



# Extended Object Tracking using Hierarchical Truncation Measurement Model with Automotive Radar

**2020 International Conference on Acoustics, Speech, and Signal Processing**

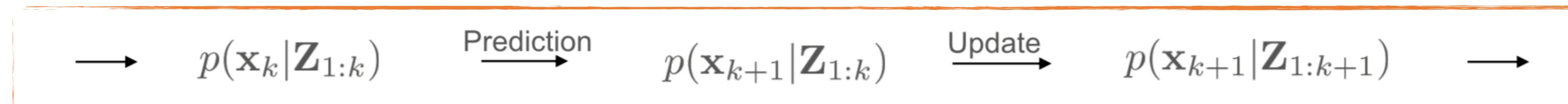
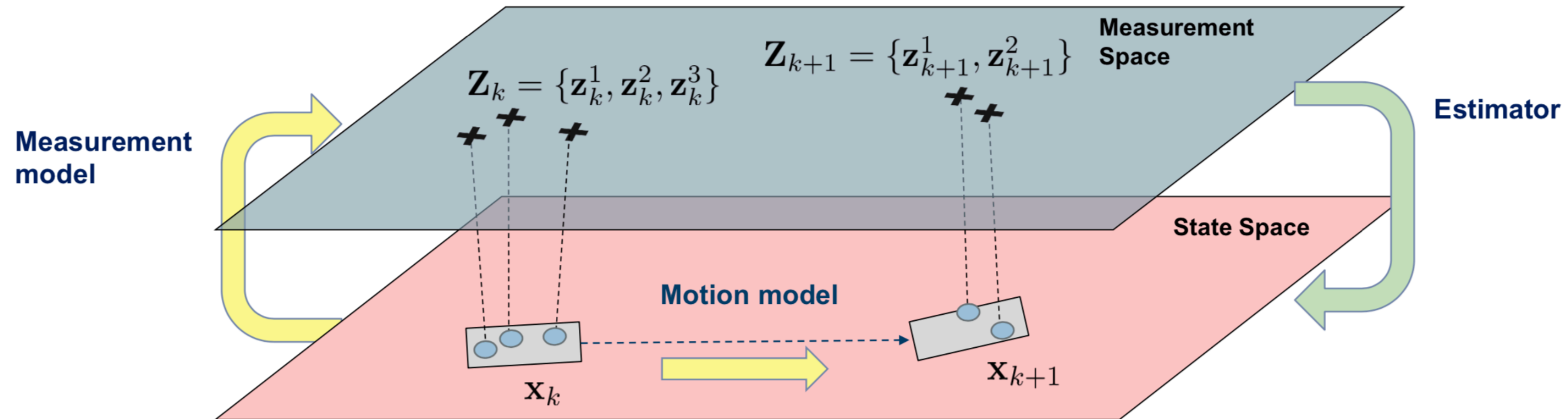
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This work is done during his internship at MERL.**

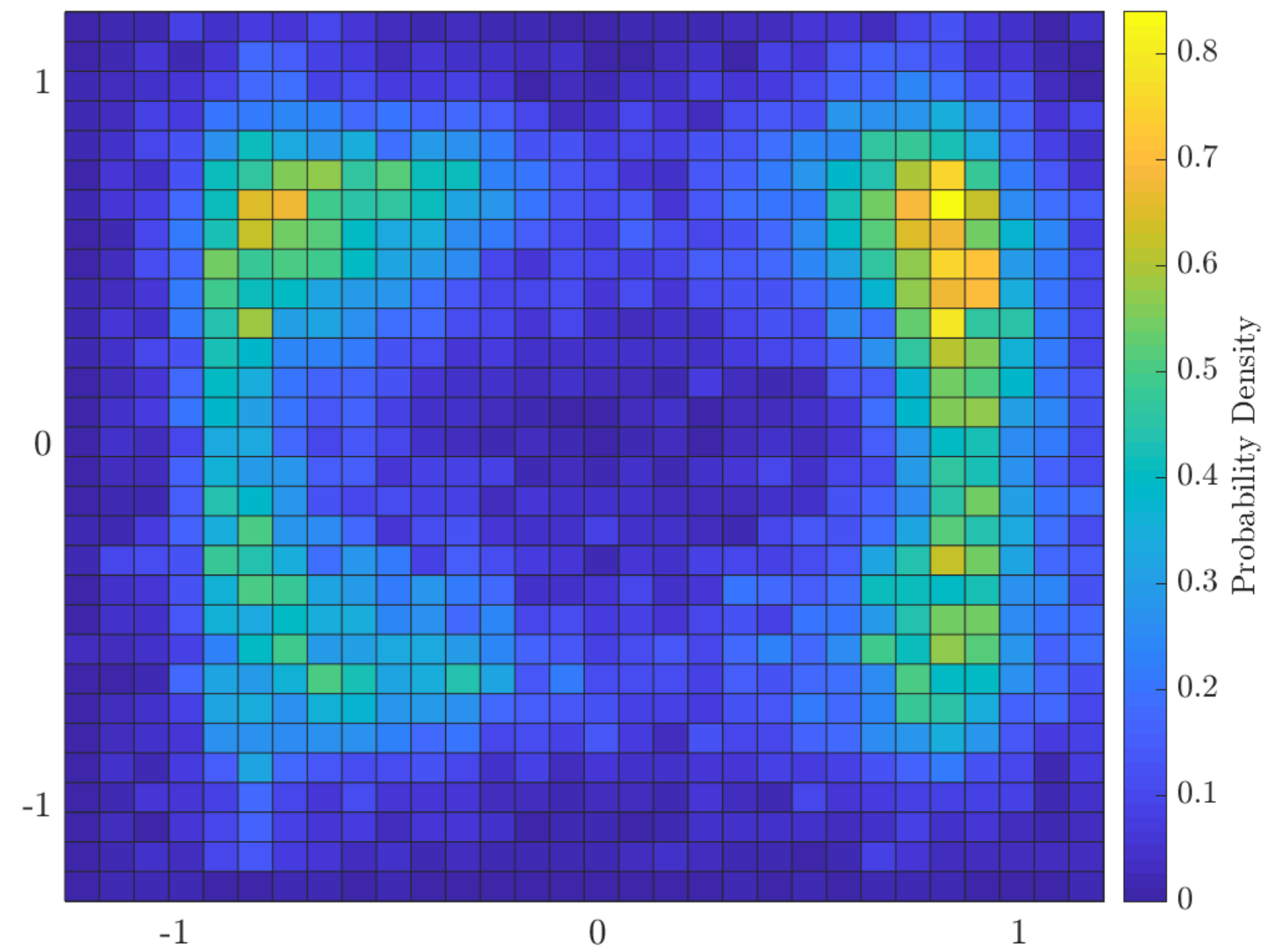
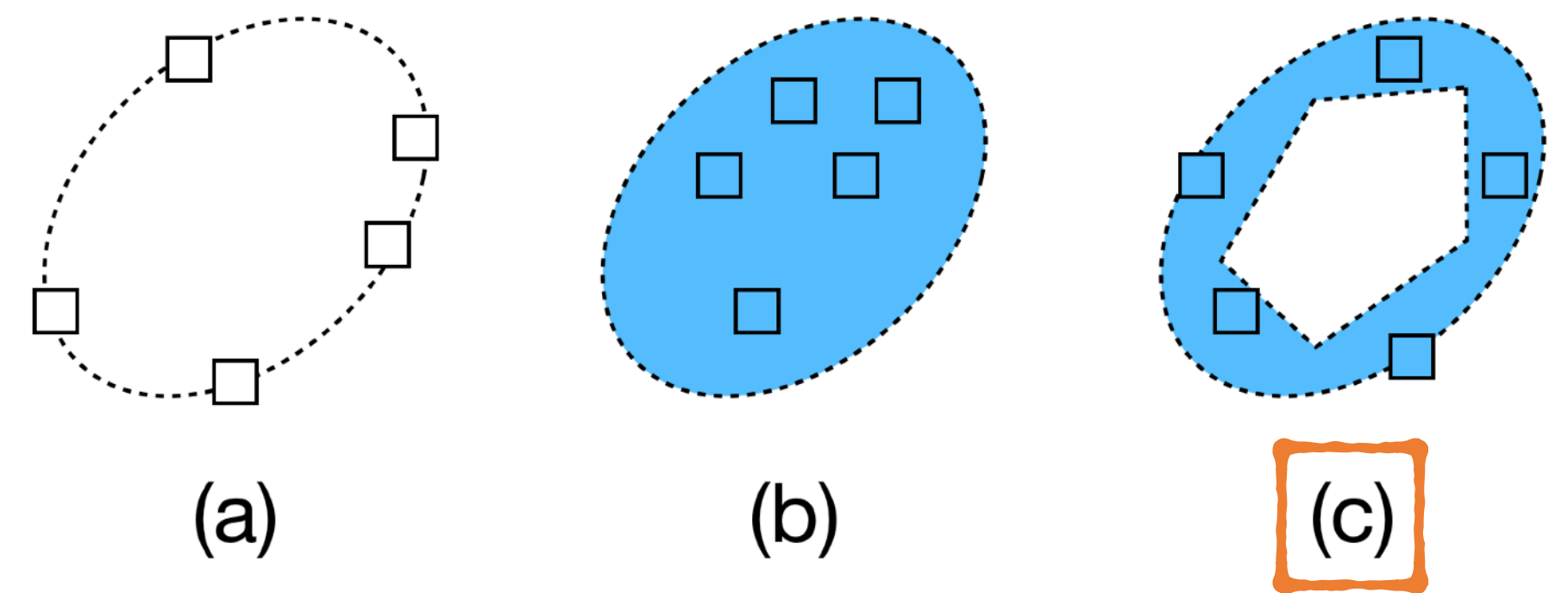
# Topic: Extended Object Tracking

- **Point object** generates at most **one** measurement per time step.
- **Extended object** generates **multiple measurements** per time step.
- Object state of interest: position, kinematic state (velocity, heading, etc.) and **extent state** (shape and size).
- Recursive Bayesian estimation:



# Motivations

- Real-world automotive radar measurements are typically distributed around edges of rigid objects (e.g., vehicles) with a certain volume.
- Common spatial models: (a) contour model, (b) surface model, are generally not applicable.
- Surface-volume model (c) capture the spatial characteristics of automotive radar measurements.
- Random matrix approach is a prominent example of surface model; it assumes elliptical object shape and is simple to implement.
- Can we leverage on the random matrix approach and the spatial characteristics of automotive radar measurements?



Histogram of accumulated Radar point cloud in unit coordinates, extracted from nuScenes dataset.

# Main Contributions

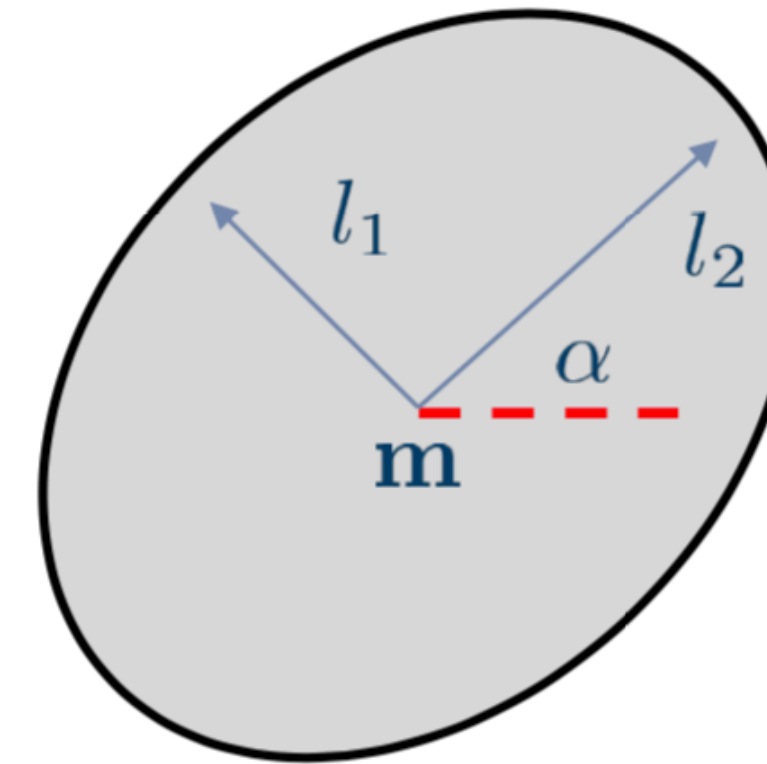
- A new surface-volume model, ***the Hierarchical Truncated Gaussian measurement model***, that resembles the spatial characteristics of real-world automotive radar measurements.
- A new random matrix based extended object tracking algorithm tailored to the new surface-volume model.
- ★ Integrating the new surface-volume model into random-matrix approach enables light-weight, realistic method implementable on automotive ECU.



# Modeling

## Dynamic model

- **Location and kinematic state:** vector  $x$ .
  - Constant velocity, Coordinated turn, etc.
  - Gaussian pdf:  $N(x; m, P)$ .
- **Extent state:** SPD matrix  $X$ .
  - Elliptic shape.
  - Extent typically has constant size, rotating during turns.
  - Inverse-Wishart pdf:  $IW(X; \nu, V)$ .



$$= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} l_1^2 & 0 \\ 0 & l_2^2 \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$



Implicit ellipse equation:  
 $(\mathbf{y} - \mathbf{m})^T V^{-1} (\mathbf{y} - \mathbf{m}) = 1$

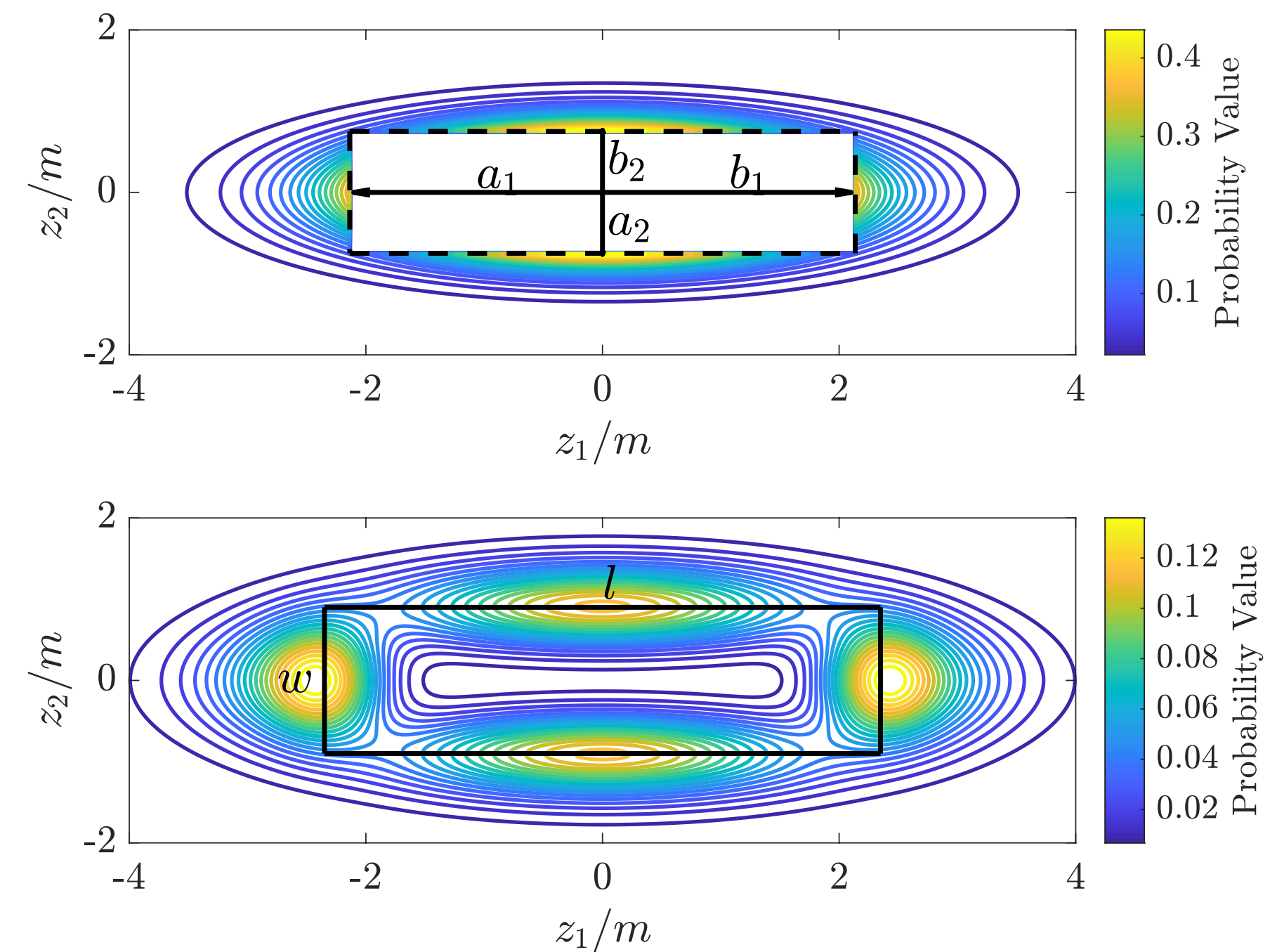
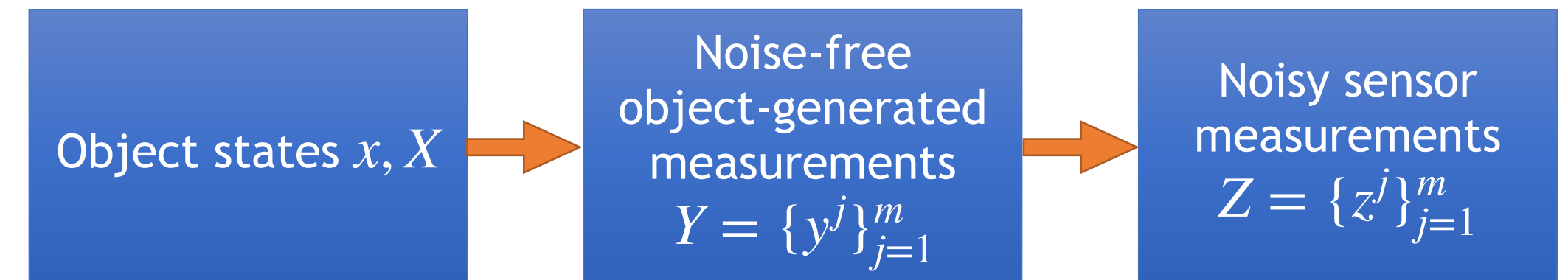
Object length and width obtained from eigen-decomposition of  $X$ .

# Modeling

## Measurement model

- Noisy sensor detection  $z$  stems from noise-free measurement source  $y$ .
- Measurement source pdf: Truncated Gaussian  $p(y | x, X) = TN(y; h(x), X, D)$ .
- Sensor noise pdf: Gaussian  $p(z | y) = N(z; y, R)$ .
- Hierarchical Truncated Gaussian measurement likelihood:  

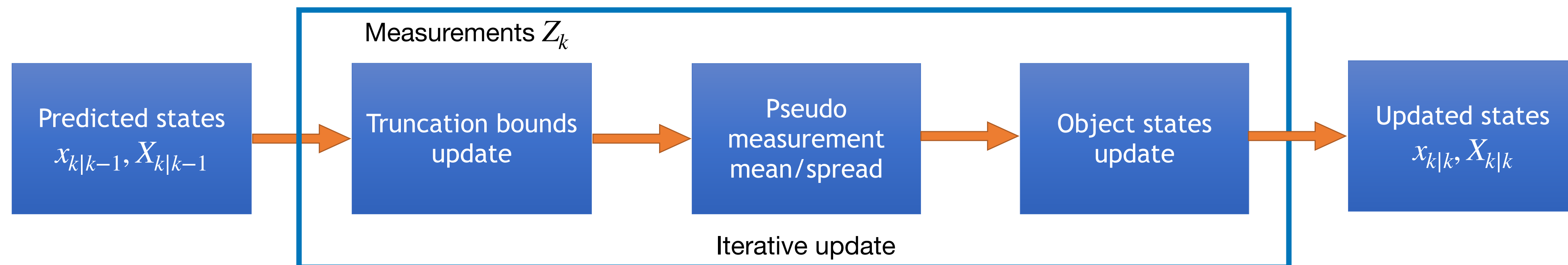
$$p(z | x, X) = \int N(z; y, R) TN(y; h(x), X, D) dy.$$
- Setting truncation bounds to  $+\infty$  to model **self-occlusion** feature, i.e., partial-view measurements.



Pdfs of Truncated Gaussian and Hierarchical truncated Gaussian.

# Problem Formulation and Solution

- **Objective:** Recursively calculate the posterior  $p(x_{k|k}, X_{k|k} | Z^k) \approx N(x_{k|k}; m_{k|k}, P_{k|k})IW(X_{k|k}; v_{k|k}, V_{k|k})$ .
- **Challenges:**
  - Measurement statistics are biased → Random Matrix approach: object states updated in a *Kalman-filter-like* fashion using *mean/spread of Gaussian distributed measurements*, may not yield good tracking performance.
  - Truncation bounds need to be estimated.
- **Proposed solution:**
  - Construct Gaussian-distributed pseudo-measurement statistics.
  - Formulate the estimation of the truncation bounds as an optimization problem.
  - Use an EM-type algorithm to ***iteratively update object states and truncation bounds***.



# Proposed Update Method

## Pseudo-measurement statistics

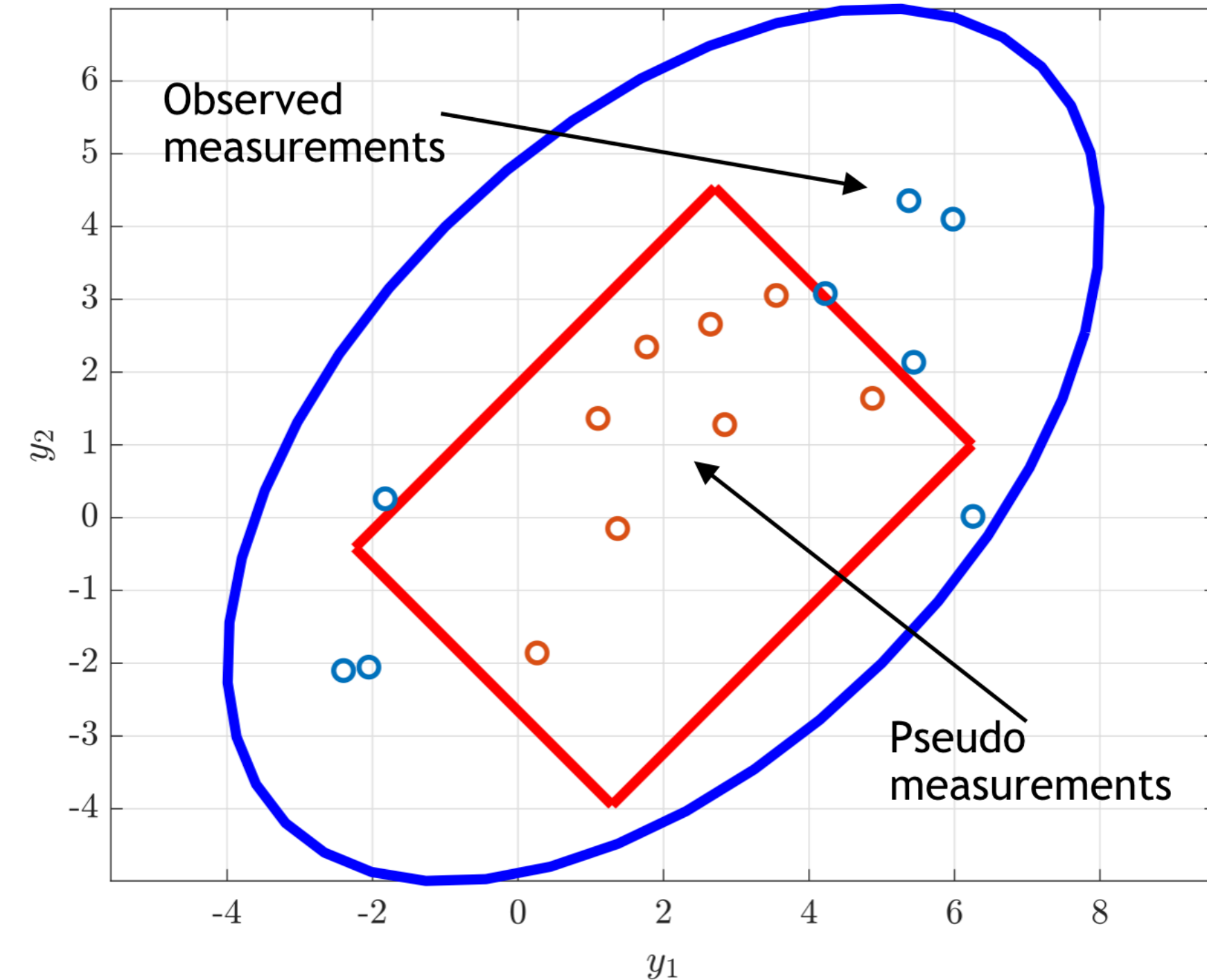
- Compute **sample** measurement mean  $\bar{z}_k$  and spread  $\bar{Z}_k$ .
- Compute **analytical** mean  $\bar{z}_k^c$  and spread  $\bar{Z}_k^c$  of Hierarchical Truncated Gaussian distribution

$$p(z_k^c | x_k, X_k) = \int N(z_k; y_k, R) TN(y_k; h(x_k), X_k, R^2 \setminus D_k) dy_k$$

- Gaussian-distributed pseudo measurement mean/spread can be constructed as the **weighted sum** of the sample and the analytical mean/spread, respectively weighted by

$$c_{D_k} = \int_{D_k} TN(y_k; h(x_k), X_k, D_k) dy_k \text{ and}$$

$$1 - c_{D_k} = \int_{R^2 \setminus D_k} TN(y_k; h(x_k), X_k, R^2 \setminus D_k) dy_k.$$



$$\frac{\text{\#Observed measurements}}{\text{\#Pseudo measurements}} = \frac{c_{D_k}}{1 - c_{D_k}}.$$



# Proposed Update Method

## Truncation bounds estimation

- **Objective:** Find the ML estimates of the truncation bounds

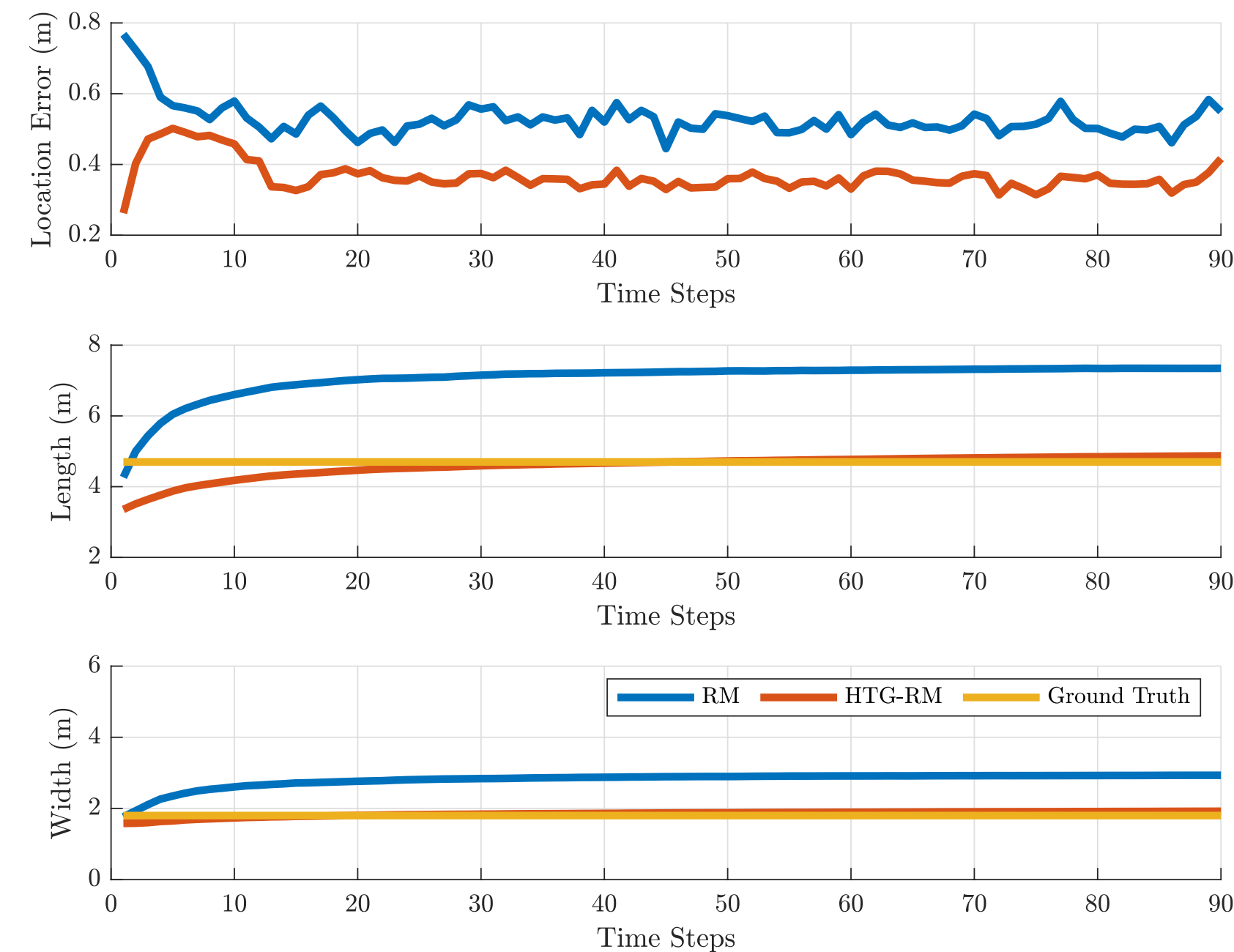
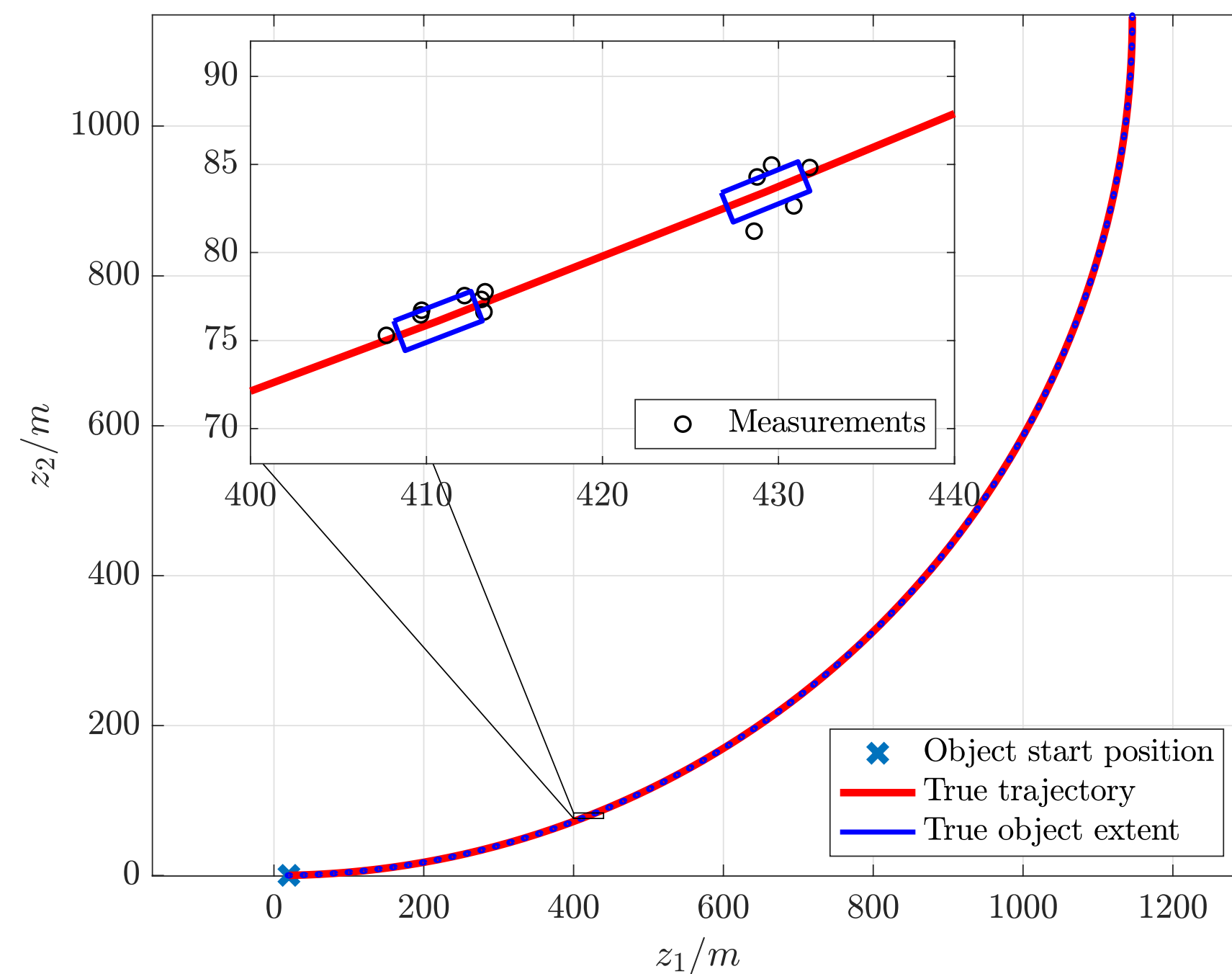
$$\arg \min_{a_k, b_k} \sum_{z_k \in Z_k} -\log p(z_k | x_k, X_k).$$

- **Challenges:** Computational demanding for online estimation.
- **Proposed solution:**
  - Decompose the joint ML estimation problem into up to four **decoupled** ML estimation problems using **expectation-maximization clustering**.
  - For each subproblem (a **univariate** constrained optimization problem), find the ML estimate using standard root-finding algorithm.

# Simulation Results

## Performance evaluation with ideal measurement model

- Rectangular object (4.7-m long and 1.8-m wide) moves following coordinated turn motion model.
- Object detections drawn from truncated uniform distribution and corrupted with Gaussian noise.
- Number of detections is Poisson distributed with mean 8.

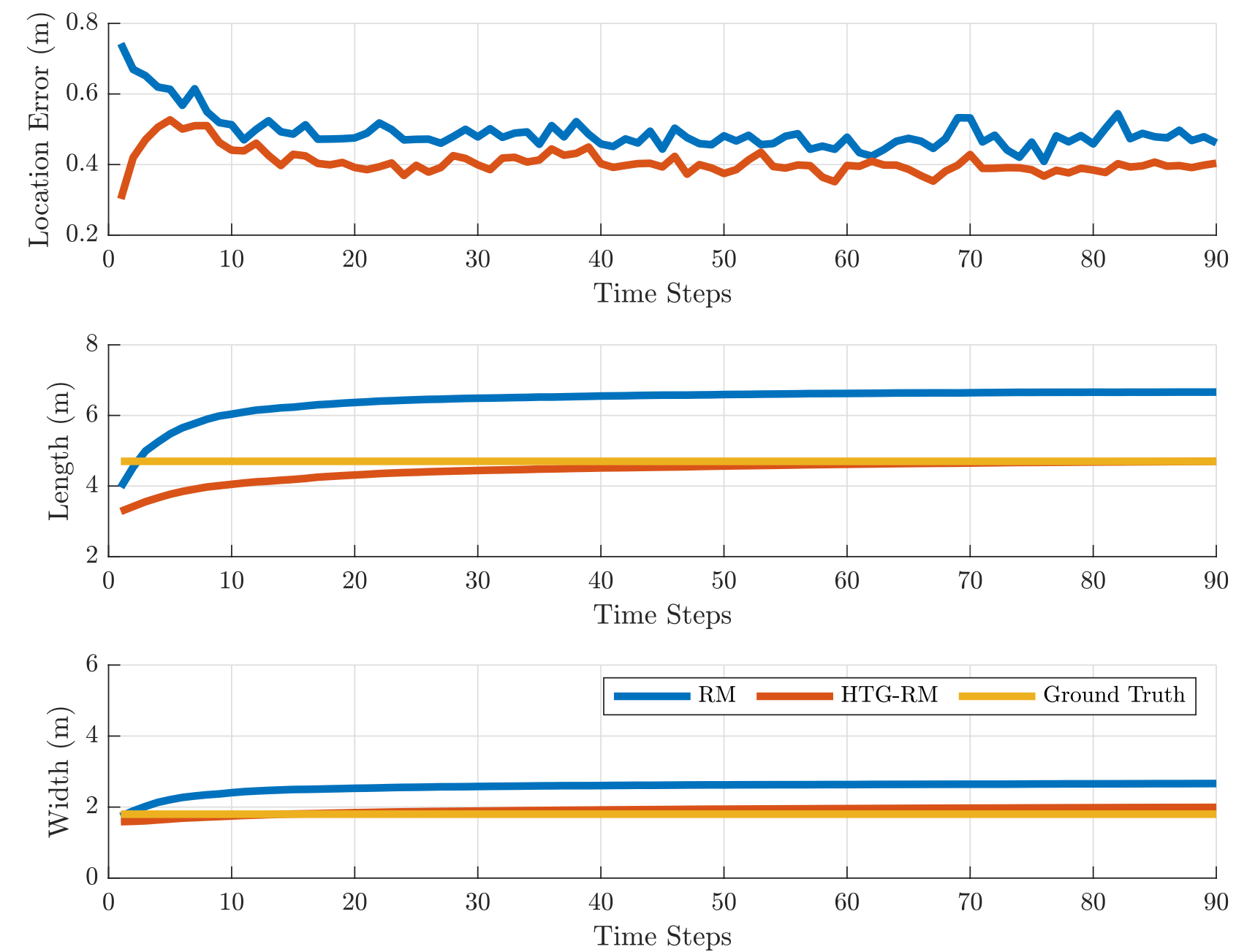
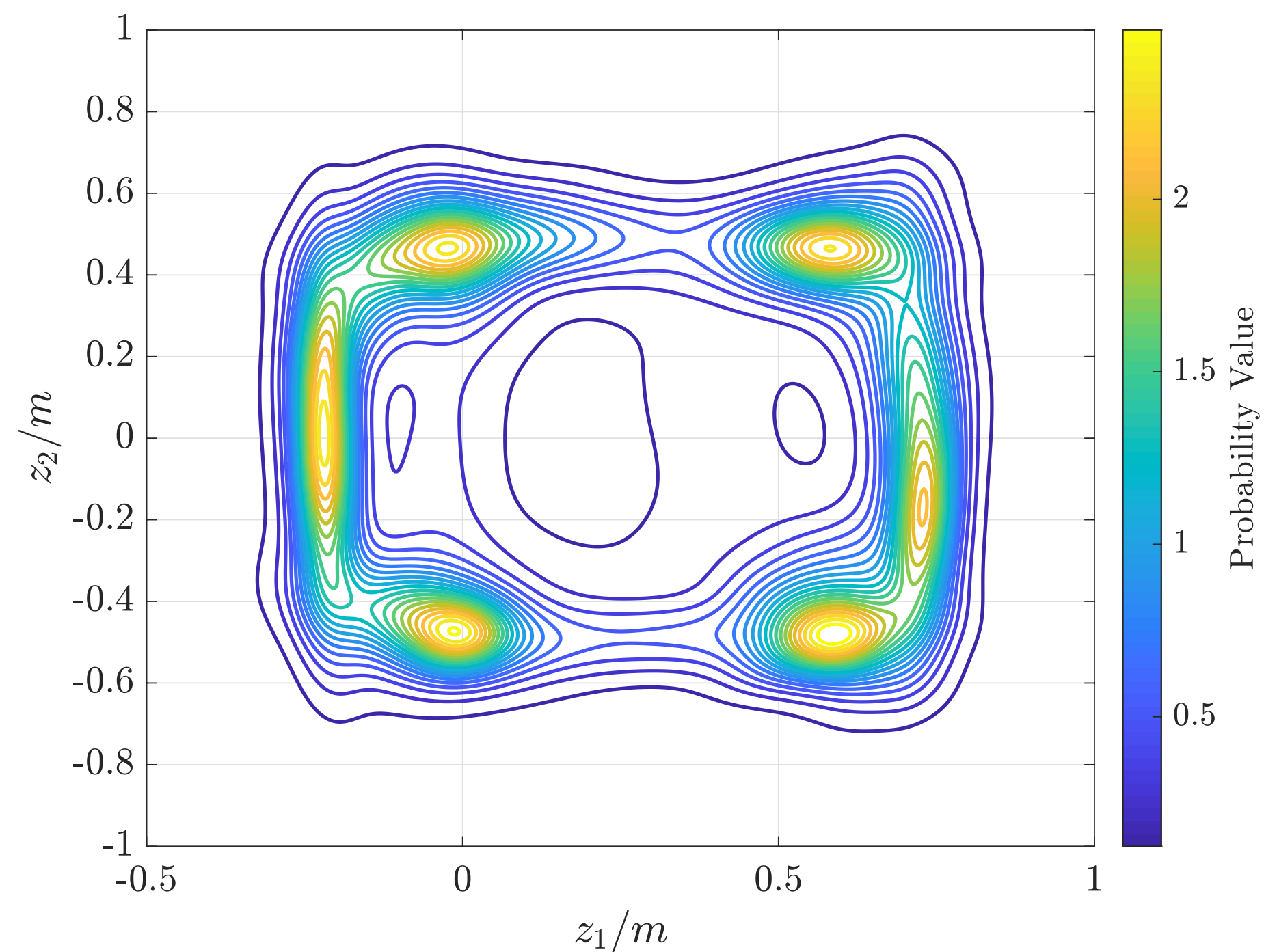


# Simulation Results

## Performance evaluation with measurement model mismatch

- Object detections drawn from an offline trained variational Radar model (student's t mixture) [Scheel 2018].
- Number of detections is Poisson distributed with mean 8.

Pdf of offline trained variational radar model



# Validation with MathWorks Measurements

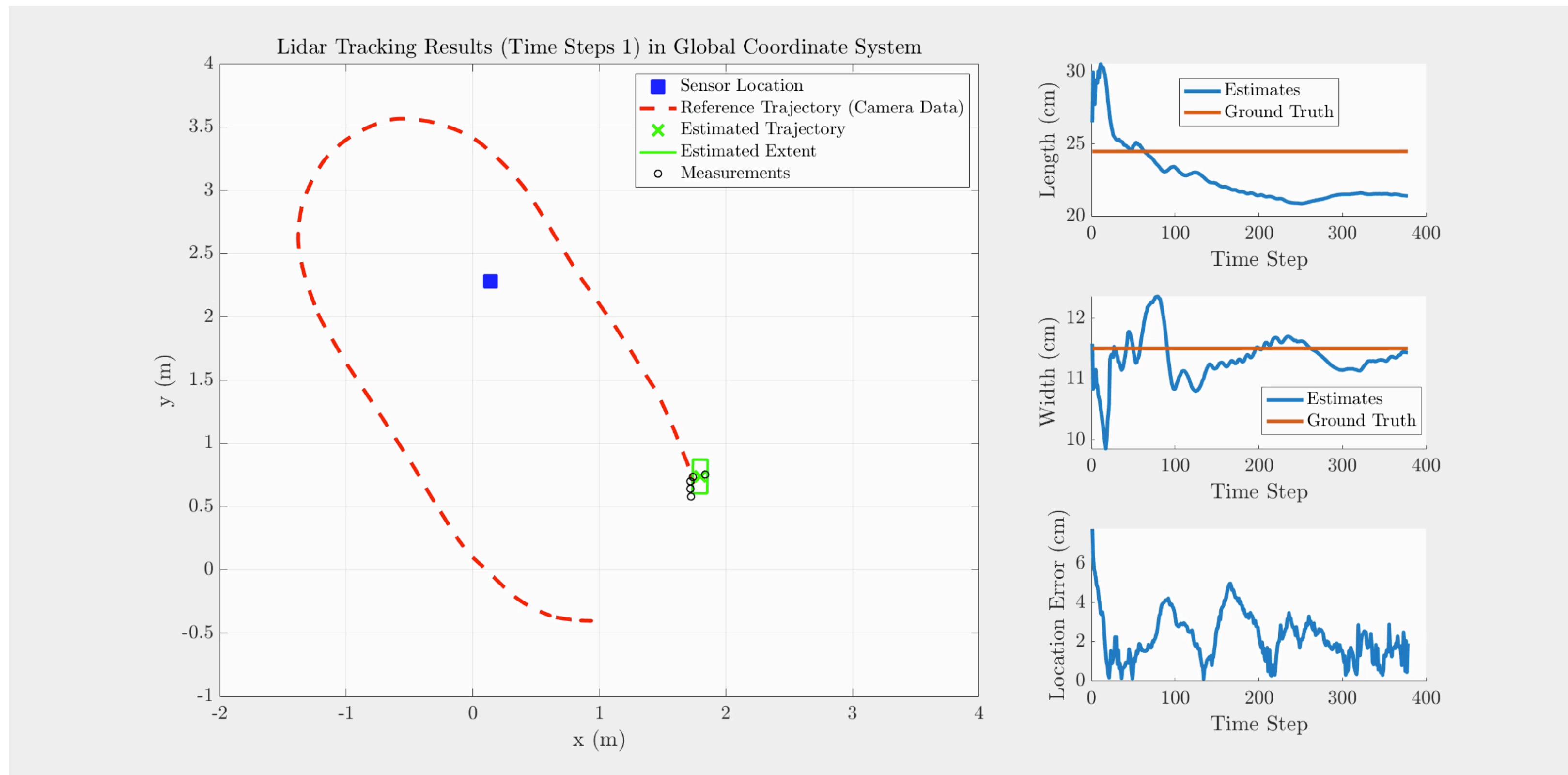
- Tracking scenario and synthetic radar measurements generated using MathWorks Automated Driving Toolbox.
- Multiple long-range, medium-range and short-range automotive radars mounted on the ego vehicle.





# Validation with In-house Hamster Lidar Data

- Hamster: Ackermann steering, velocity control, IMU, **camera (reference)**, Lidar, wheel encoders, GPS.
- Lidar sensor is fixed and one Hamster car is moving. In general, Lidar measurements lie on the edges of the object.



# Summary

- Proposed a new surface-volume model, ***the Hierarchical Truncated Gaussian measurement model***, which resembles the spatial characteristics of real-world automotive radar measurements.
- Developed a new Random Matrix based extended object tracking algorithm tailored to the new measurement model.
- Simulation results validate and demonstrate the effectiveness of the proposed approach.