An Optimal Symmetric Threshold Strategy for Remote Estimation over the Collision Channel

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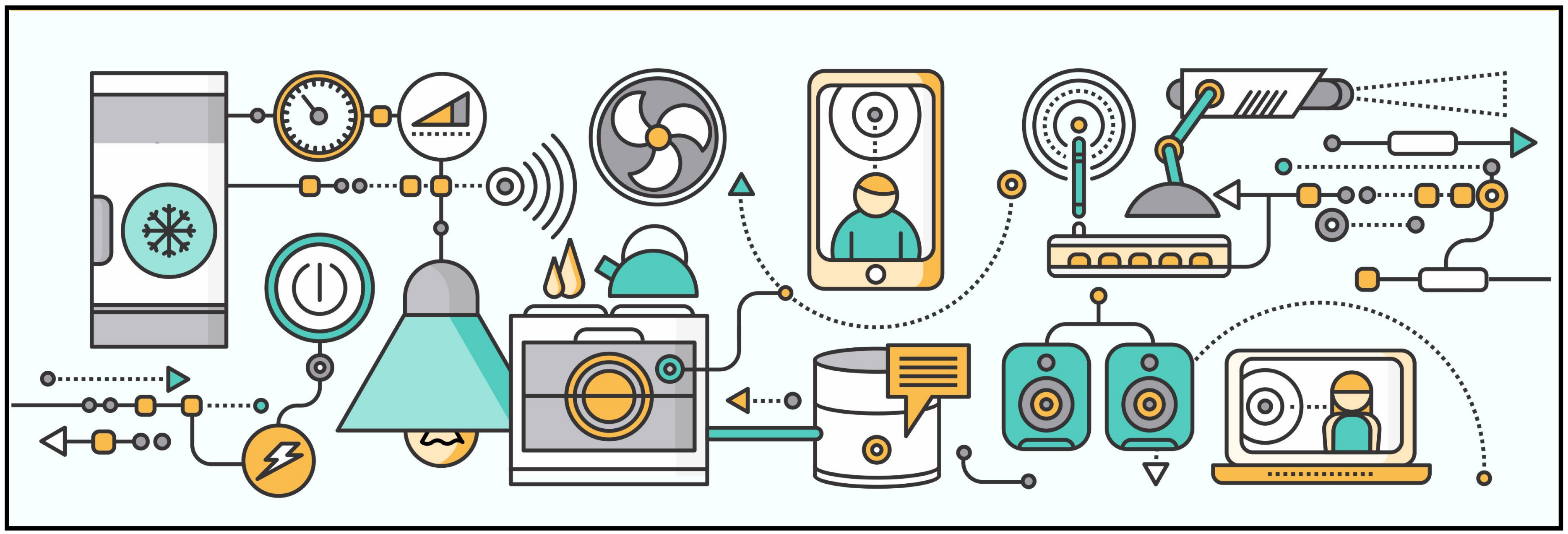
Introduction

Motivation: Modern large-scale wireless distributed systems share limited bandwidth and should operate with minimal delay.



Channel Model: limited bandwidth → subset of users communicate at any given time, collisions can occur

Introduction

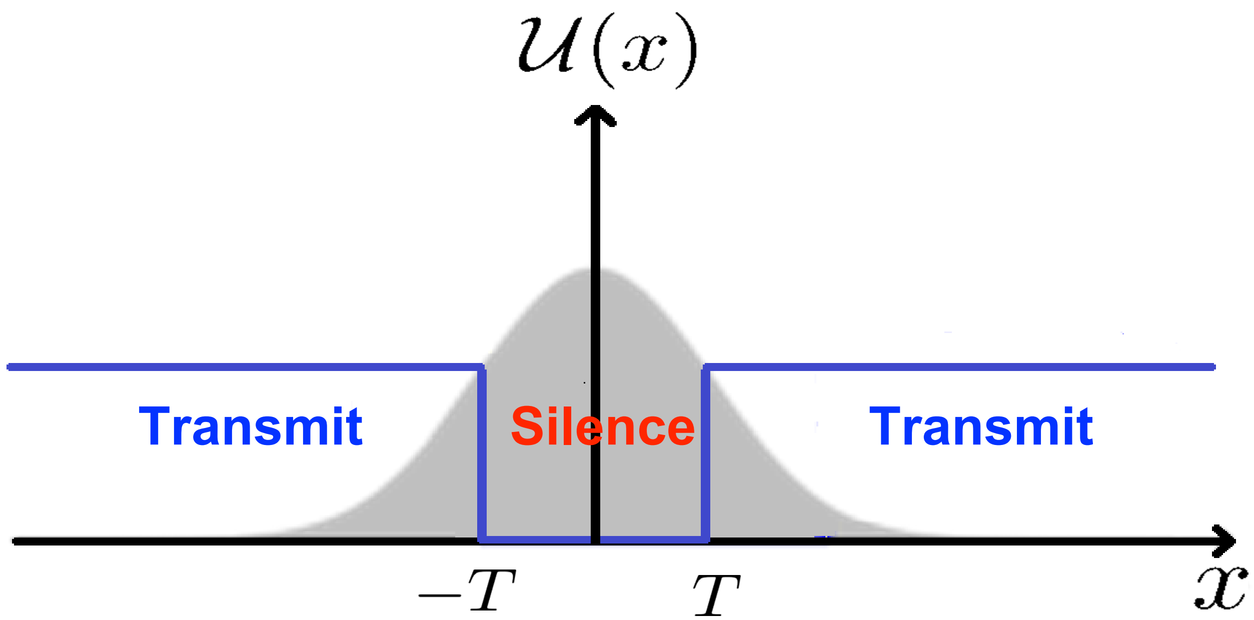
Prior work:

- systems with a single sensor
    - a limited number of transmissions [Imer&Basar'10]
    - costly communication [Lipsa&Martins'11]
    - energy harvesting sensor [Nayyar et. al'13], [Leong et. al'18]
- systems with multiple sensors
    - discrete random variables [Vasconcelos&Martins'19]
    - shared channel [Ding et. al'17]
- systems with feedback
    - based on the feedback [Huang&Dey'06], [Dogandzic&Qiu'08]
    - based on its observation and feedback quality [Michelusi&Mitra'15]

Introduction

Most relevant work:

- Vasconcelos & Martins, Optimal estimation over the collision channel, IEEE TAC 2017
    - **asymmetric** threshold strategies under one-shot transmission
- Vasconcelos & Mitra, Observation-driven scheduling for remote estimation of two gaussian random variables, IEEE TCNS 2019
    - sensors with largest magnitude observations should be transmitted
    - centralized system→ lower bound of new scheme

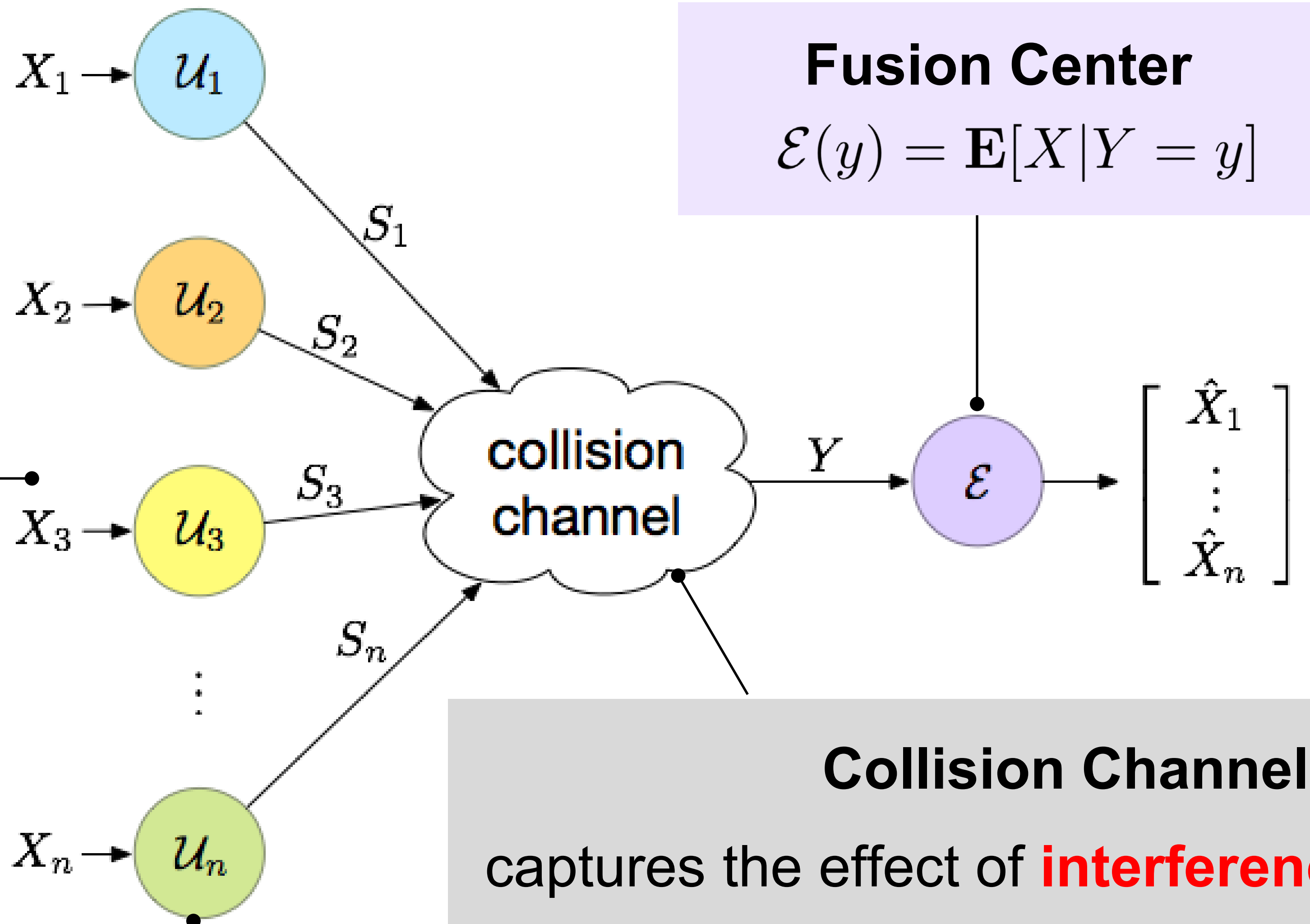


Threshold policy

Our Goal: Design a **decentralized threshold** strategy that minimizes the mean-squared error

Basic framework

Sensors



Fusion Center

$$\mathcal{E}(y) = \mathbf{E}[X|Y = y]$$

Observations

$$X_i \sim f_X$$

$$X_i \perp\!\!\!\perp X_j$$

$$f_X(x) > 0,\ x \in \mathbb{R}$$

$$f_X(x) = f_X(-x)$$

Collision Channel

captures the effect of **interference**

more than $\boldsymbol{k}$ transmissions results in a **collision**

Threshold Policy

$$U_i \in \{0, 1\}$$

Stay silent $|X_i| < T$

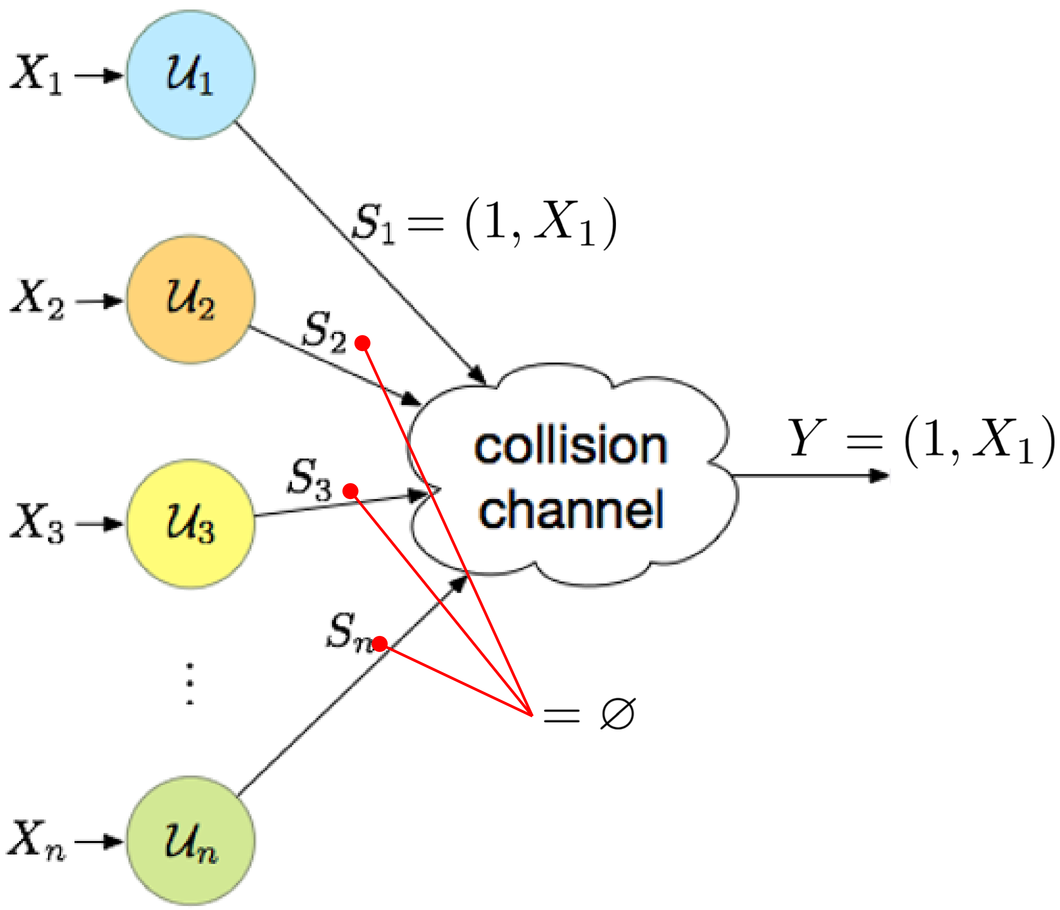
Transmit $|X_i| \geq T$

each sensor independently compares measurement to threshold

Collision channel

Reliable transmission

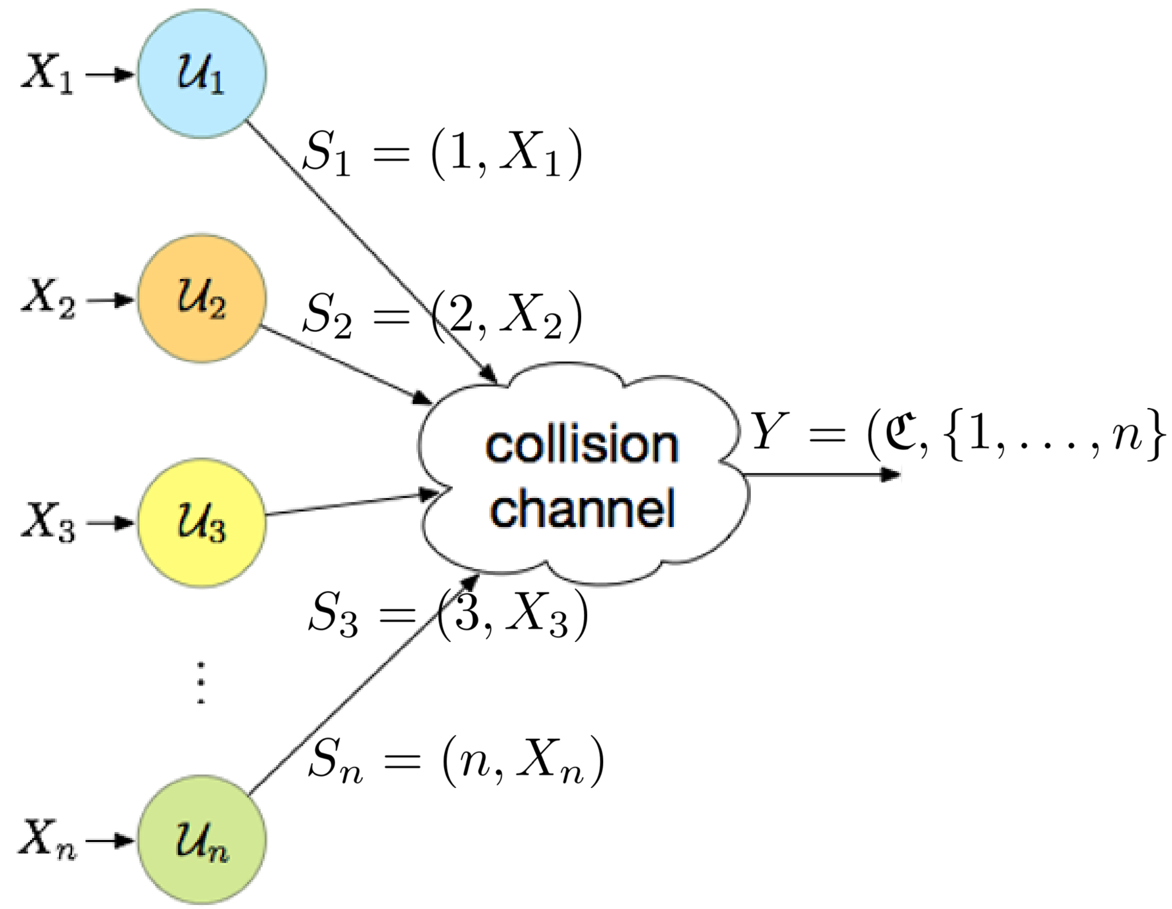
$$\sum_{i=1}^{n} U_i \leq k$$



$$Y = \{(i, X_i) | i \in \mathbb{S}\}$$

Collision

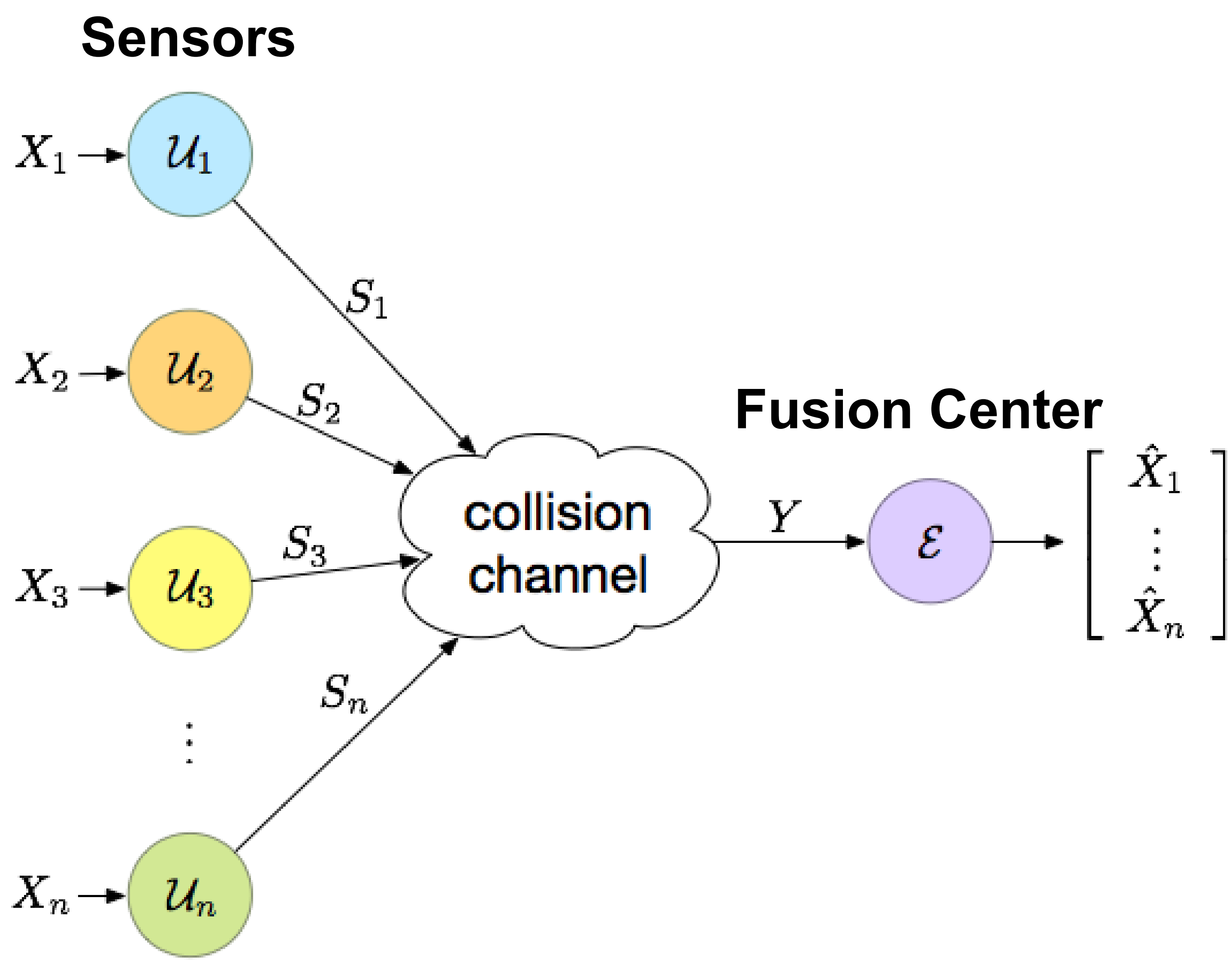
$$\sum_{i=1}^{n} U_i \geq k + 1$$



$$Y = (\mathfrak{C}, \mathbb{S})$$

$$\mathbb{S} = \{i | U_i = 1\}$$

Estimation over the collision channel



Problem

Find an optimal threshold $T^\star$ that minimizes MSE

MSE

$$\mathcal{J}_{n,k}(T) \triangleq \frac{1}{n}\mathbf{E}\left[\sum_{i=1}^{n}\left(X_i - \hat{X}_i\right)^2\right]$$

Main results

Estimator

$$\hat{X}_i = \begin{cases} X_i \mathbf{1}(|X_i| \geq T) & \text{if } \sum_{i=1}^n U_i \leq k \\ 0 & \text{if } \sum_{i=1}^n U_i \geq k+1 \end{cases}$$

Cost Function

$$\mathcal{J}(T) = \mathbf{E}\left[X^2\right] - \mathbf{E}\left[X^2 \mathbf{1}(|X| \geq T)\right] F_{n,k}(T)$$

cdf for an order statistic

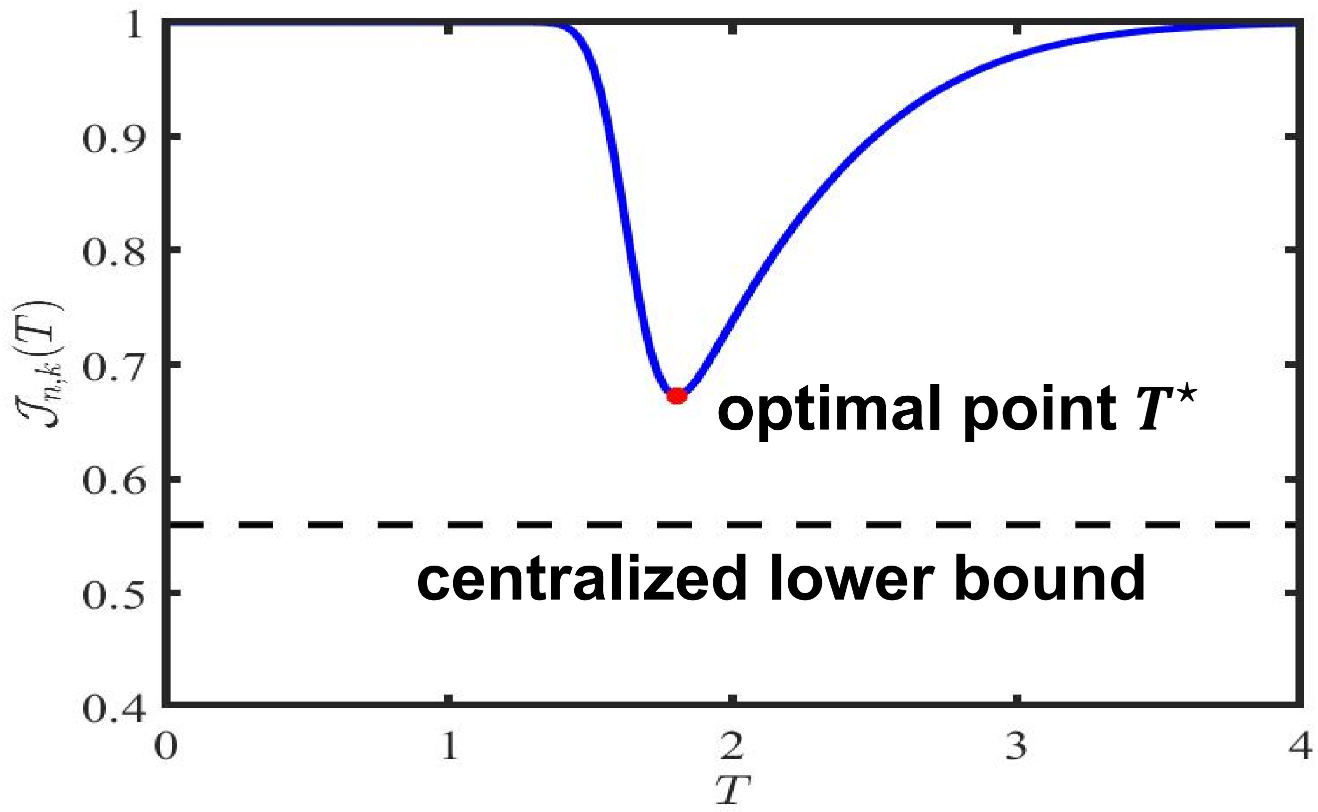
$$F_{n,k}(T) = \sum_{j=0}^{k-1} \binom{n-1}{j} (F_Z(T))^{n-j-1}(1-F_Z(T))^j, \; Z = |X|$$

results based on **order statistics**, hold for **arbitrary** symmetric distributions

Main results

Theorem

There exists a unique threshold: $T^{\star} = \mathrm{argmin}_{T \in \mathbb{R}} \mathcal{J}_{n,k}(T)$



$n = 200, k = 20$

$X_i \sim \mathcal{N}(0, 1)$

Sketch of Proof:

Step 1: Compute derivative

Step 2: Prove there is a unique local minimum

Sketch of Proof

Step 1

$$\mathcal{J}'(T) = f_{n,k}(T) \underbrace{\left[\frac{T^2 f_Z(T) F_{n,k}(T)}{f_{n,k}(T)} - \mathbf{E}[X^2 \mathbf{1}(|X| \geq T)] \right]}_{\doteq h(T)}$$

Step 2

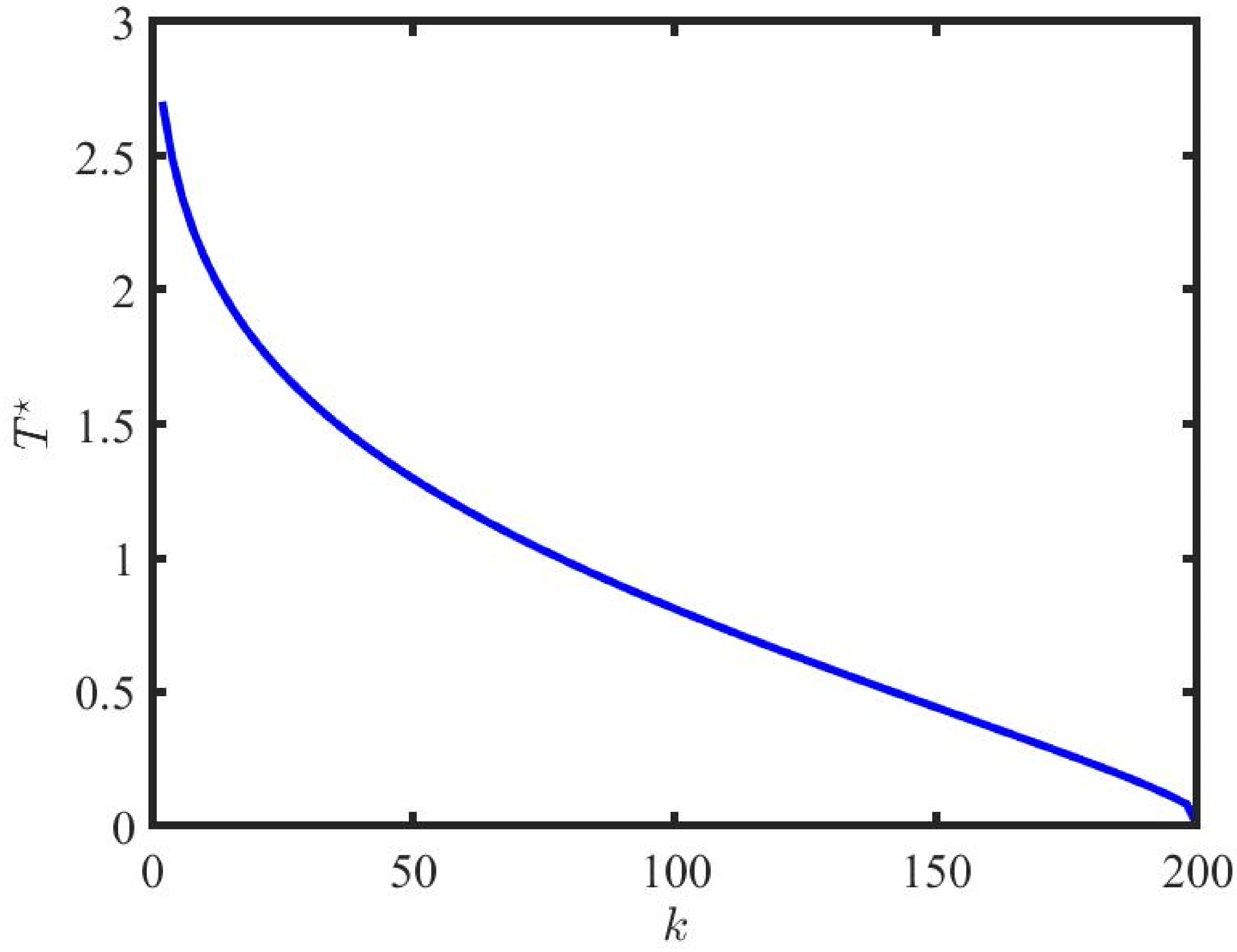
1) $f_{n,k}(T) = k \begin{pmatrix} n-1 \\ k \end{pmatrix} (F_Z(T))^{n-k-1} (1 - F_Z(T))^{k-1} f_Z(T) > 0$

2) $h(T)$ is a strictly increasing continuous function of T for $T \in [0, +\infty)$
$h(0^+) < 0$ and $h(\infty) > 0$

⇩

There is a unique threshold such that $\mathcal{J}'(T^\star) = 0$ and $T^\star = \text{argmin}_T \mathcal{J}(T)$

Optimal threshold vs channel capacity

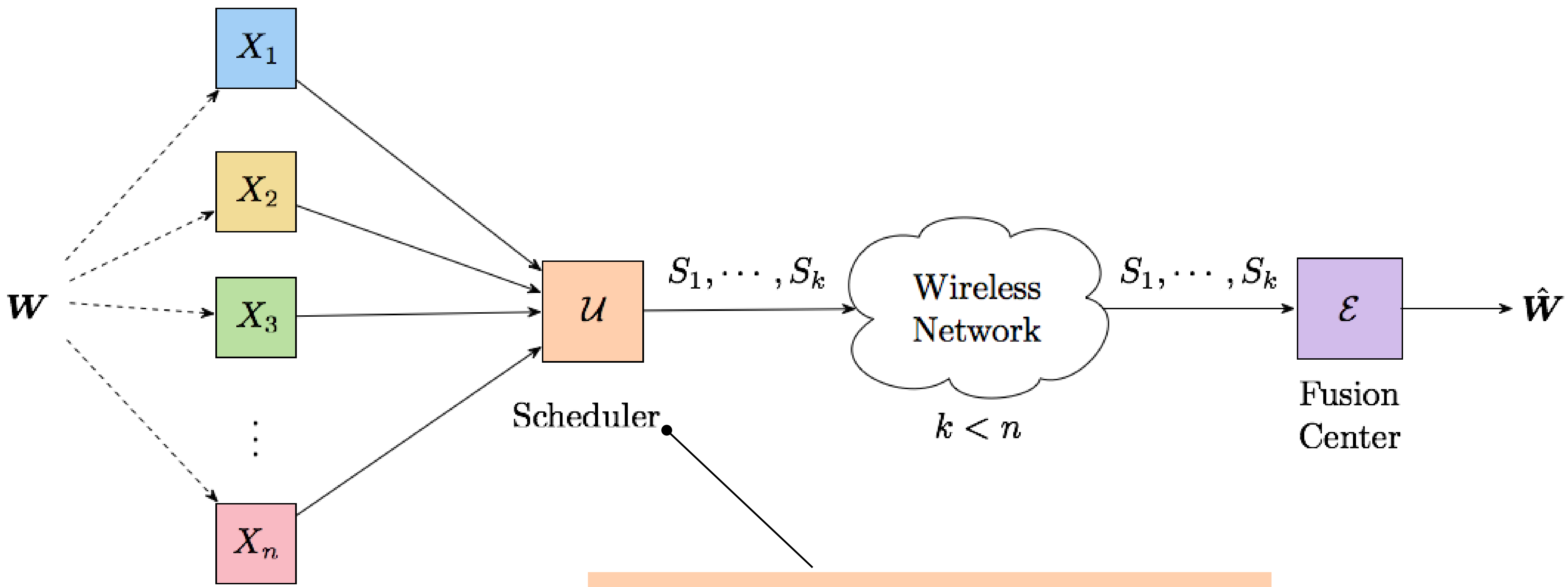


$n = 200$

$X_i \sim \mathcal{N}(0, 1)$

- Optimal threshold $T^\star$ decreases as collision capacity k increases
- When channel supports all packets, optimal threshold is zero

Centralized lower bound



selects largest k measurements

Optimal performance of this system is a lower bound to the decentralized problem

1. Vasconcelos & Mitra, Observation-driven scheduling for remote estimation of two gaussian random variables, IEEE TCNS 2019

Centralized lower bound

$$\mathcal{J}_L = \frac{1}{n} \sum_{i=k+1}^{n} \mathbf{E}\left[Z_{(i)}^2\right]$$

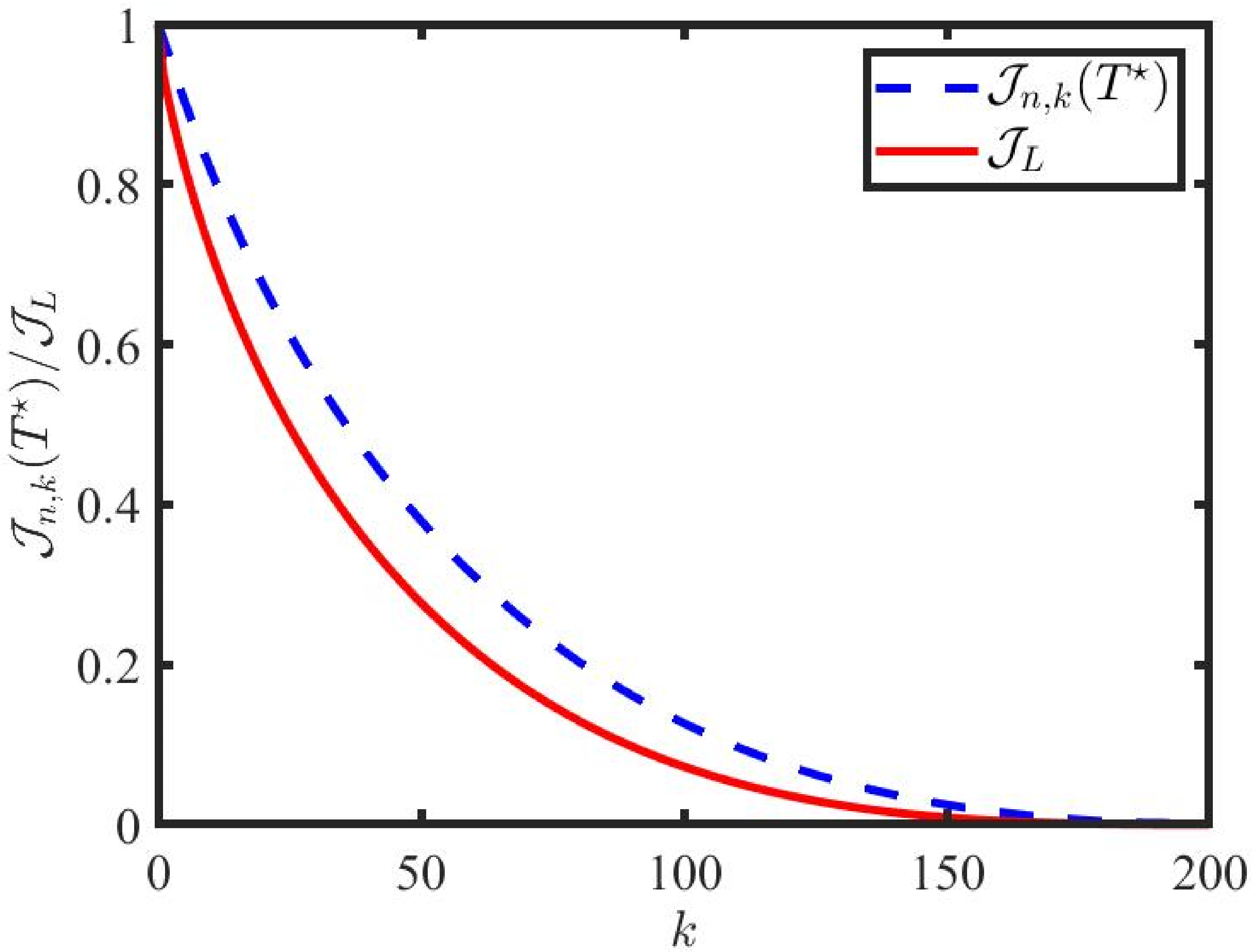
$Z_{(i)}$ denotes the i-th largest value in $\{|X_i|\}_{i=1}^n$

$$\mathbf{E}\left[Z_{(t)}^2\right] = \frac{\int_0^\infty z^2 F_Z(z)^{n-t} \left(1 - F_Z(z)\right)^{t-1} f_Z(z) dz}{\mathrm{B}(n-i+1, i)}$$

$Z = |X|$

beta function

Decentralized vs Centralized



$n = 200$

$X_i \sim \mathcal{N}(0,1)$

- As capacity k increases, MSE decreases
- Decentralized scheme performs close to centralized scheme

Summary & ongoing work

1. Summary:

Analyzed the optimal symmetric threshold policy for remote estimation problem

Provided existence of unique optimal threshold

2. Ongoing work:

Consider local communication

Analysis for unknown (or imprecise) probabilistic models

- Quantile-based decentralized scheme

Thank you!