A Low-Latency Successive Cancellation Hybrid Decoder for Convolutional Polar Codes

Yu Wang, Shikai Qiu, Lirui Chen, Qinglin Wang,
Yang Zhang, Cang Liu, Zuocheng Xing

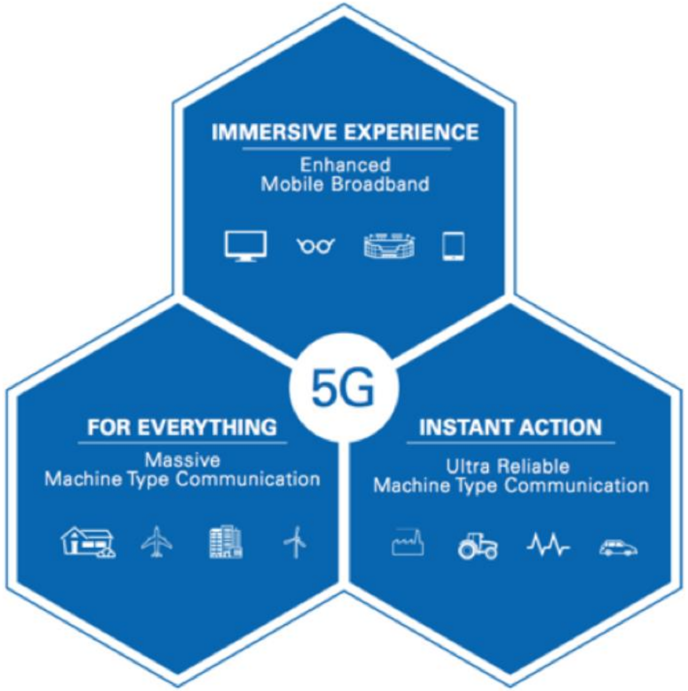
National Laboratory for Parallel and Distributed Processing
College of Computer
National University of Defense Technology

2020 IEEE International Conference on Acoustics, Speech and Signal Processing
Virtual Conference, May 7 2020

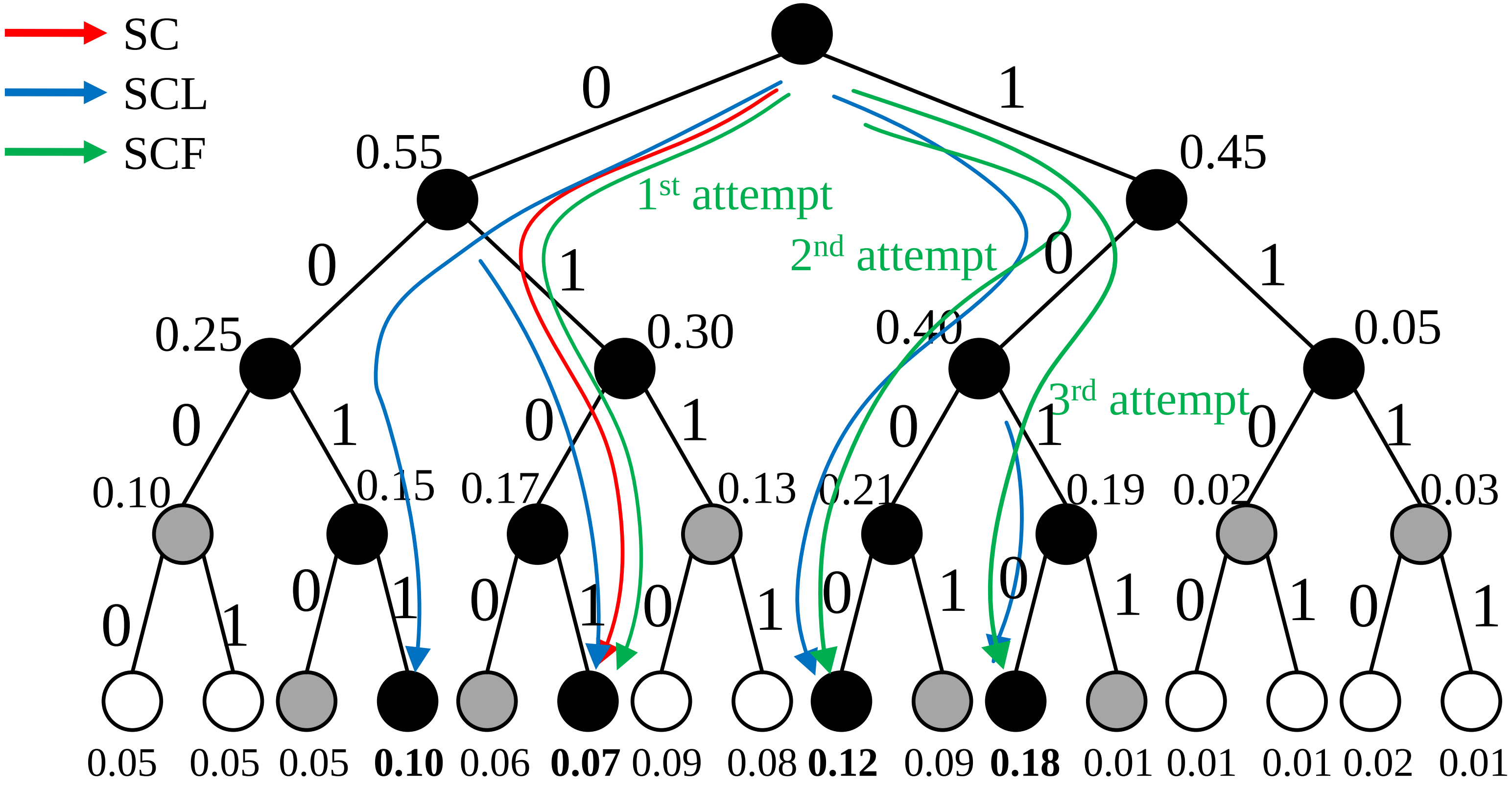
Outline

1. SCL and SCF Decoding
2. SCL-SCF Hybrid Decoding Algorithm
3. Convolutional Polar Codes
4. Construction of Critical Set
5. Simulation Results

SCL and SCF Decoding

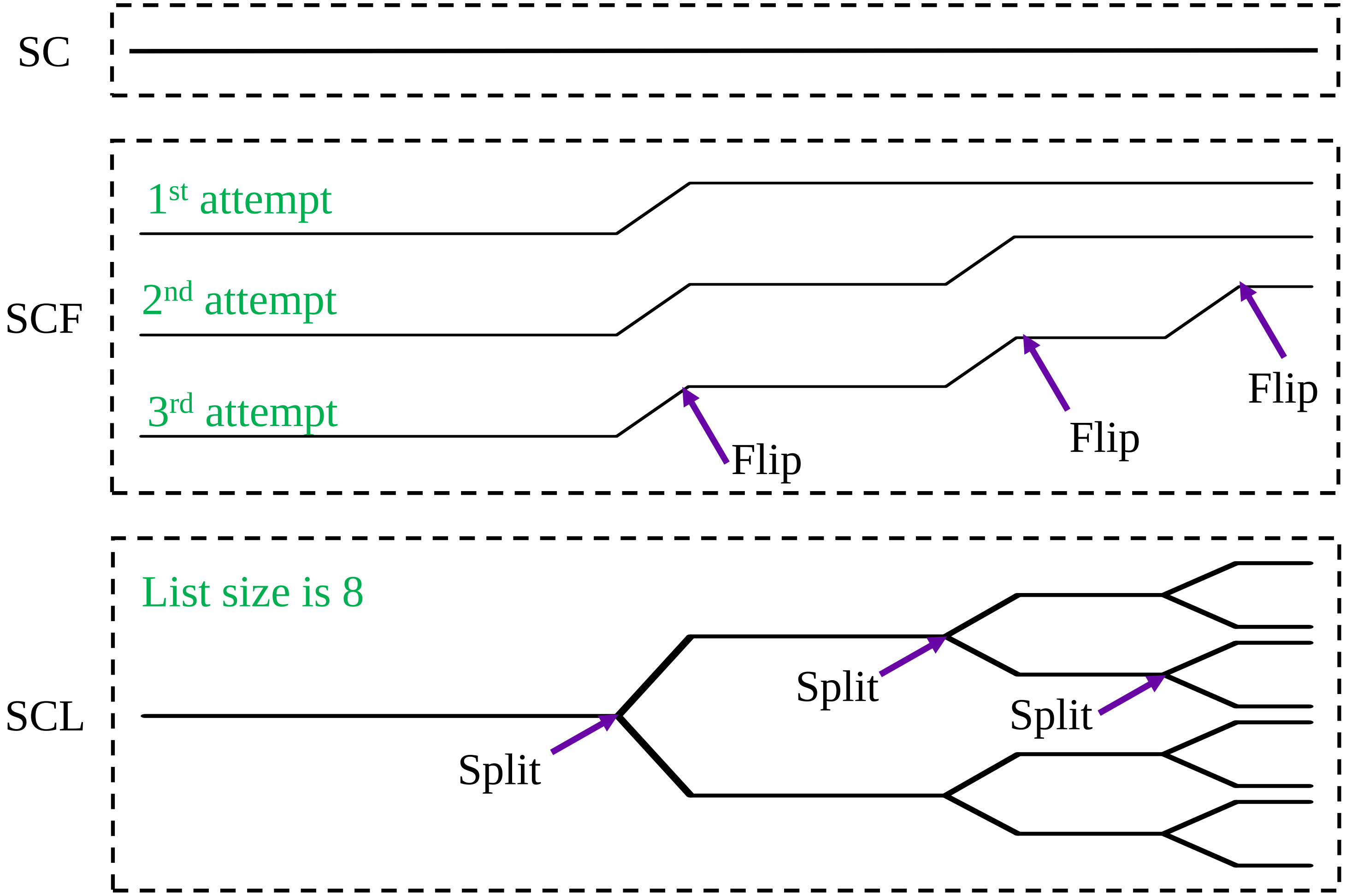


Binary tree representation of SC, SCL and SCF decoding



- SC decoding is the deep first searching method → Low complexity
- SCL have much better performance → Increased complexity, Long decoding latency
- SCF have improved performance and low complexity
- D-SCF can correct multiple erroneous bits → Variable decoding latency

Line representation of SCF decoding



SC decoding following the deep first searching method can be viewed as a line.

Flip to change the decoding trajectory

SCF decoding finds a valid codeword at the cost of **time.**

Split to change the decoding trajectory

SCL decoding finds a valid codeword at the cost of **space.**

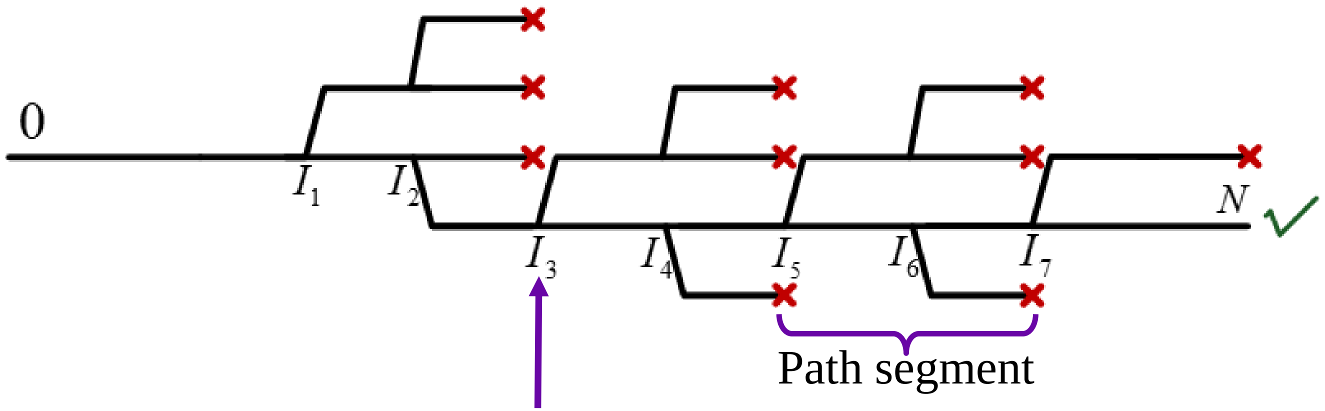
Similarity

SCL decoding can be viewed as a parallel multi-bit flipping decoder, which selects **the most L possible multi-bit flipping combinations** at each information bit.

Algorithm 1 Hybird decoding with critical set

procedure HYBIRD DECODER($y_1^N, S_c, \mathcal{I}$,P)
  $|\mathcal{P}| \leftarrow \{1\}$
  $PM \leftarrow 0$
  for $j \leftarrow 1$ **to** $|\mathcal{P}|$ **do**
    for $i \leftarrow 1$ **to** N **do**
      $LLR_j^{(i,n)} \leftarrow$recursivelyCalc(y_1^N)
      if $i \notin \mathcal{I}$ **then** $\hat{u}_j^i \leftarrow 0$, update(PM_j)
      else
        if $LLR_j^{(i,n)} \geq 0$ **then**
          $\hat{u}_j^i \leftarrow 0$, update(PM_j)
        else $\hat{u}_j^i \leftarrow 1$, update(PM_j)
        end if
      end if
      if $i \in S_c$ **then**
        if $|\mathcal{P}| = P$ **then** $\mathcal{P} \leftarrow \text{argmin}_{j \in |\mathcal{P}|} PM_j$
        end if
        $p_{flip} \leftarrow$ duplicatePath(p_j), $|\mathcal{P}| \leftarrow 2\,|\mathcal{P}|$
        $\hat{u}_{flip}^i \leftarrow 1 - \hat{u}_j^i$, update($PM_{flip}$)
      end if
    end for
  end for
  $p^* \leftarrow \text{argmin}_{j \in |\mathcal{P}|} PM_j$
  return $\hat{u}_{p^*}$
end procedure

Use path metric to indicate whether the flipping is correct or not.



Perform pruning before path splitting

$\mathcal{O}(PN\log_2 N + S / P)$

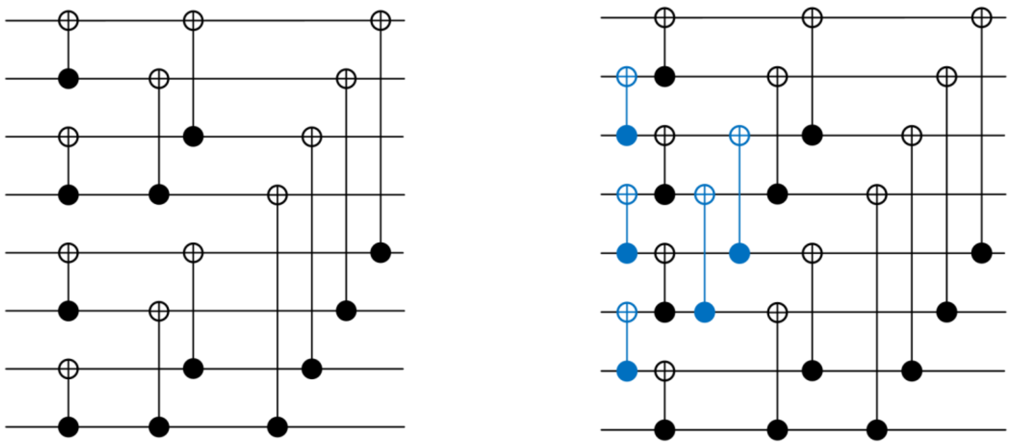
Num. of path segments

Path metric is a time-lagged indicator.

Need an accumulation of path metric to indicate the flipping.

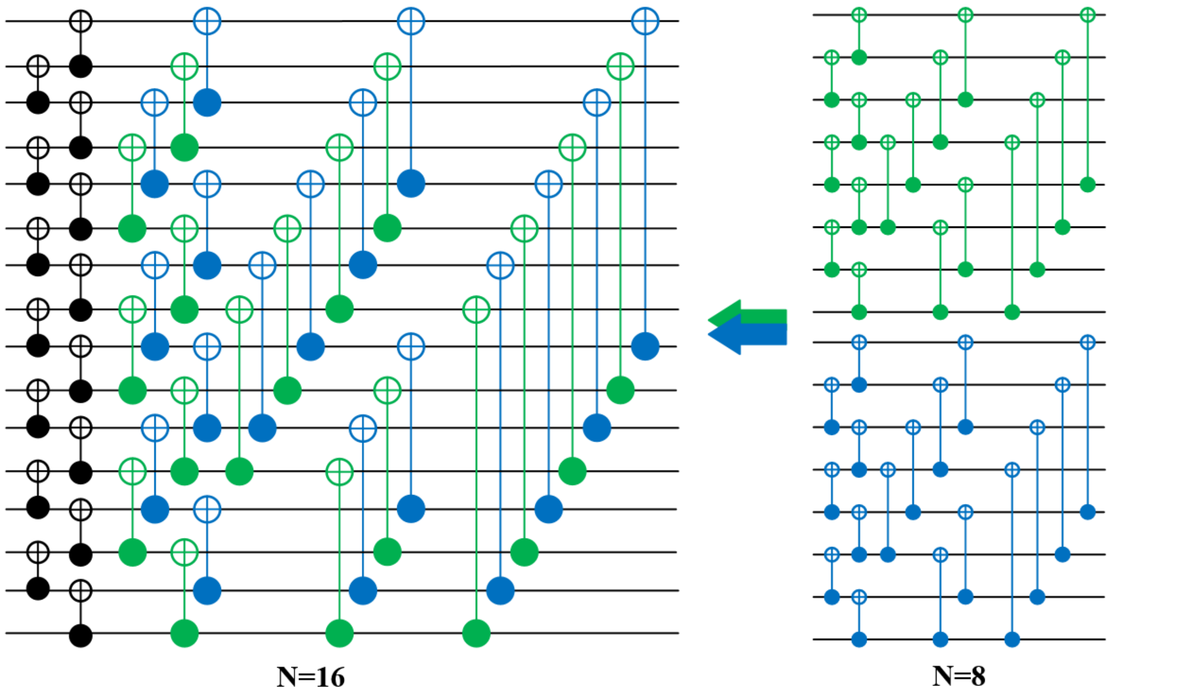
Less path segments improve accuracy of path metric indication.

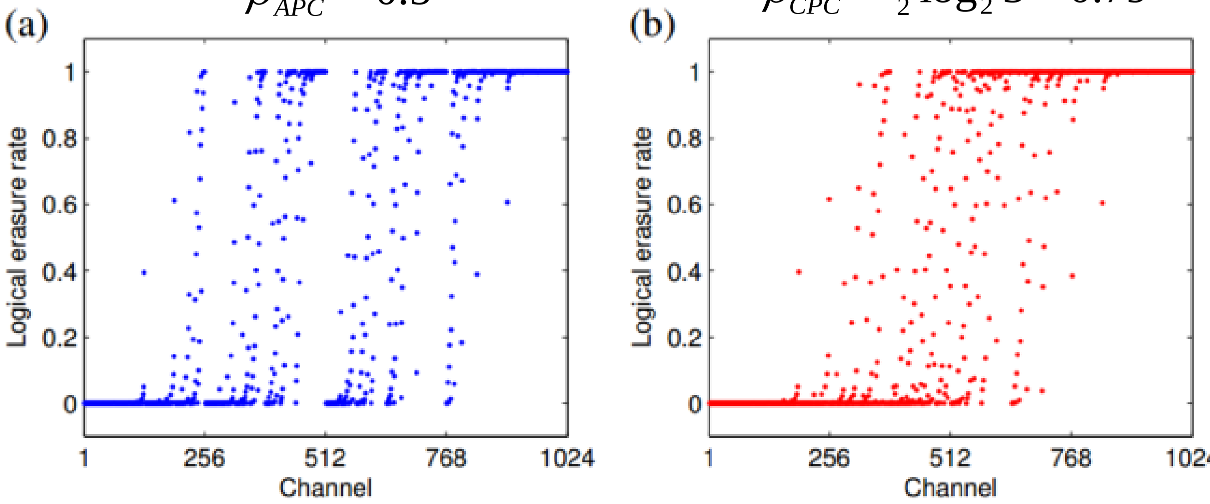
Need a fixed and efficient critical set S_c.



Arikan Polar Codes (APC) Convolutional Polar Codes (CPC)[1]

$\beta_{APC} = 0.5$ $\beta_{CPC} = \frac{1}{2}\log_2 3 \approx 0.79$





The polarization results with 50% erasure channel.

Matrix format[2]: $c = uG_N$ $G_N = \begin{bmatrix} G_{N/2} & 0 \\ F^{\otimes(n-1)^T} G_{N/2} & G_{N/2} \end{bmatrix}$ $G_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

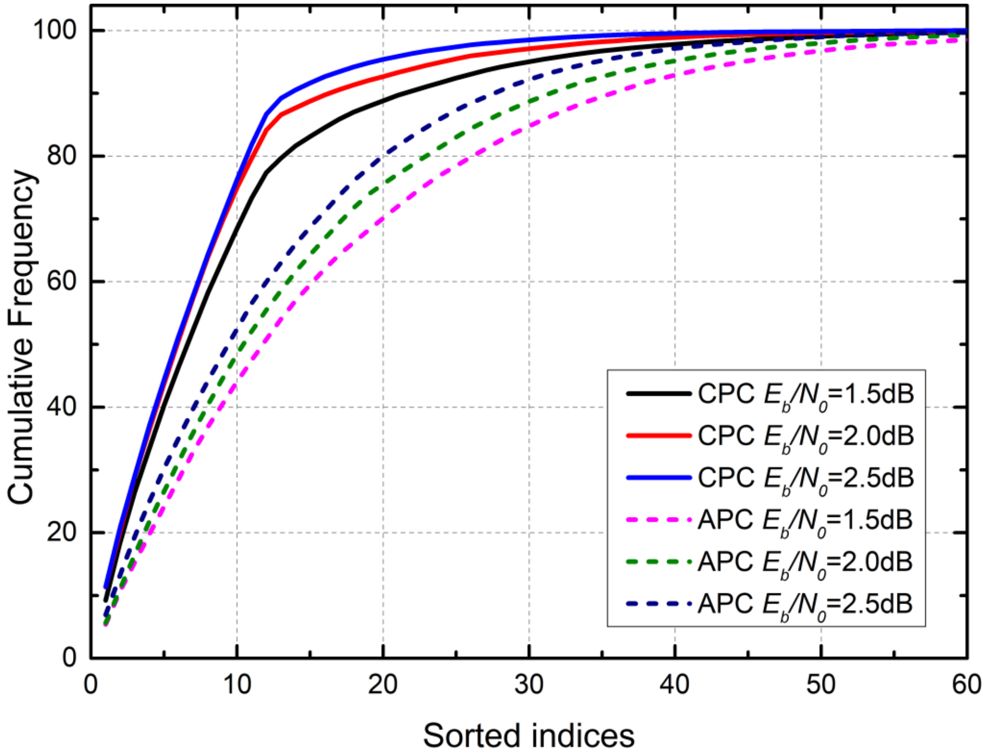
Construction Method: Monte-Carlo Simulation

Hard decision: $\hat{u}_i = h(L_i) = \begin{cases} u_i & \text{if } i \in \mathcal{A}^c \\ \dfrac{1-\operatorname{sgn}(L_i)}{2} & \text{if } i \in \mathcal{A} \end{cases}$ $L_i = \log \dfrac{W_N^{(i)}\left(y_1^N, u_1^{i-1} \mid 0\right)}{W_N^{(i)}\left(y_1^N, u_1^{i-1} \mid 1\right)}$

[1] Andrew James Ferris, Christoph Hirche, and David Poulin, "Convolutional polar codes," 2017,arXiv:1704.00715.

[2] H. Saber, Y. Ge, R. Zhang, W. Shi and W. Tong, "Convolutional Polar Codes: LLR-based Successive Cancellation Decoder and List Decoding Performance," *2018 IEEE International Symposium on Information Theory (ISIT)*, Vail, CO, 2018, pp. 1480-1484.

APC(256,86)



SC-Oracle decoder is used to evaluate the error frequency[1]

Conclusion: The channel-induced errors of CPCs are more concentrated than that of APCs.

CPCs are more suitable for our proposed decoding scheme.

Table 1. The size of critical set for CPC $N = 256$ and different code rate at various E_b/N_0 points.

E_b [dB]	Rate(R) 1/6	1/4	1/3	1/2	2/3
1.0	49	67	84	107	157
1.5	41	55	68	90	127
2.0	38	49	56	76	112
2.5	33	38	44	59	85
3.0	24	29	29	32	64

Conclusion: $S_{\text{high-SNR}} \in S_{\text{low-SNR}}$

$$S_{\text{low-R}} \in S_{\text{high-R}}$$

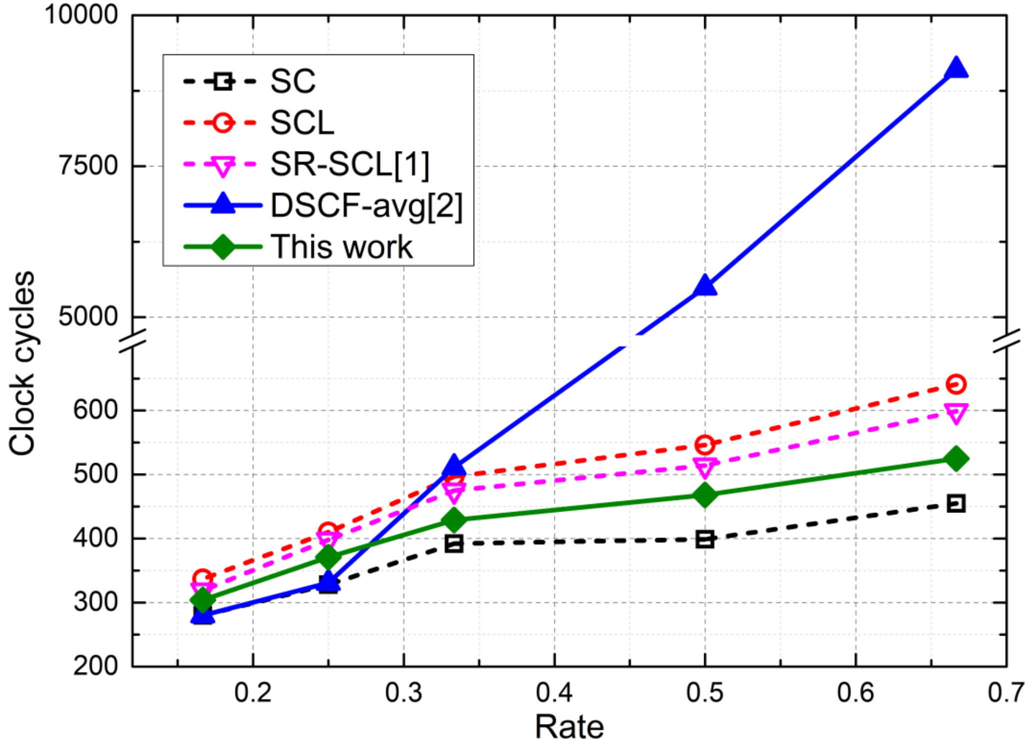
Different code lengths have different critical sets.

$$N = \{128, 256, 512, 1024\}$$

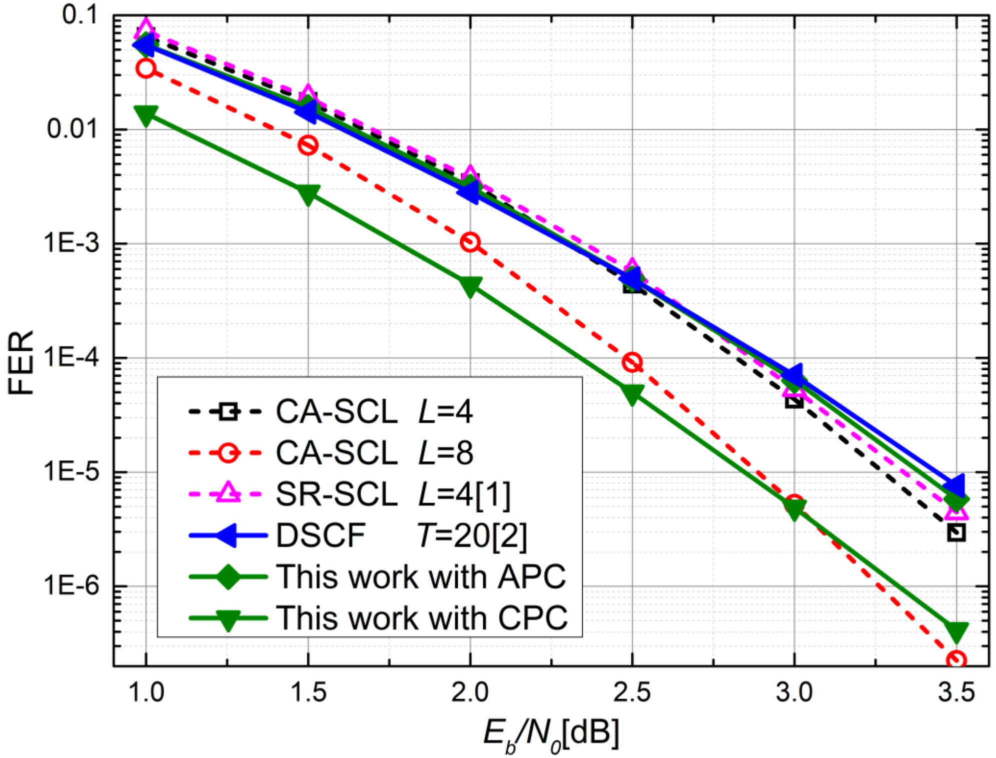
As for one code length, a single critical set is preferable.

[1] C. Condo, F. Ercan and W. J. Gross, "Improved successive cancellation flip decoding of polar codes based on error distribution," 2018 IEEE Wireless Communications and Networking Conference Workshops (WCNCW), Barcelona, 2018, pp. 19-24.

$E_b/N_0 = 1.0\text{dB}, N = 256, T = 20, R \in \{1/6, 1/4, 1/3, 1/2, 2/3\}$



PC(256,86)



$$L_{\text{SC}} = 2N + \frac{N}{64}\log_2\left(\frac{N}{256}\right) - \sum_{i=0}^{\log_2 N}\left\lfloor \frac{b}{2^i} \right\rfloor \left\lceil \frac{2^i}{64} \right\rceil$$

$$L_{\text{SCL}} = L_{\text{SC}} + k + F_C \qquad L_{\text{hybrid}} = L_{SC} + \lceil S/P \rceil + F_C$$

Hybrid decoding with APCs is comparable with CA-SCL(L=4).

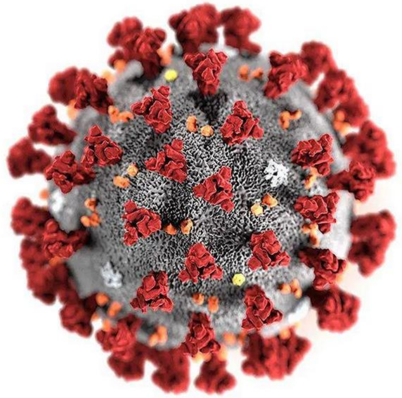
Hybrid decoding with CPCs is superior to CA-SCL(L=8).

Get performance gain from: 1. High polarization rate of CPC
2. Proposed hybrid decoding scheme

[1] C. Gao, R. Liu, B. Dai and X. Han, "Path Splitting Selecting Strategy-Aided Successive Cancellation List Algorithm for Polar Codes," in IEEE Communications Letters, vol. 23, no. 3, pp. 422-425, March 2019.
[2] L. Chandesris, V. Savin and D. Declercq, "Dynamic-SCFlip Decoding of Polar Codes," in IEEE Transactions on Communications, vol. 66, no. 6, pp. 2333-2345, June 2018.

Thanks!

One world, one fight!



COVID-19