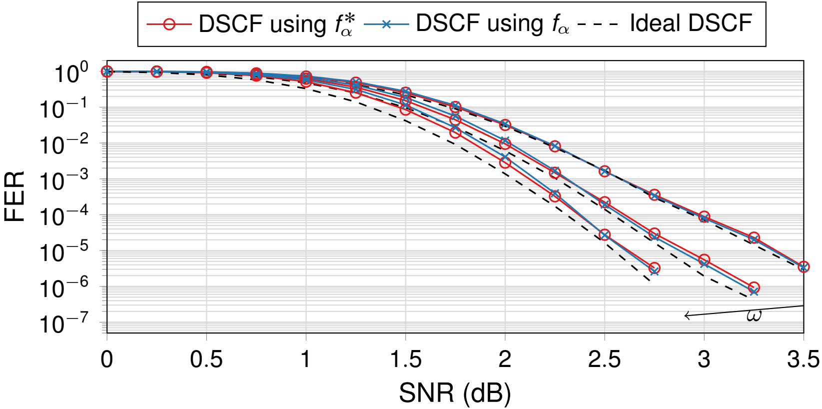
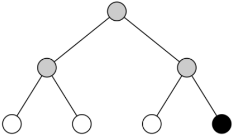


# Incorporation of Special Nodes into DSCF

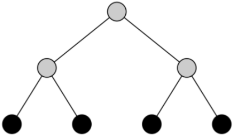
- Simplification of the metric made DSCF a potential algorithm towards a practical implementation.

- Special bit-patterns (nodes) of interest: Repetition (Rep), and Rate-1 nodes.

- Examples:



Repetition (Rep)



Rate-1

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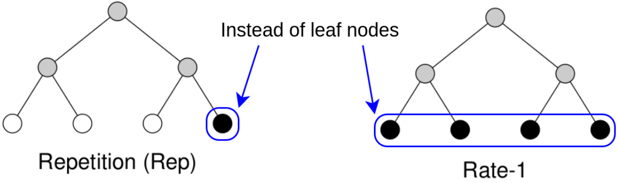

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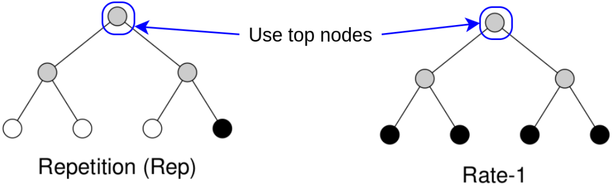

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Recall the original DSCF metric:

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Let us split it as follows:

$$M_\alpha(\mathcal{E}_\omega) = \underbrace{|L^0[\mathcal{E}_{\omega-1}]_{i_\omega}|}_{M'(L)} + \underbrace{\sum_{j \in \mathcal{E}_{\omega-1}} |L^0[\mathcal{E}_{\omega-1}]_j| + S_\alpha(\mathcal{E}_\omega)}_{M''(L)}.$$

- $M'(L)$ is the *instantaneous value* obtained from the node, on the spot.
- $M''(L)$ is the *accummulative value*, formed over the course of the decoding.

# Incorporation of Special Nodes into DSCF

$$M(L) = M'(L) + M''(L).$$

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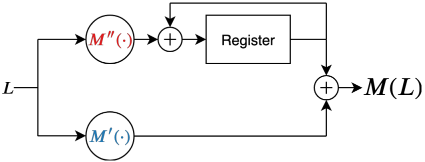


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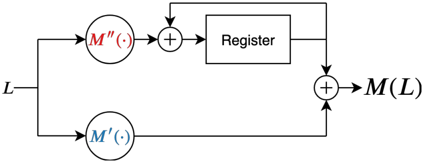



- **Main idea:** Do the same with special nodes involved.

# Involving Rep Nodes into DSCF

- The only information bit in a Rep node is repeated over at the top-level.
- The same LLR can be obtained by summing all top-node LLRs:

$$L_{\mathrm{Rep}} = \left| \sum_{i \in N_v} L^S[\mathcal{E}_{\omega-1}]_i \right|$$

Therefore;

- $L_{\mathrm{Rep}}$ is directly used to create $M'(L)$ and update $M''(L)$.

# Involving Rate-1 Nodes into DSCF

- Rate-1 nodes are uncoded sequences (no redundancy).
- As a result, all top-node indices are considered for bit-flipping individually.

For each top-node index $i$:

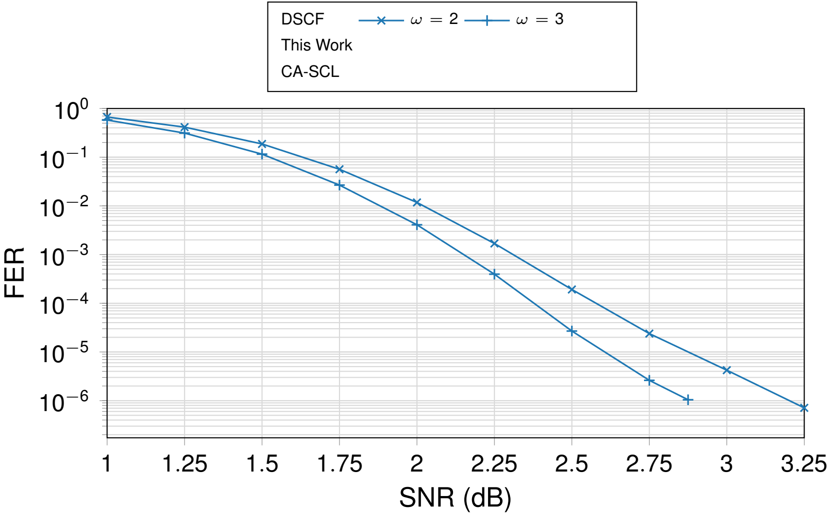
$$L_{\text{Rate-1},i} = |L^S[\mathcal{E}_{\omega-1}]_i|$$

For a Rate-1 node of size $N_v$;

- $N_v$ $M'(L)$ values are created for each top-node index.
- $M''(L)$ is updated once per Rate-1 node using $N_v$ LLRs.

# Results - Performance

- $PC(1024, 512)$, CRC length $C = 16$ bits, $T_{\max} \in \{40, 200\}$ for $\omega \in \{2, 3\}$.



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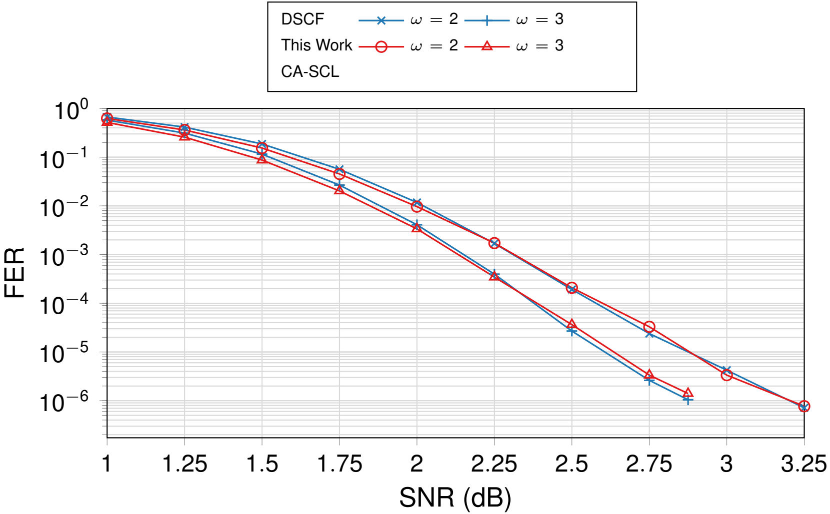

# Results - Performance

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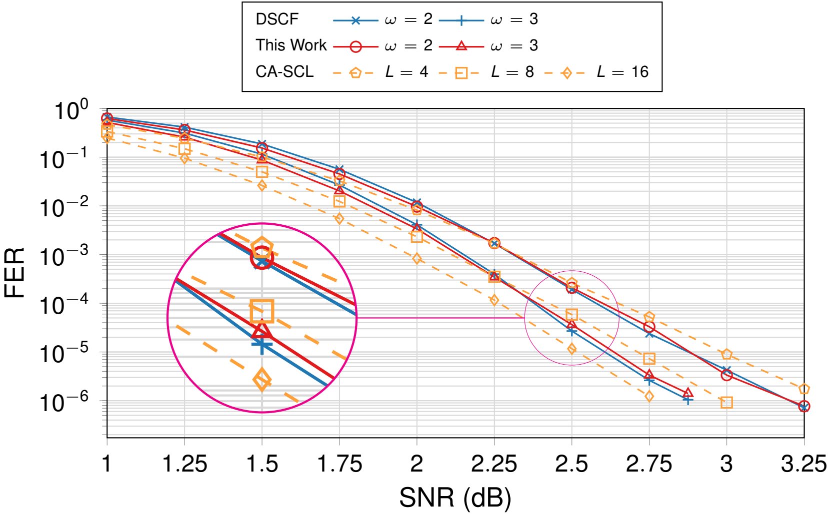
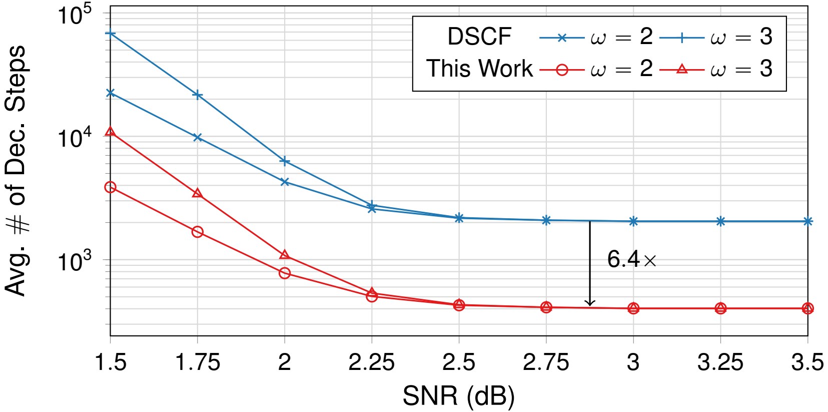


# Results - Computational Complexity

- $PC(1024, 512)$, CRC length $C = 16$ bits, $T_{\max} \in \{40, 200\}$ for $\omega \in \{2, 3\}$.



# Conclusion

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- We showed how to:
  - Replace transcendental computations with a simple threshold
  - Implement some special bit patterns into DSCF to speed up the decoding.
- When all simplifications are applied:
  - The FER is equivalent to the original DSCF algorithm,
  - Expensive operations (log, exp, $\times$) are eliminated,
  - Average number of decoding steps is reduced by up to $6.4\times$.

Thank you for your attention!