Active Control of Line Spectral Noise with Simultaneous Secondary Path Modeling Without Auxiliary Noise

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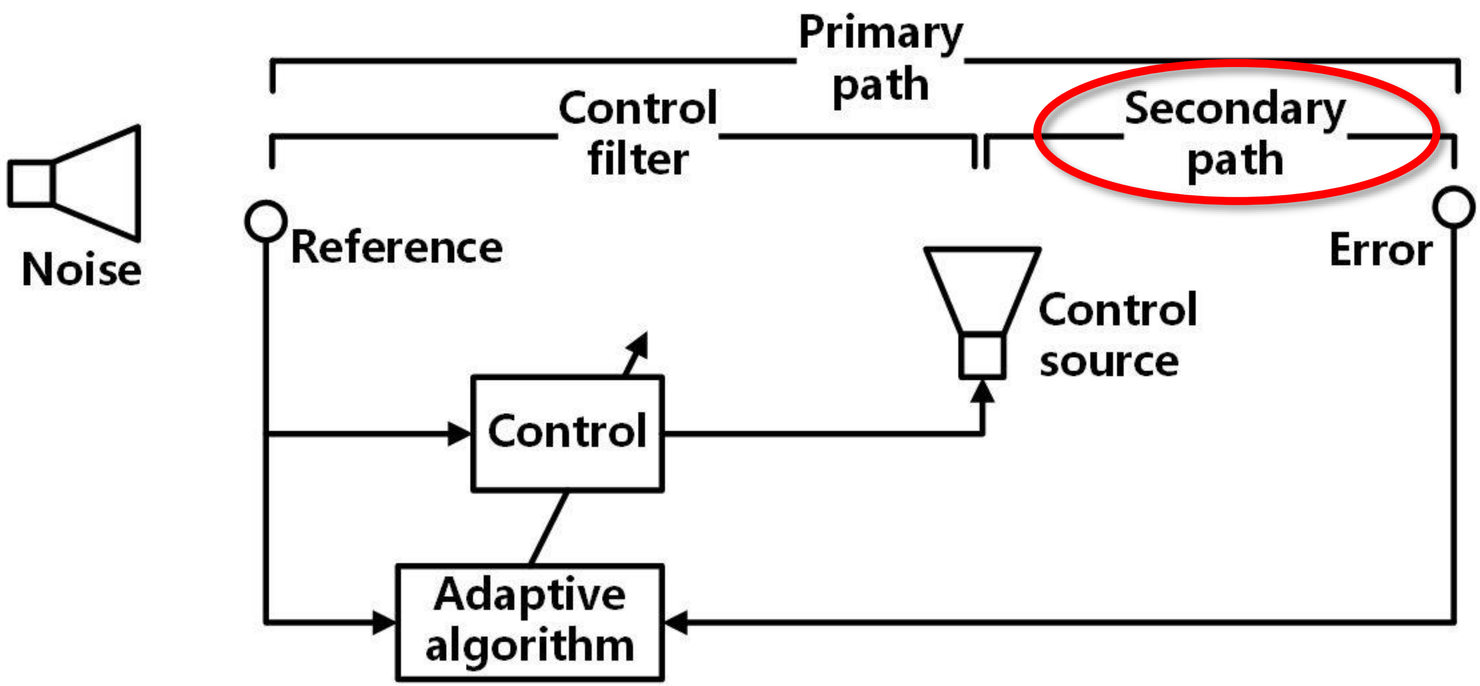
Outlines

- Background
- Theory
- Simulations
- Conclusions

Background

- Scheme of ANC



Background

- Off-line modeling
  - **pros** Easy implementation
  - **cons** Incapable of tracking varying secondary path
  - **cons** Have to be executed before ANC
- On-line modeling
  - **pros** Theoretically good tracking ability
  - **cons** Require additive noise
  - **cons** Some methods suspend the control process
- Proposed method
  - **pros** Simple structure
  - **pros** No additive noise

S. Elliott, *Signal processing for active control*. Elsevier, 2000.
C. Y. Chang, et.al., *Appl. Acoust.*, 2018.

Background

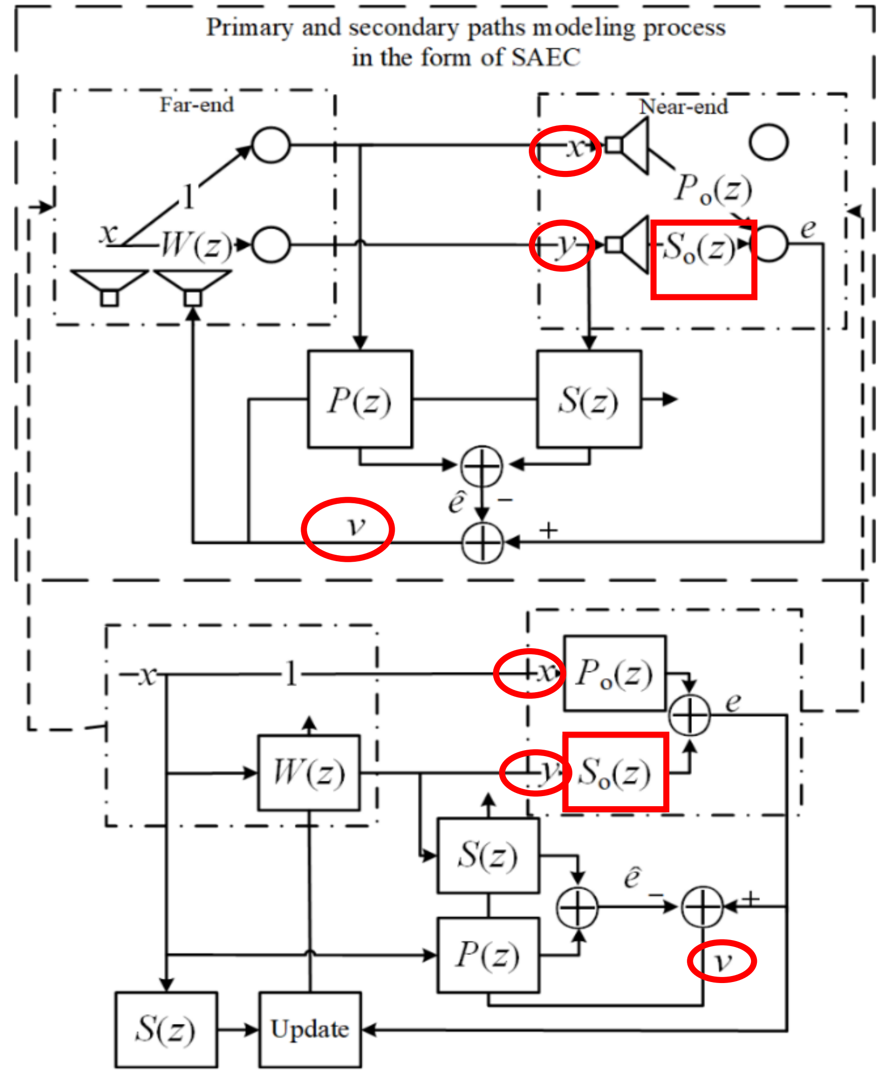
- Previous work
  - **Proof** of the effectiveness of the proposed simple structure to **model** and **control simultaneously** under the basic **assumption** that the primary noise is **not line spectral process**

- This work
  - **Proof** of the effectivity of the proposed simple structure to **model** and **control simultaneously** under the basic assumption that the primary noise is **line spectral process**

M. Hu, et.al., *J. Acoust. Soc. Am.*, 2019

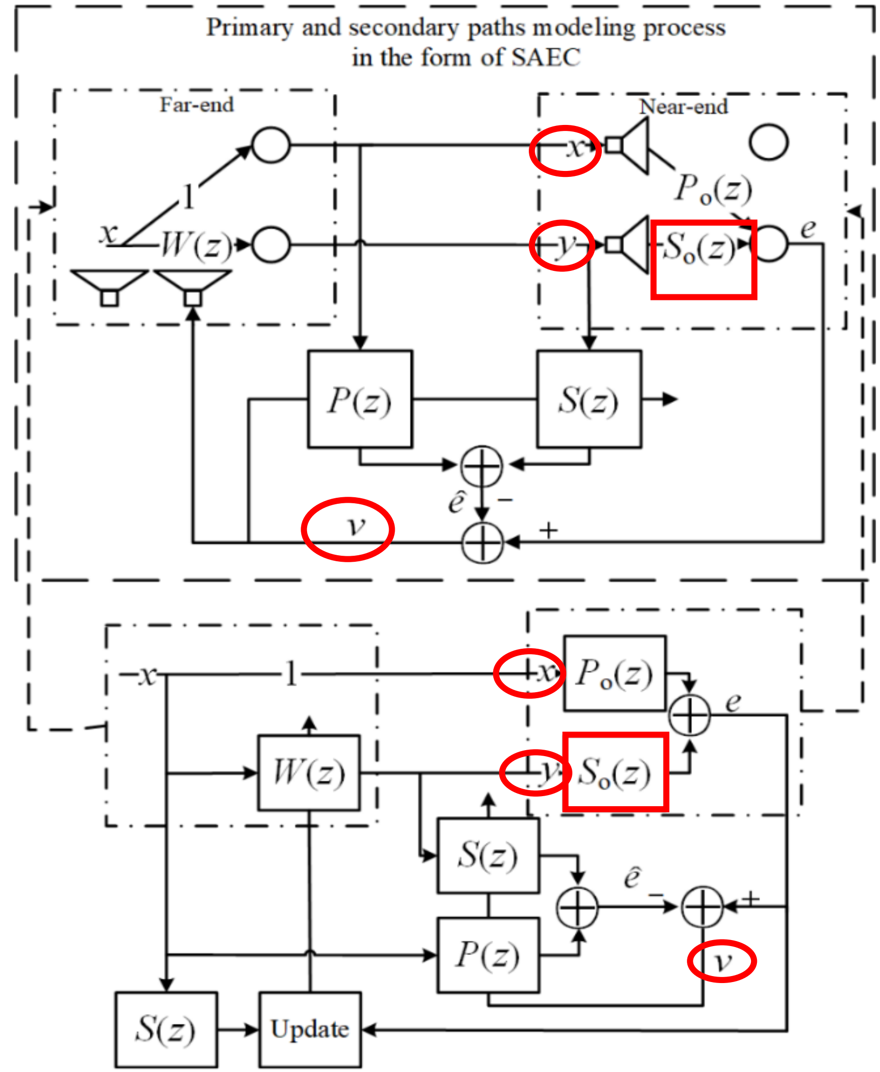
Theory

- Note the connection between SAEC and ANC
- Analysis of non-uniqueness problem of SAEC is helpful in ANC
  - Control filter acts as a time-varying transfer function



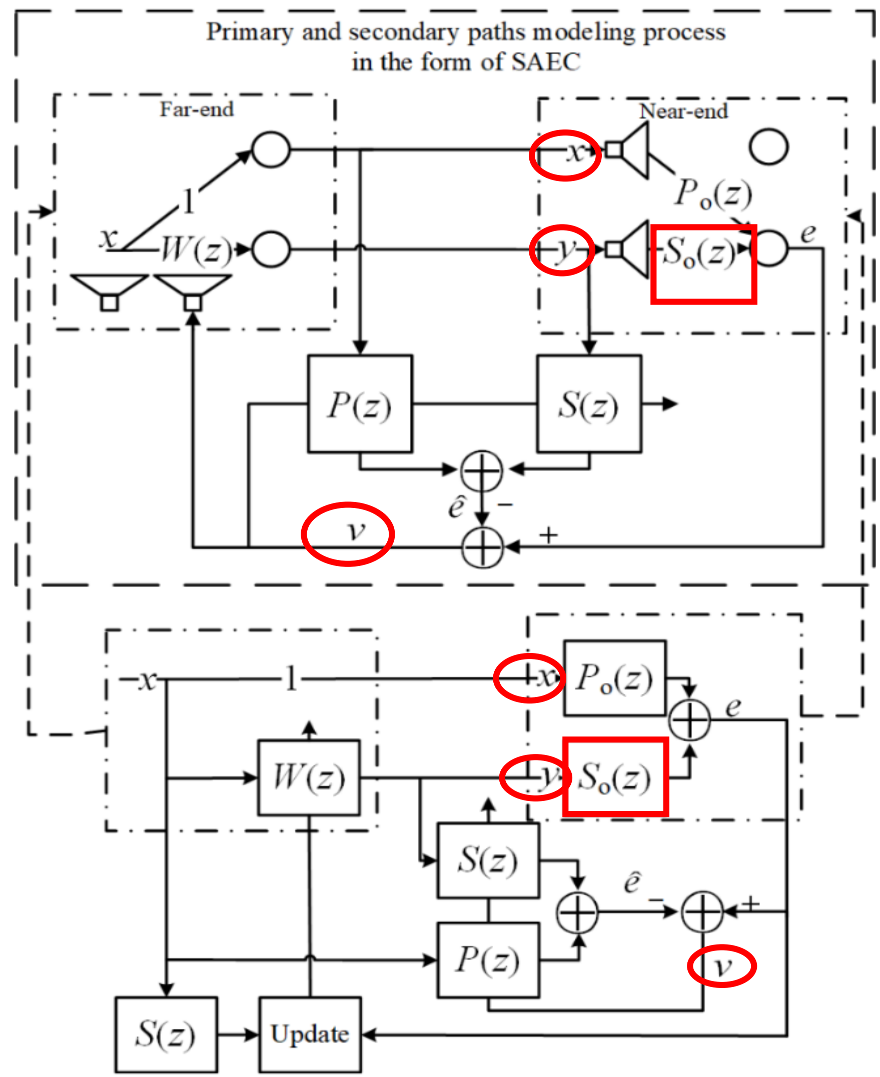
Theory

- The goal is to prove that the joint auto-correlation matrix of *x* and *y* is full-rank
  - *x* is the reference signal
  - y is the output of the control source
  - x and y are the modeling inputs of the primary and secondary path
  - Full rank joint auto-correlation matrix of *x* and *y* leads to a wiener function of unique solution while minimizing the cost *v*, $E\left[v^2\right]$



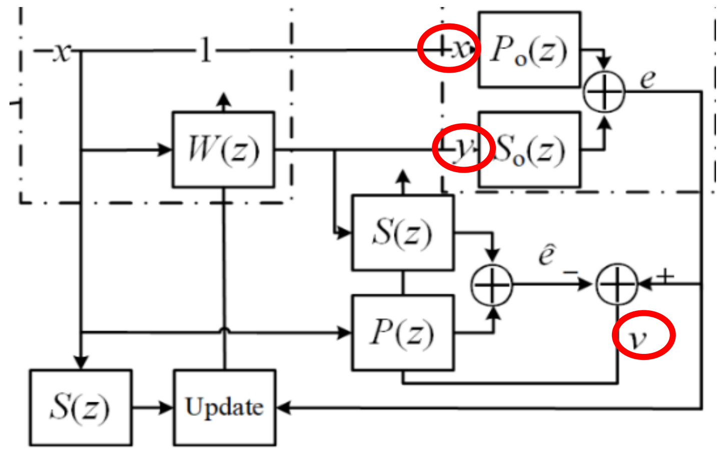
Theory

- Assumption
  - The reference noise is of line spectral property and composed of *N*/2 frequencies
  - The impulse responses of *P*, *S*, *W* are all set as *N*
- Used facts
  - 1. Time-varying control filter *W*(z)
  - 2. Signal with non-zero power spectrum at *N*/2 frequencies is full-rank for a correlation matrix of dimension $N \times N$
  - 3. The eigenvalues of the circulant matrix are determined by the Fourier transform of its first row



P. P. Vaidyanathan, The Theory of Linear Prediction, vol. 2, no. 1. 2007
B. Bamieh, "Discovering the Fourier Transform: A Tutorial on Circulant Matrices, Circular Convolution, and the DFT," 2018.

Theory



- Cost function

$$E\left[v^2\right]=\mathbf{c}^{\mathrm{T}}E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right]\mathbf{c}$$

- where (assume noise with N/2 frequencies)

$$\mathbf{c}=\mathbf{G}^{\mathrm{T}}(\mathbf{p}_{\mathrm{o}}-\mathbf{p})+\mathbf{W}^{\mathrm{T}}(\mathbf{s}_{\mathrm{o}}-\mathbf{s}) \qquad \mathbf{G}=[\mathbf{I}\quad \mathbf{0}]_{N\times 2N}$$

$$\tilde{\mathbf{x}}(n)=\left[\mathbf{x}^{\mathrm{T}}(n)\quad \mathbf{x}^{\mathrm{T}}(n)\right]^{\mathrm{T}} \qquad \mathbf{x}(n)=\left[x(n)\quad \cdots \quad x(n-N+1)\right]^{\mathrm{T}}$$

$$\mathbf{W}=\left[\begin{array}{cccc:cccc} w(1) & w(2) & \cdots & w(N) & 0 & \cdots & 0 & 0 \\ 0 & w(1) & w(2) & \cdots & w(N) & \ddots & \vdots & 0 \\ \vdots & 0 & \ddots & \cdots & \cdots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & w(1) & w(2) & \cdots & w(N) & 0 \end{array}\right]$$

Theory

- Take the derivative of the cost function with respect to **s**

$$\frac{\partial E\left[v^2\right]}{\partial \mathbf{s}} = -2\mathbf{W}E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right]\mathbf{c}$$

- Initialize **w** as a non-zero vector, then **W** has full row rank

$$\mathbf{W}E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right]\mathbf{c} = 0 \Leftrightarrow E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right]\mathbf{c} = 0$$

- Utilize time-varying property of **w**

$$\mathbf{c}=\mathbf{G}^{\mathrm{T}}\left(\mathbf{p}_{\mathrm{o}} - \mathbf{p}\right) + \mathbf{W}^{\mathrm{T}}\left(\mathbf{s}_{\mathrm{o}} - \mathbf{s}\right)$$

$$\mathbf{c}_{*}=\mathbf{G}^{\mathrm{T}}\left(\mathbf{p}_{\mathrm{o}} - \mathbf{p}\right) + \mathbf{W}_{*}^{\mathrm{T}}\left(\mathbf{s}_{\mathrm{o}} - \mathbf{s}\right)$$

$$E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right]\mathbf{c} = 0, E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right]\mathbf{c}_{*} = 0$$

Fact 1

Time-varying control filter *W*(z)

Theory

- Subtraction of $E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right]\mathbf{c}=0$ and $E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right]\mathbf{c}_{*}=0$ leads to

$$E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right](\mathbf{c}-\mathbf{c}_{*})=E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right]\tilde{\mathbf{W}}^{\mathrm{T}}(\mathbf{s}_{\mathrm{o}}-\mathbf{s})=0$$

$$\tilde{\mathbf{W}}=(\mathbf{W}-\mathbf{W}_{*})$$

- Divide $\tilde{\mathbf{W}}$ into left and right part $\tilde{\mathbf{W}}_{(1)}$ and $\tilde{\mathbf{W}}_{(2)}$
- $E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right]$ can be denoted as

$$E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right]=\begin{bmatrix}\mathbf{R} & \mathbf{R}\\ \mathbf{R} & \mathbf{R}\end{bmatrix} \quad \mathbf{W}=\left[\begin{array}{cccc:cccc} w(1) & w(2) & \cdots & w(N) & 0 & \cdots & 0 & 0\\ 0 & w(1) & w(2) & \cdots & w(N) & \ddots & \vdots & 0\\ \vdots & 0 & \ddots & \cdots & \cdots & \ddots & 0 & \vdots\\ 0 & \cdots & 0 & w(1) & w(2) & \cdots & w(N) & 0 \end{array}\right]$$

$\tilde{\mathbf{W}}_{(1)}$ $\tilde{\mathbf{W}}_{(2)}$

$$\mathbf{R}=E\left[\mathbf{x}\mathbf{x}^{\mathrm{T}}\right]$$

Theory

- $$\mathbf{R}\left(\tilde{\mathbf{W}}_{(1)}+\tilde{\mathbf{W}}_{(2)}\right)^{\mathrm{T}}\left(\mathbf{s}_{\mathrm{o}}-\mathbf{s}\right)=0$$

Fact 2

$$\Updownarrow$$

Signal with non-zero power spectrum at *N*/2 frequencies is full-rank for a correlation matrix of dimension $N \times N$

- $$\left(\tilde{\mathbf{W}}_{(1)}+\tilde{\mathbf{W}}_{(2)}\right)^{\mathrm{T}}\left(\mathbf{s}_{\mathrm{o}}-\mathbf{s}\right)=0$$
- Where

$$\tilde{\mathbf{W}}_{(1)}+\tilde{\mathbf{W}}_{(2)}=\begin{bmatrix} \tilde{w}(1) & \tilde{w}(2) & \cdots & \tilde{w}(N) \\ \tilde{w}(N) & \tilde{w}(1) & \cdots & \tilde{w}(N-1) \\ \vdots & \ddots & \ddots & \vdots \\ \tilde{w}(2) & \tilde{w}(3) & \cdots & \tilde{w}(1) \end{bmatrix}$$

- is full-rank

Fact 3

The eigenvalues of the circulant matrix are determined by the Fourier transform of its first row

Simulations

Initializing x, y, u, h, f, Q, λ, μ, and w

for $n = 0, 1, 2, \ldots$ do

(a) for $i = 0$ to $N - 1$, $y(n - i) = \mathbf{w}^{\mathrm{T}}\mathbf{x}(n - i)$; stack $\mathbf{x}(n)$ and $\mathbf{y}(n)$ into $\mathbf{u}(n)$;

(b) **modeling process** using the RLS algorithm

$\mathbf{k} = \mathbf{Q}\mathbf{u}(n) / (\lambda + \mathbf{u}^{\mathrm{T}}(n)\mathbf{Q}\mathbf{u}(n))$

$\mathbf{h} = \mathbf{h} + \mathbf{k}(e(n) - \mathbf{h}^{\mathrm{T}}\mathbf{u}(n))$

$\mathbf{Q} = (\mathbf{Q} - \mathbf{k}\mathbf{u}^{\mathrm{T}}(n)\mathbf{Q}) / \lambda$

assign the last N taps of **h** to **s**;

(c) for $i = 0$ to $N - 1$, $f(n - i) = \mathbf{s}^{\mathrm{T}}\mathbf{x}(n - i)$;

(d) **control process** using the LMS algorithm

$\mathbf{w} = \mathbf{w} - 2\mu\mathbf{f}(n)e(n)$.

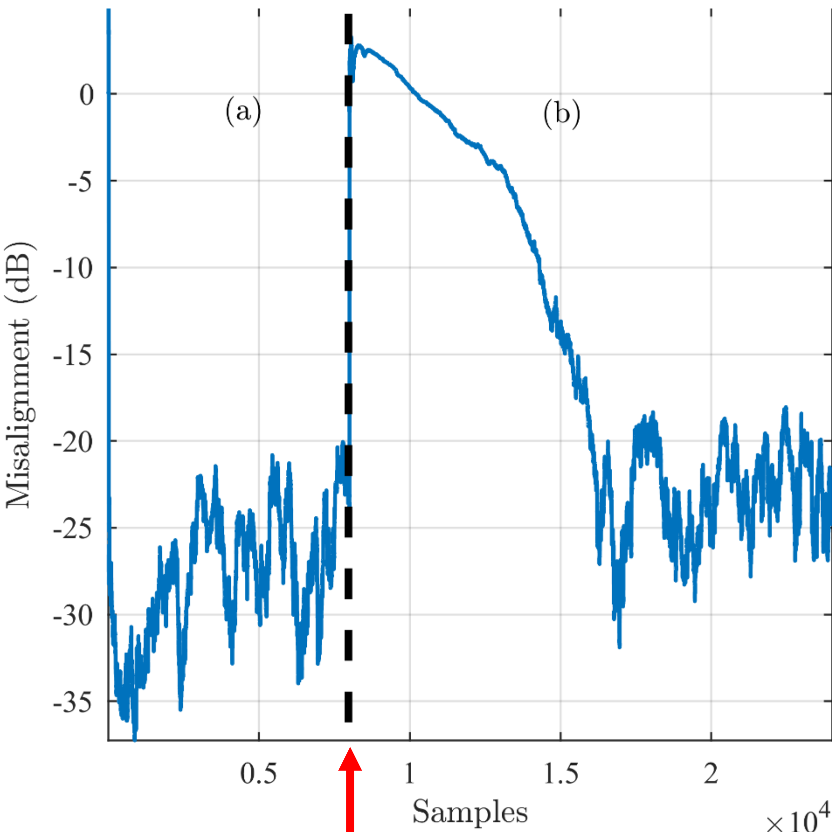
Excited frequency number is 3 so N = 6

Forgetting factor of RLS is 0.999

Learning rate μ = 0.01

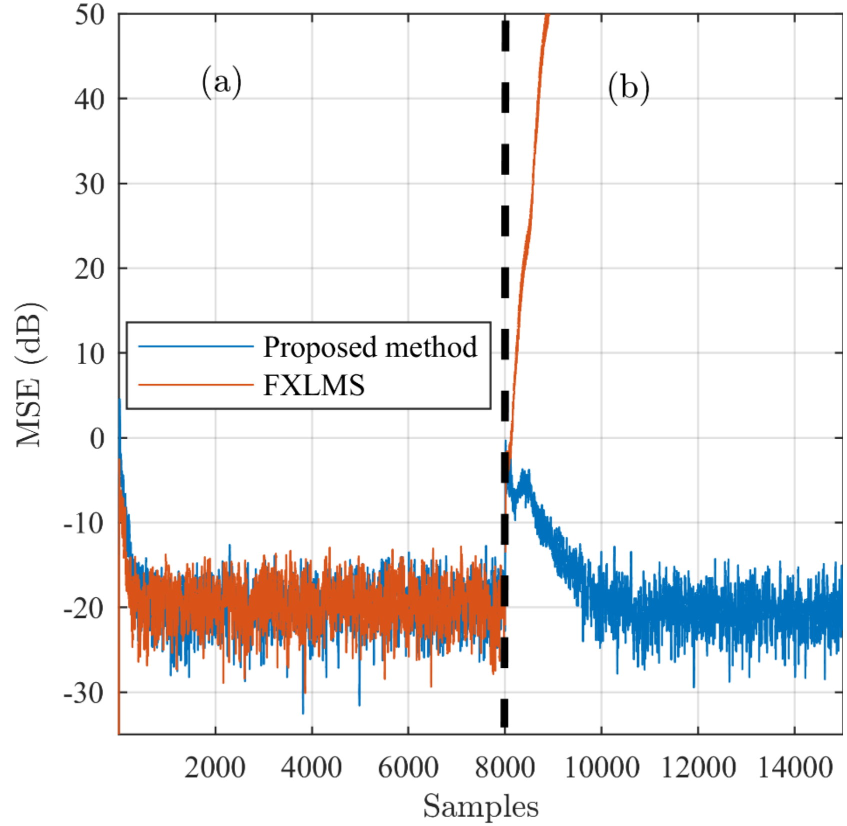
w is initialized as zero except for 0.001 at the first tap

Simulations



Model of State (a) is measured in the listening room of the Institute of Acoustics, Nanjing University

Disturb State (a) by adding random noise with SNR = 20 dB to give a different set of transfer functions, denoted by State (b)



Conclusions

- For the noise of line spectral process, the modeling process of the secondary path that uses only the output of the control filter is possible.
- The secondary path is guaranteed to converge to the optimum as long as the number of non-zero spectrum frequencies is known and the control filter is set as twice this number.

Thanks for your attention!