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Cross-domain Joint Dictionary Learning (XDJDL) for ECG Reconstruction from PPG

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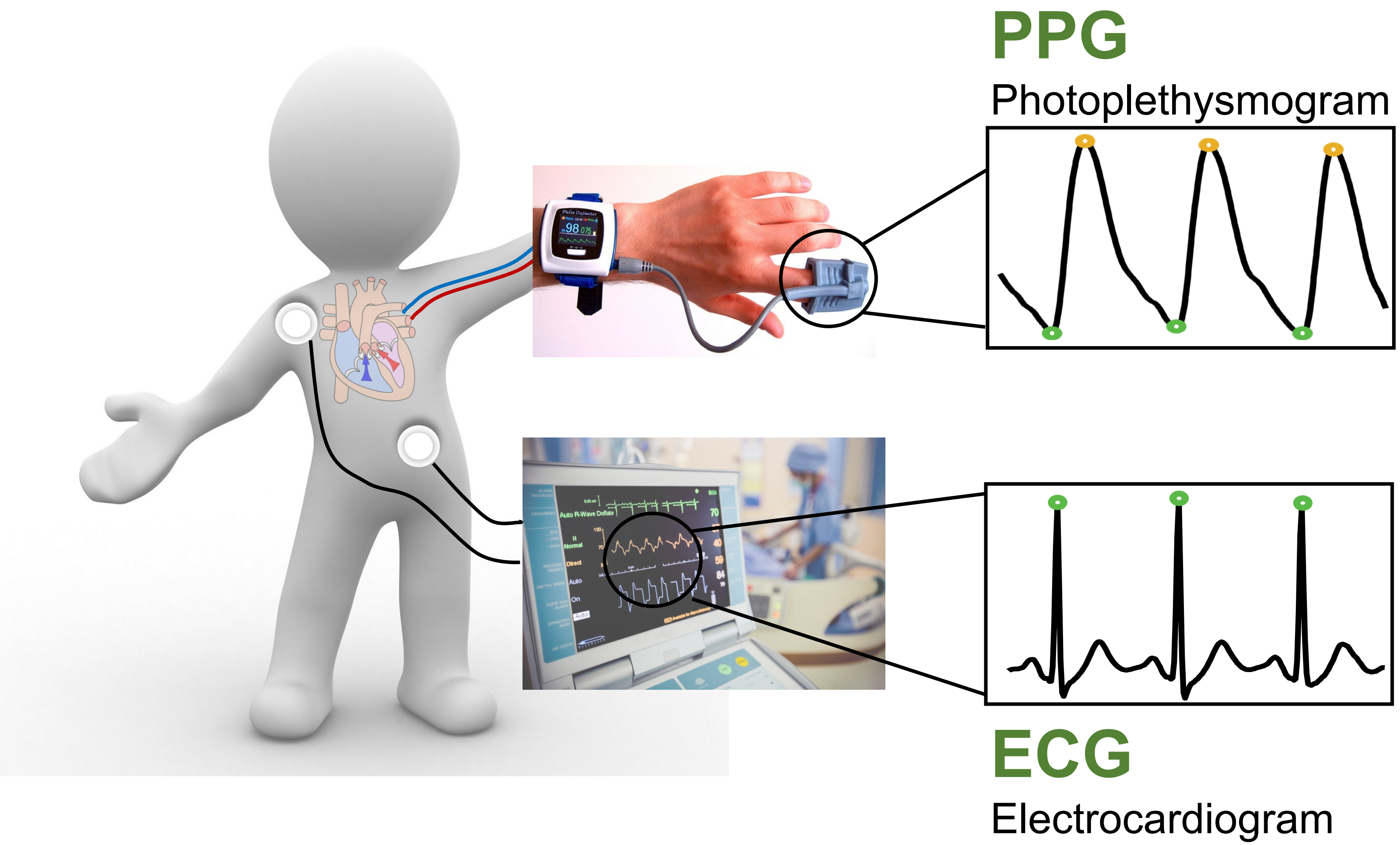
Why ECG reconstruction from PPG: Benefits & Physiological model



Optical sensing

Less direct cardiac information

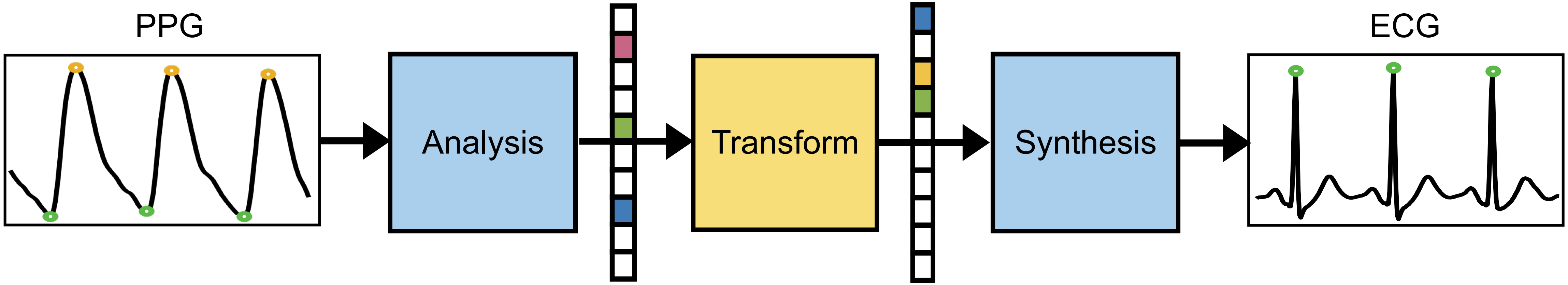
More user-friendly and cheaper

Bio-electrical signal

Clinical gold standard

Restrictive; uncomfortable

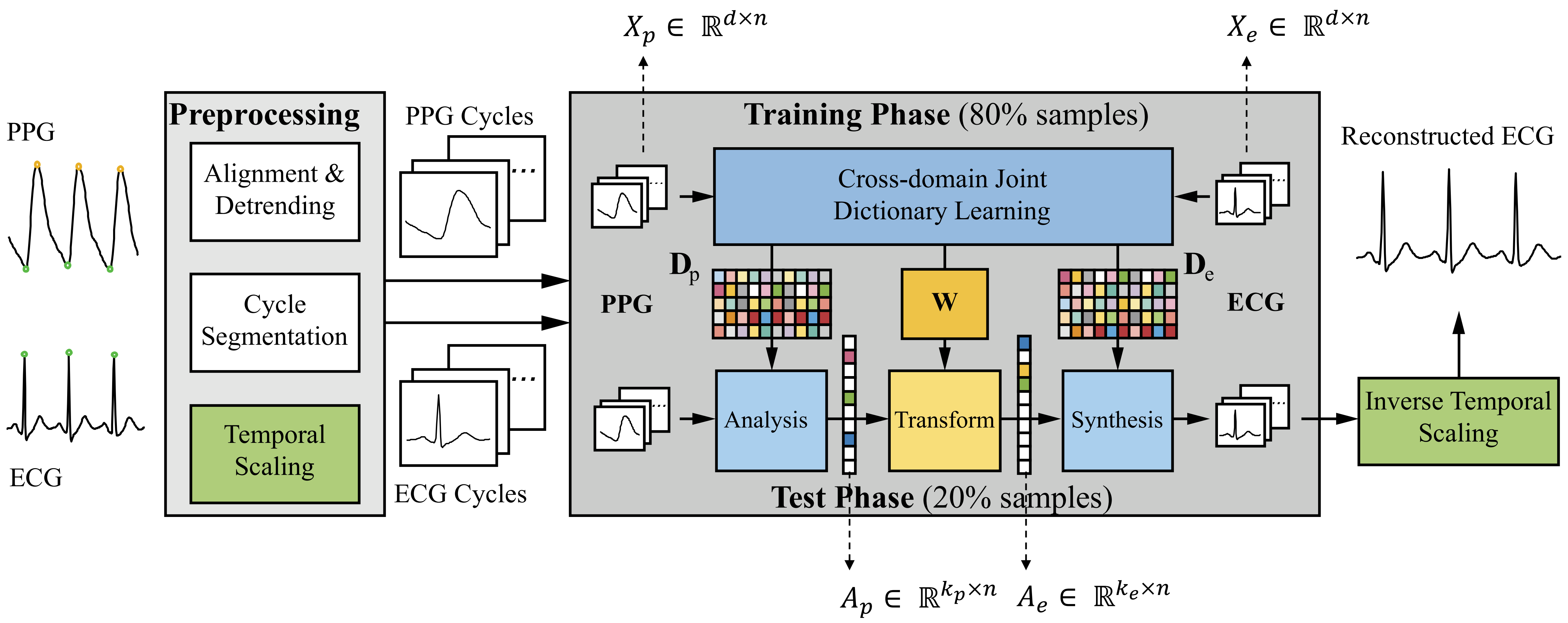
Why joint dictionary learning



- More representative and adaptive than universal dictionary, e.g. DCT [1]
- Better explain the physiological nature between PPG and ECG

[1] Q. Zhu, et al. "ECG reconstruction via PPG: A pilot study," *IEEE EMBS International Conference on Biomedical & Health Informatics (BHI)*, 2019.

Framework Overview



Mathematical model

$$\min_{\substack{\mathbf{D}_e,\mathbf{A}_e,\mathbf{D}_p,\\ \mathbf{A}_p,\mathbf{W}}} \underbrace{\|\mathbf{X}_e - \mathbf{D}_e\mathbf{A}_e\|_F^2 + \alpha\|\mathbf{X}_p - \mathbf{D}_p\mathbf{A}_p\|_F^2}_{\text{Data fidelity terms}} + \underbrace{\beta\|\mathbf{A}_e - \mathbf{W}\mathbf{A}_p\|_F^2}_{\text{Mapping fidelity term}} \quad \text{s.t.} \underbrace{\begin{array}{l}\|\mathbf{a}_{p,j}\|_0 \le t_p, \quad j = 1, ..., n \\ \|\mathbf{a}_{e,j}\|_0 \le t_e, \quad j = 1, ..., n\end{array}}_{\text{Sparsity constraint}}$$

In matrix form,

$$\begin{array}{c} \\ \min\limits_{\substack{\mathbf{D}_e,\mathbf{A}_e,\mathbf{D}_p,\\ \mathbf{A}_p,\mathbf{W}}} \end{array} \begin{array}{c} X \\ \left\| \begin{pmatrix} \mathbf{X}_e \\ \sqrt{\alpha}\mathbf{X}_p \\ \mathbf{0} \end{pmatrix} \right. \end{array} - \begin{array}{c} D \\ \begin{pmatrix} \mathbf{D}_e & \mathbf{0} \\ \mathbf{0} & \sqrt{\alpha}\mathbf{D}_p \\ -\sqrt{\beta}\mathbf{I} & \sqrt{\beta}\mathbf{W} \end{pmatrix} \end{array} \begin{array}{c} A \\ \left. \begin{pmatrix} \mathbf{A}_e \\ \mathbf{A}_p \end{pmatrix} \right\|_F^2 \end{array}$$

$$s.t. ||\mathbf{a}_{e,j}||_0 \le t_e, \quad j = 1, ..., n.$$
$$||\mathbf{a}_{p,j}||_0 \le t_p, \quad j = 1, ..., n.$$

Optimization

Step 1: sparse coding

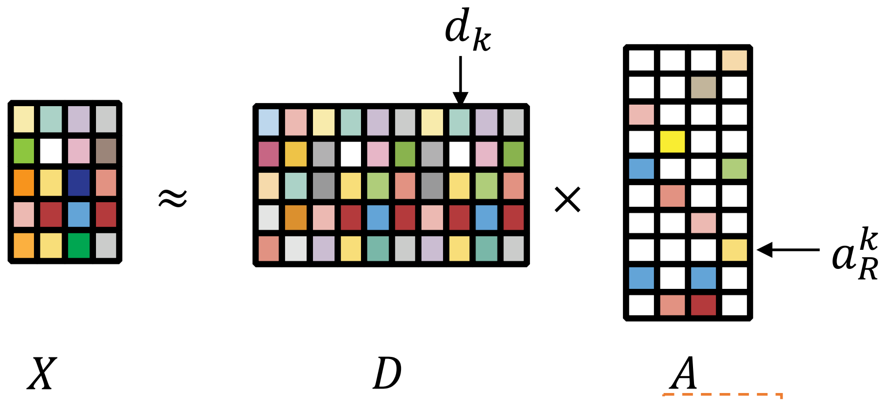
$$\min_{\mathbf{A}} \|\mathbf{X}-\mathbf{DA}\|_F^2 ,$$
$$\text{s.t. } \|\mathbf{a}_{+,j}\|_0 \le t_e, \quad j=1,\dots,n$$
$$\|\mathbf{a}_{-,j}\|_0 \le t_p, \quad j=1,\dots,n$$

Step1-1 [1] :

$$\min_{\mathbf{A}} \|\mathbf{X}-\mathbf{DA}\|_F^2$$
$$\text{s.t. } \|\mathbf{a}_j\|_0 \le t_e + t_p, \quad j=1,\dots,n$$

Step1-2:
Modify the non-zero entries to ensure the local sparsity constraints

Step 2: dictionary update



where $X=\begin{pmatrix} X_e \\ \sqrt{\alpha}X_p \\ 0 \end{pmatrix}$, $D=\begin{pmatrix} D_e & \boxed{0} \\ \boxed{\begin{matrix} 0 \\ -\sqrt{\beta}I \end{matrix}} & \begin{matrix} \sqrt{\alpha}D_p \\ \sqrt{\beta}W \end{matrix} \end{pmatrix}$, $A=\begin{pmatrix} A_e \\ A_p \end{pmatrix}$

Update the kth atom in dictionary using SVD [2]

$$\mathbf{R}_k \triangleq \mathbf{X} - \sum_{j\neq k} \mathbf{d}_j \mathbf{a}_R^j \quad \Rightarrow \quad \begin{aligned} \mathbf{R}_k &= \mathbf{U\Sigma V^T} \\ \mathbf{d}_k &= \mathbf{U}(:,1) \\ \mathbf{a}_R^k &= \mathbf{\Sigma}(1,1)\mathbf{V^T}(1,:) \end{aligned}$$

[1] Orthogonal matching pursuit (OMP) is applied here for sparse coding.
Tropp, Joel A., and Anna C. Gilbert. "Signal recovery from random measurements via orthogonal matching pursuit." IEEE Transactions on information theory (2007)

[2] M Aharon, M Elad, A Bruckstein. "K-SVD: An algorithm for designing overcomplete dictionaries for sparse representation". IEEE Transactions on Signal Processing(2006)

Optimization

Step 1: sparse coding

$$\min_{\mathbf{A}} \|\mathbf{X} - \mathbf{D}\mathbf{A}\|_F^2 ,$$
$$\text{s.t. } \|\mathbf{a}_{+,j}\|_0 \leq t_e, \quad j = 1, ..., n$$
$$\|\mathbf{a}_{-,j}\|_0 \leq t_p, \quad j = 1, ..., n$$

Step1-1 [1] :

$$\min_{\mathbf{A}} \|\mathbf{X} - \mathbf{D}\mathbf{A}\|_F^2$$
$$\text{s.t. } \|\mathbf{a}_j\|_0 \leq t_e + t_p, \quad j = 1, ..., n$$

Step1-2:
Modify the non-zero entries to ensure the local sparsity constraints

Step 2: dictionary update

Step 2-1:

$$< \mathbf{D}_e^*, \mathbf{A}_e^* >= \operatorname*{argmin}_{\mathbf{D}_e, \mathbf{A}_e} \|\mathbf{X}_e - \mathbf{D}_e \mathbf{A}_e\|_F^2$$

Step 2-2:

$$< \mathbf{D}_p^*, \mathbf{A}_p^*, \mathbf{W}^* >= \operatorname*{argmin}_{\mathbf{D}_p, \mathbf{A}_p, \mathbf{W}} \left\| \begin{pmatrix} \sqrt{\alpha}\mathbf{X}_p \\ \sqrt{\beta}\mathbf{A}_e^* \end{pmatrix} - \begin{pmatrix} \sqrt{\alpha}\mathbf{D}_p \\ \sqrt{\beta}\mathbf{W} \end{pmatrix} \mathbf{A}_p \right\|_F^2$$

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Experimental results: Dataset composition

Mini-MIMIC (selected data from MIMIC III database [1])

- ICU data with various ECG morphologies

Cardiovascular diseases		# patients	# cycles
Congestive Heart Failure		7	7075 (20.6%)
Myocardial Infarction (MI)	ST-segment Elevated	3	2962(8.7%)
	Non-ST-segment Elevated	4	4144(12.1%)
Hypotension		7	8281(24.2%)
Coronary Artery Disease		12	11781(34.4%)
Total		33	34243(100%)

CHF: congestive heart failure
STEMI: ST-segment elevated Myocardial infarction
NSTEMI: non-ST segment elevated Myocardial infarction
HYPO: hypotension
CAD: coronary artery disease

[1] J., Alistair EW, et al. "MIMIC-III, a freely accessible critical care database." Scientific data, 2016.

Quantitative results: XDJDL vs DCT

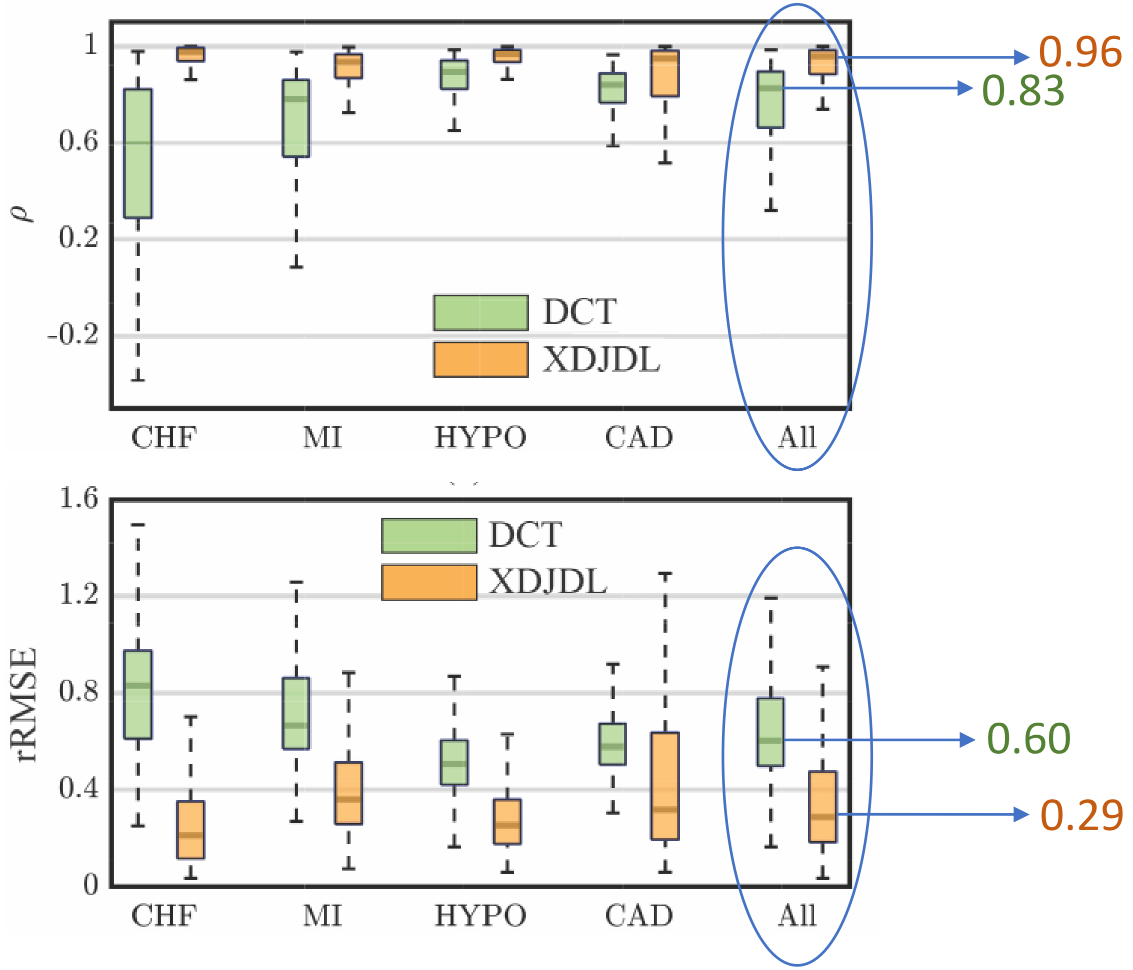
- Pearson correlation coefficient

$$\rho = \frac{(\mathbf{y}_{\text{test}} - \bar{y}_{\text{test}})^{\mathrm{T}}(\hat{\mathbf{y}}_{\text{test}} - \bar{\hat{y}}_{\text{test}})}{\|\mathbf{y}_{\text{test}} - \bar{y}_{\text{test}}\|_2 \, \|\hat{\mathbf{y}}_{\text{test}} - \bar{\hat{y}}_{\text{test}}\|_2}$$

- Relative root mean square error

$$\mathrm{rRMSE} = \frac{\|\mathbf{y}_{\text{test}} - \hat{\mathbf{y}}_{\text{test}}\|_2}{\|\mathbf{y}_{\text{test}}\|_2}$$

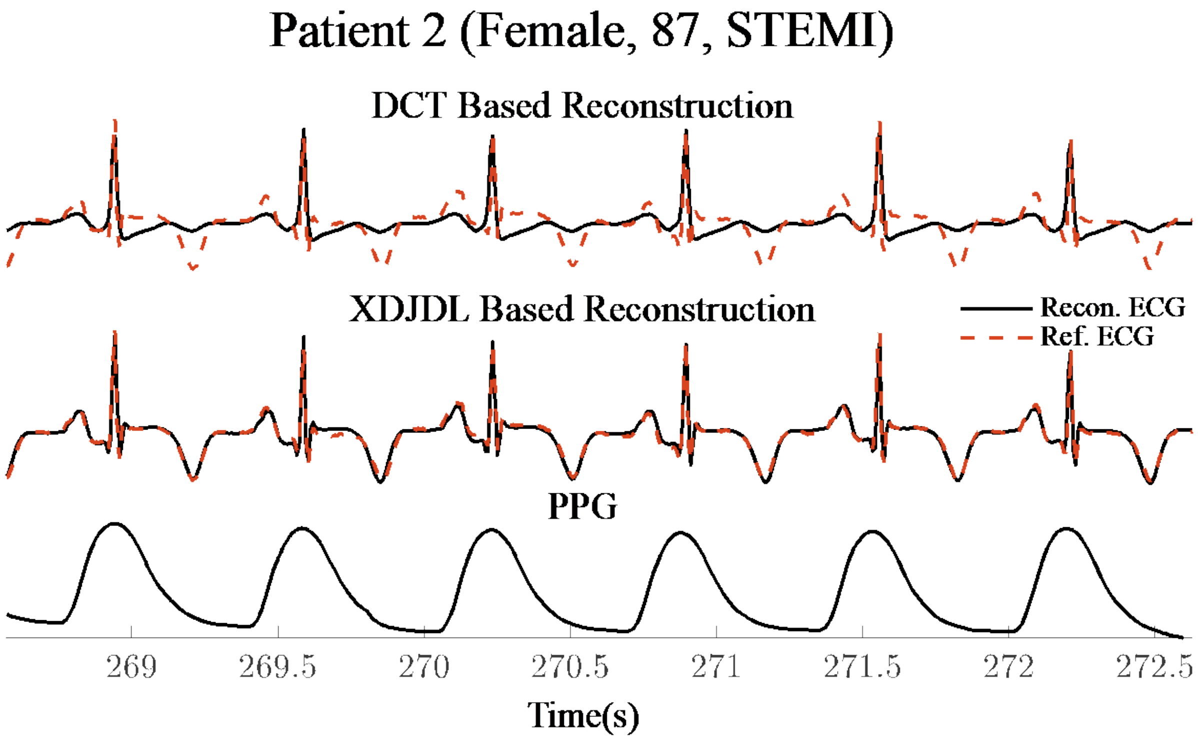
Reconstruction Scheme		DCT [1]	XDJDL **(proposed)**
ρ	$\hat{\mu}$	0.71	**0.88**
	$\hat{\sigma}$	0.31	0.23
$rRMSE$	$\hat{\mu}$	0.67	**0.39**
	$\hat{\sigma}$	0.26	0.31

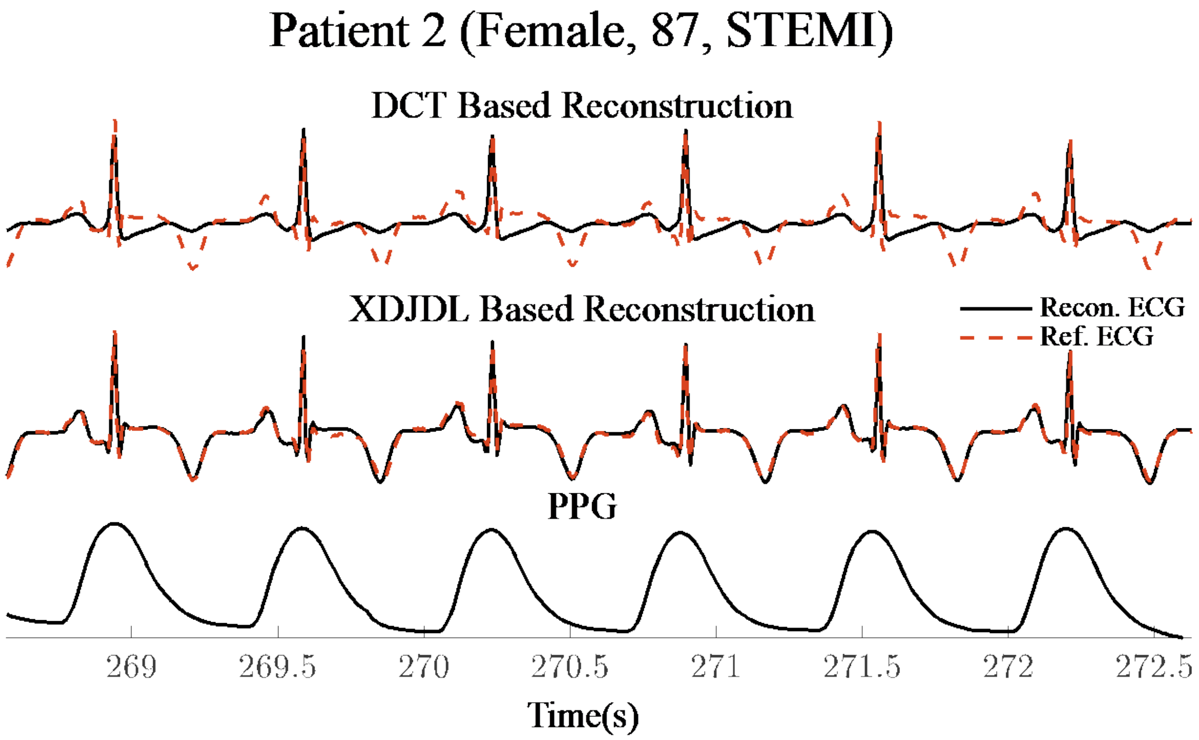


CHF: congestive heart failure
MI: Myocardial infarction
HYPO: hypotension
CAD: coronary artery disease

[1] Q. Zhu, et al. "ECG reconstruction via PPG: A pilot study," BHI, 2019.

Experimental results: Visualization for Reconstruction Performance





- Pearson correlation coefficient:

  DCT: 0.19
  XDJDL: 0.94

- Pearson correlation coefficient:

  DCT: 0.46
  XDJDL: 0.87

More models for comparison

Model	Sparsity constraint	Linear mapping between codes?	Optimization methods
DCT [1]	NA	✓	ADMM
Cross-view projective dictionary learning (CPDL) [2]	NA	✗	ADMM
Super resolution via sparse representation (ScSR) [3]	L1	✗	LARS + MOD
Semi-coupled dictionary learning (SCDL) [4]	L1	✓	LARS + MOD
Coupled K-SVD dictionary training (CDL) [5]	L0	✗	OMP + SVD
Cross-domain joint dictionary learning (XDJDL) (**proposed**)	L0	✓	OMP + SVD

[1] Q. Zhu, et al. "ECG reconstruction via PPG: A pilot study," *BHI,* 2019.
[2] S. Li, et al. "Cross-view projective dictionary learning for person re-identification." *IJCAI*, 2015.
[3] J. Yang, et al. ''Image super-resolution via sparse representation''. *IEEE Transactions on Image Processing*, 2010.
[4] S. Wang, et al. "Semi-coupled dictionary learning with applications to image super-resolution and photo-sketch synthesis." *CVPR,* 2012.
[5] J. Xu, et al. "Coupled K-SVD dictionary training for super-resolution." *ICIP,* 2014.

ADMM: alternating direction method of multipliers
LARS: least angle regression
MOD: method of directions
OMP: orthogonal matching pursuit
SVD: singular vector decomposition

Quantitative results: More models for comparison

Reconstruction Scheme	ρ			rRMSE		
	$\hat{\mu}$	$\hat{\sigma}$	med	$\hat{\mu}$	$\hat{\sigma}$	med
DCT [1]	0.71	0.31	0.83	0.67	0.26	0.60
CPDL [2]	0.74	0.31	0.85	0.63	0.35	0.56
ScSR [3]	0.82	0.23	0.89	0.54	0.21	0.52
SCDL [4]	0.83	0.21	0.89	0.52	0.22	0.49
CDL [5]	0.85	0.25	0.95	0.49	0.51	0.34
XDJDL (**proposed**)	**0.88**	0.23	0.96	**0.39**	0.31	0.29

[1] Q. Zhu, et al. “ECG reconstruction via PPG: A pilot study,” *BHI,* 2019.
[2] S. Li, et al. "Cross-view projective dictionary learning for person re-identification." *IJCAI*, 2015.
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[5] J. Xu, et al. "Coupled K-SVD dictionary training for super-resolution.” *ICIP,* 2014.

Conclusion and Perspectives

- Proposed the cross-domain joint dictionary learning model
- Improved ECG reconstruction accuracy from PPG on Mini-MIMIC
- Long-term & user-friendly ECG monitoring via PPG sensor
- (future work) Broader range of subjects with improved accuracy

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