Adaptive Matched Filter using Non-Target Free Training Data

ICASSP 2020

Aref Miri Rekavandi, Abd-Krim Seghouane, Robin J. Evans

The university of Melbourne

May, 2020

Outline

Model

Observations follow a general linear model (GLM) as

$$\mathbf{y} = \mathbf{H}\theta + \mathbf{n}$$

where

- $\mathbf{y} \in \mathbb{C}^N$
- $\mathbf{H} \in \mathbb{C}^{N \times t}$ [known]
- $\theta \in \mathbb{C}^t$ [unknown]
- $\mathbf{n} \in \mathbb{C}^N \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$ with unknown covariance

Another K training data denoted by $\{\mathbf{n}_k\}_{k=1}^K$ is available for the estimation of covariance.

Problem Definition

Now problem is to decide between

$$\mathcal{H}_0 = \begin{cases} \mathbf{y} = \mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}) \\ \mathbf{y}_k = \mathbf{n}_k \sim (1-\epsilon)\mathcal{CN}(\mathbf{0}, \mathbf{R}) + \epsilon\mathcal{CN}(\mathbf{H}\beta, \mathbf{R}) \end{cases}$$

and

$$\mathcal{H}_1 = \begin{cases} \mathbf{y} = \mathbf{H}\boldsymbol{\theta} + \mathbf{n} \sim \mathcal{CN}(\mathbf{H}\boldsymbol{\theta}, \mathbf{R}) \\ \mathbf{y}_k = \mathbf{n}_k \sim (1-\epsilon)\mathcal{CN}(\mathbf{0}, \mathbf{R}) + \epsilon\mathcal{CN}(\mathbf{H}\beta, \mathbf{R}) \end{cases}$$

Classical Detectors

GLRT framework for the case of $\epsilon = 0$ (signal free) is

$$T_{GLRT} = \frac{\max_{\boldsymbol{\theta},\mathbf{R}} f_1(\mathbf{y}|\mathbf{R})}{\max_{\mathbf{R}} f_0(\mathbf{y}|\mathbf{R})},$$

where the likelihood function in both hypotheses are maximized over all unknown parameters. This leads to

$$T_{GLRT} = \frac{\mathbf{y}^\dagger \mathbf{S}^{-1}\mathbf{H}(\mathbf{H}^\dagger \mathbf{S}^{-1}\mathbf{H})^{-1}\mathbf{H}^\dagger \mathbf{S}^{-1}\mathbf{y}}{1 + \mathbf{y}^\dagger \mathbf{S}^{-1}\mathbf{y}},$$

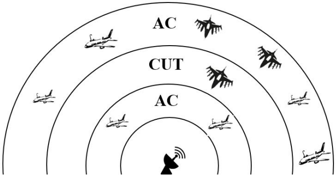
where $\mathbf{S} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{y}_k \mathbf{y}_k^\dagger$ is the sample covariance estimated using training data.

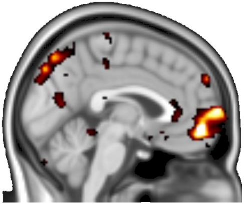
Classical Detectors

Using an ad-hoc approach, first we assume the covariance is known and maximize the likelihood function over $\boldsymbol{\theta}$. At the end, the covariance will be replaced with the sample covariance. This leads to adaptive matched filter (AMF) given by

$$T_{AMF} = \mathbf{y}^{\dagger}\mathbf{S}^{-1}\mathbf{H}(\mathbf{H}^{\dagger}\mathbf{S}^{-1}\mathbf{H})^{-1}\mathbf{H}^{\dagger}\mathbf{S}^{-1}\mathbf{y}.$$

Challenge:





From KL divergece to $\alpha-$divergence

Classical detectors are based on the maximum likelihood solution which in large samples regime ($K \to \infty$) is equivalent to

$$\hat{\boldsymbol{\lambda}}_{ML} = \arg\max_{\boldsymbol{\lambda}} \frac{1}{K} \sum_{i=1}^{K} \log\left(f\left(y_i, \lambda\right)\right) = \arg\min_{\boldsymbol{\lambda}} KL\left(f(\mathbf{y}, \boldsymbol{\lambda}^*), f(\mathbf{y}, \boldsymbol{\lambda})\right),$$

As an alternative we propose to use the $\alpha-$divergence defined as

$$D_\alpha\left(g(\mathbf{y}, \lambda) \parallel f(\mathbf{y}, \omega)\right) = \frac{1}{\alpha(\alpha-1)}\left[\int g(\mathbf{y}, \lambda)^\alpha f(\mathbf{y}, \omega)^{1-\alpha} d\mathbf{y} - 1\right],$$

to develop the test. When $\alpha \to 1$, $D_\alpha(.) \to KL(.)$.

Proposed Robust AMF

We adopt the AMF approach. But for estimation of the covariance matrix in second step, we use following expression

$$\hat{\mathbf{R}} = \arg\min_{\mathbf{R}} D_\alpha \left(f(\mathbf{y}, \mathbf{R}), g_e(\mathbf{y}; \mathbf{y}_1, \cdots, \mathbf{y}_K) \right),$$

where $g_e(\mathbf{y}; \mathbf{y}_1 \cdots \mathbf{y}_K) = \frac{1}{K} \sum_{k=1}^{K} \delta(\mathbf{y} - \mathbf{y}_k)$ is the empirical denisity. The solution gives:

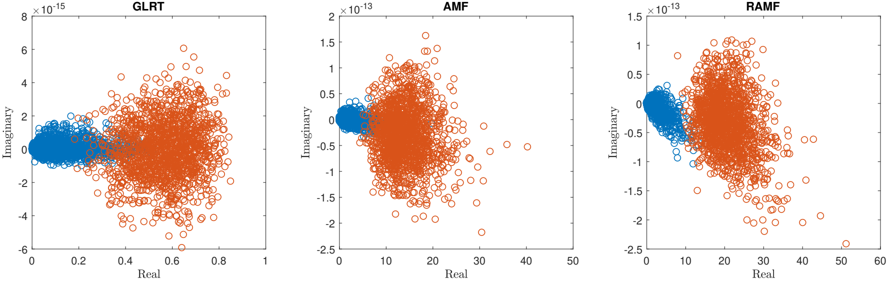
$$\hat{\mathbf{R}} = \frac{1}{\sum_{k=1}^{K} w_k} \sum_{k=1}^{K} w_k \mathbf{y}_k \mathbf{y}_k^\dagger,$$

where $w_k = \exp\{-(1-\alpha)\mathbf{y}_k^\dagger \mathbf{R}^{-1} \mathbf{y}_k\}$. This leads to

$$T_{RAMF} = \mathbf{y}^\dagger \hat{\mathbf{R}}^{-1} \mathbf{H} (\mathbf{H}^\dagger \hat{\mathbf{R}}^{-1} \mathbf{H})^{-1} \mathbf{H}^\dagger \hat{\mathbf{R}}^{-1} \mathbf{y}.$$

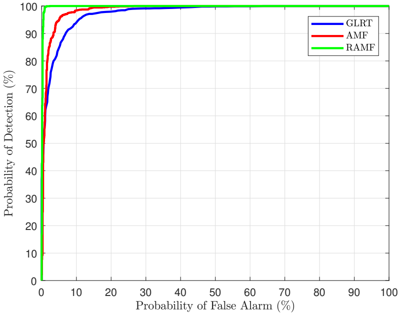
Results

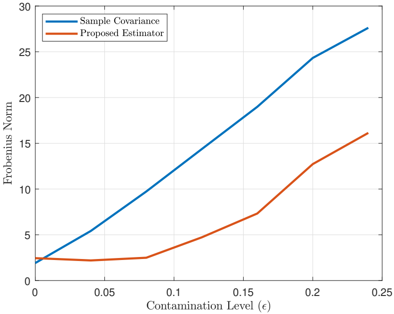
In simulation, $\mathbf{R}$ is an exponentially correlated covariance matrix with one-lag correlation coefficient 0.9, i.e., the $(i,j)-$th element of $\mathbf{R}$ is set to $0.9^{|i-j|}$. SNR=25dB, $N=10$, $K=40$, $\epsilon=0.15$ and $t=2$. Scatter plots of $\mathcal{H}_0$ (blue) and $\mathcal{H}_1$ (red) are shown below.



Results

Receiver operating characteristic plot (left) and accuracy of the robust estimator (right) for different ϵ values:






Questions?
Thanks for your attention!