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Spatial Active Noise Control Based on Kernel Interpolation with Directional Weighting

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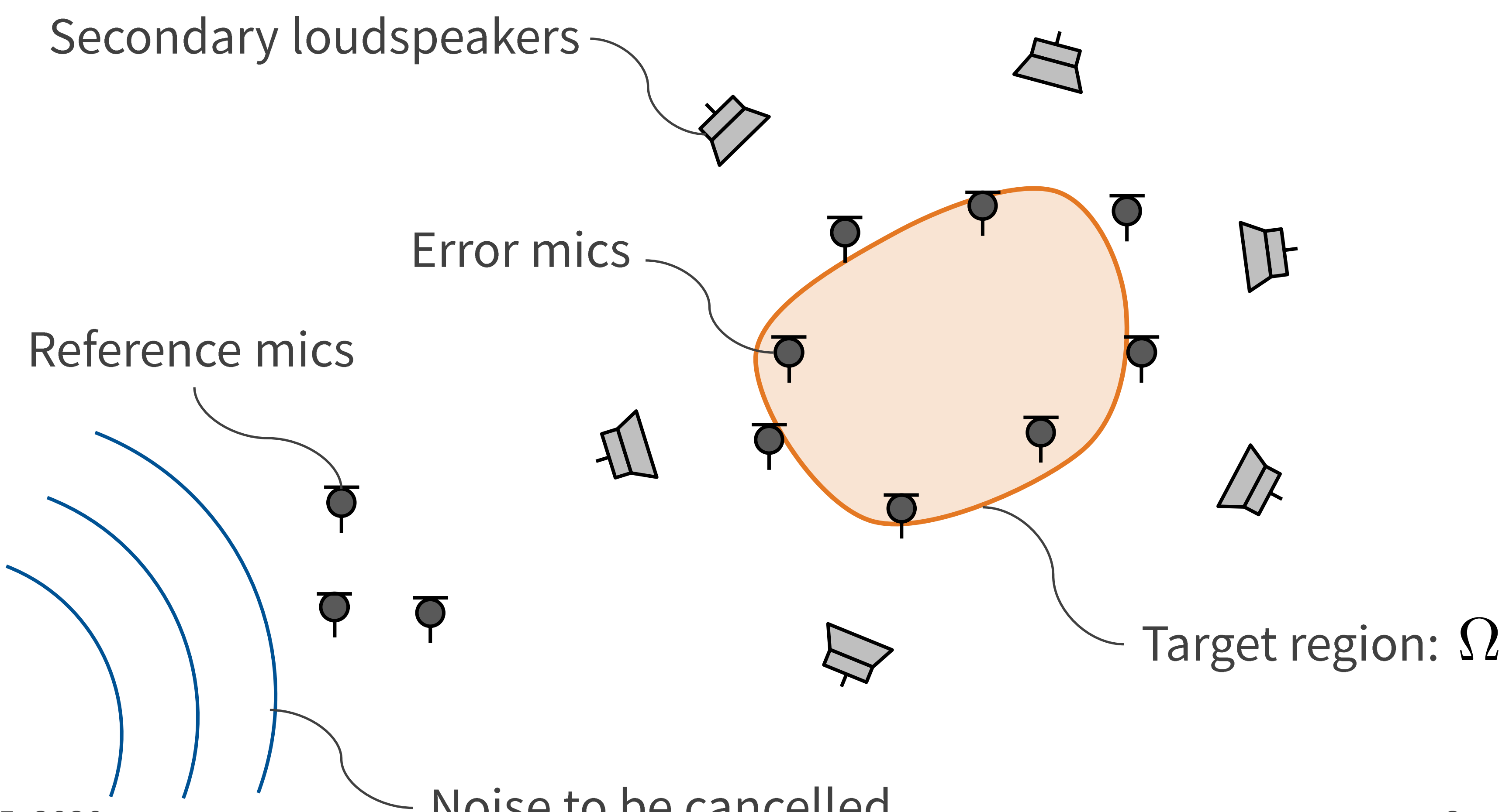
Spatial Active Noise Control

- Spatial active noise control (ANC)
  - Noise cancellation in space with multiple loudspeakers
  - Focused on 2D problem, but can be extended to 3D case



Multipoint pressure control

- Multipoint pressure control (MPC) in freq domain
  - Reducing noise power at discrete error microphones

Cost function of MPC:

$$\mathcal{L}_{\mathrm{MPC}} = \|\boldsymbol{e}(\omega)\|^2 = \sum_{m=1}^{M} |e_m(\omega)|^2$$

# of error mics

Signals of error mics at ω :

$$\boldsymbol{e}(\omega) = [e_1(\omega), \ldots, e_M(\omega)]^{\mathsf{T}}$$

- Adaptive filtering technique is used to obtain driving signals of secondary loudspeakers for minimizing $\mathcal{L}_{\mathrm{MPC}}$
- Noise cancellation in the region between error mics is NOT guaranteed owing to point-wise cost funtion

Cost function based on regional noise power

[Ito+ ICASSP 2019]

➢ Our strategy to achieve spatial ANC

Cost function based on regional noise power:

$$\mathcal{L} = \int_{\Omega} |u(\boldsymbol{r}, \omega)|^2 \mathrm{d}\boldsymbol{r}$$

Sound pressure at $\boldsymbol{r} \in \Omega$

- Regional integral of noise power over target region Ω is used as cost function to cancel spatial noise
- Pressure distribution inside Ω must be estimated from signals of error mics $\boldsymbol{e}$ to calculate the cost function $\mathcal{L}$
- Adaptive filtering algorithm to minimize $\mathcal{L}$ estimated from $\boldsymbol{e}$ is derived

Kernel interpolation of sound field

How to estimate $u(\boldsymbol{r})$ $(\boldsymbol{r} \in \Omega)$ from $\boldsymbol{e}$ $(\in \mathbb{C}^M)$?

- Kernel ridge regression of $u(\boldsymbol{r})$

Regularization term in $\mathcal{H}$

$$\underset{u \in \mathcal{H}}{\text{minimize}} \sum_{m=1}^{M} |u(\boldsymbol{r}_m) - e_m|^2 + \lambda \|u\|_{\mathcal{H}}^2$$

Position of mth error mic

$$\hat{u}(\boldsymbol{r}) = \left((\boldsymbol{K} + \lambda \boldsymbol{I})^{-1} \boldsymbol{e}\right)^{\mathsf{T}} \boldsymbol{\kappa}(\boldsymbol{r})$$

$$\boldsymbol{K} = \begin{bmatrix} \kappa(\boldsymbol{r}_1, \boldsymbol{r}_1) & \cdots & \kappa(\boldsymbol{r}_1, \boldsymbol{r}_M) \\ \vdots & \ddots & \vdots \\ \kappa(\boldsymbol{r}_M, \boldsymbol{r}_1) & \cdots & \kappa(\boldsymbol{r}_M, \boldsymbol{r}_M) \end{bmatrix}$$: Gram matrix

$$\boldsymbol{\kappa}(\boldsymbol{r}) = \begin{bmatrix} \kappa(\boldsymbol{r}, \boldsymbol{r}_1) & \cdots & \kappa(\boldsymbol{r}, \boldsymbol{r}_M) \end{bmatrix}^{\mathsf{T}}$$

Kernel function of $\mathcal{H}$

Kernel interpolation of sound field

How to estimate $u(\boldsymbol{r})$ $(\boldsymbol{r} \in \Omega)$ from $\boldsymbol{e}$ $(\in \mathbb{C}^M)$?

- Under the assumption that possible noise source direction is uniform, kernel function $\kappa(\boldsymbol{r}, \boldsymbol{r}_m)$ should be 0th-order Bessel function in 2D case:

Wave number $(k = \omega / c)$

$$\kappa(\boldsymbol{r}, \boldsymbol{r}_m) = J_0(k\|\boldsymbol{r} - \boldsymbol{r}_m\|)$$

[Ito+ ICASSP 2019]

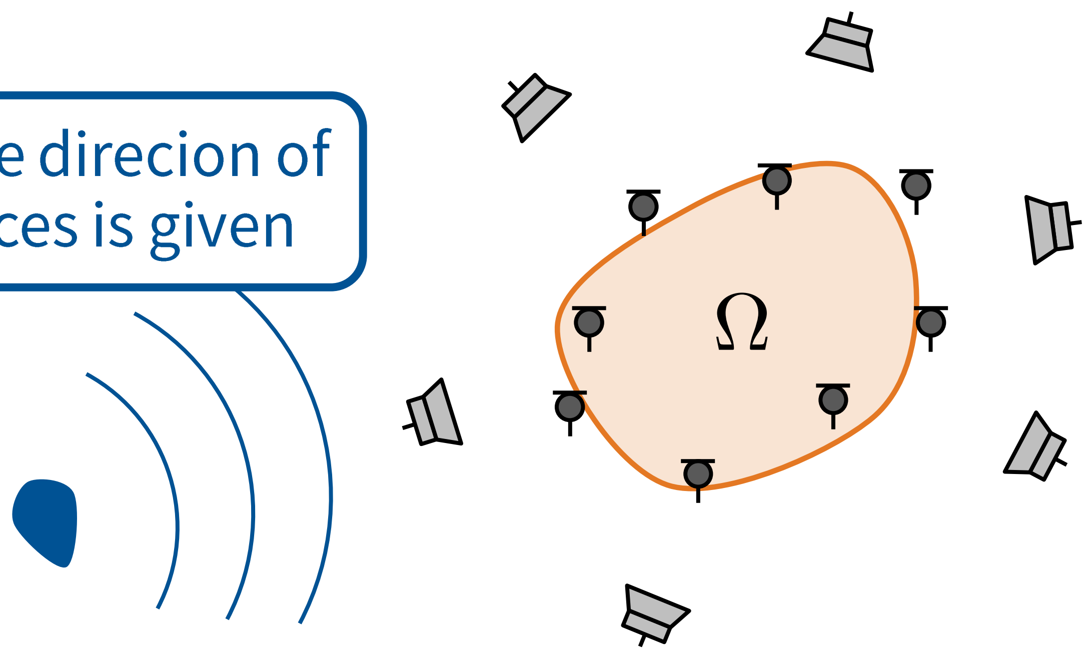
- Function interpolation with the constraint that $u(\boldsymbol{r})$ satisfies Helmholtz equation [Ueno+ IWAENC 2018]
- Correspond to sound field analysis based on spherical harmonic decomposition of infinite-order [Ueno+ IEEE SPL 2018]

Effective sound field interpolation method when noise signal possibly comes from all directions

Idea

- **Use of prior knowledge on approximate direction of primary noise sources**
  - In some practical applications, approximate direction of noise sources can be given in advance
  - Propose spatial ANC based on kernel interpolation with directional weighting to take the prior knowledge into consideration

Approximate direcion of noise sources is given



Reproducing kernel for directional weighting

- Kernel function is defined as

Planewave arriving from $\boldsymbol{\eta}$ ($\boldsymbol{k} = -k\boldsymbol{\eta}$)

$$\kappa(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{1}{2\pi} \int_S \underline{w(\boldsymbol{\eta})} e^{\mathrm{j}\boldsymbol{k}^{\mathsf{T}}(\boldsymbol{r}_1 - \boldsymbol{r}_2)} \mathrm{d}\boldsymbol{\eta}$$

Directional weighting function

  - $w(\boldsymbol{\eta})$ corresponds to directional weghting of regularization term.
  - The smaller the weighting function is, the larger the regularization term becomes.
  - Weighting function in prior arrival direction should be set larger than the other directions.

Reproducing kernel for directional weighting

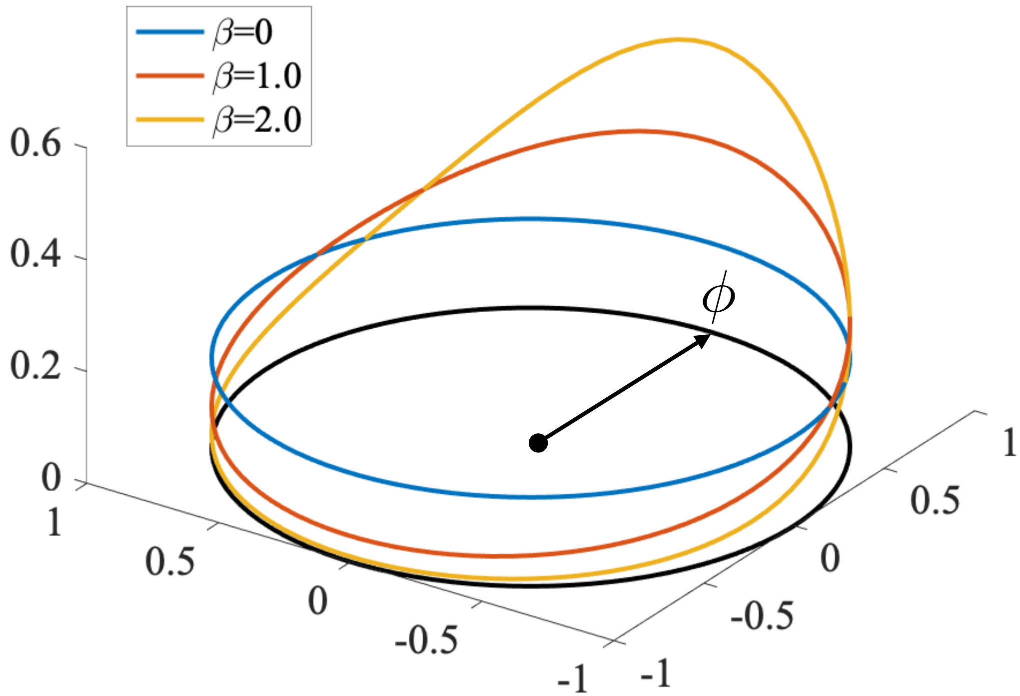
- Weighting function is defined as

$$w(\boldsymbol{\eta}) = e^{\beta \cos(\theta - \phi)}$$

θ : Angle of direction $\boldsymbol{\eta}$

ϕ : Approximate direction of noise source

β : Constant shape parameter



Reproducing kernel for directional weighting

- Kernel function is derived as

$$\kappa(\boldsymbol{r}_1, \boldsymbol{r}_2) = J_0 \left(\mathrm{j} \left[(\beta \cos\phi - \mathrm{j}kx_{12})^2 + (\beta \sin\phi - \mathrm{j}ky_{12})^2 \right]^{\frac{1}{2}} \right)$$

where $(x_{12}, y_{12})^{\mathsf{T}} := \boldsymbol{r}_1 - \boldsymbol{r}_2$

  – When $w(\boldsymbol{\eta}) = 1 \ (\beta = 0)$, $\kappa(\boldsymbol{r}_1, \boldsymbol{r}_2) = J_0\left(k\|\boldsymbol{r}_1 - \boldsymbol{r}_2\|\right)$

- Kernel ridge regression by replacing kernel function

$$\hat{u}(\boldsymbol{r}) = \left((\boldsymbol{K} + \lambda \boldsymbol{I})^{-1}\boldsymbol{e}\right)^{\mathsf{T}} \boldsymbol{\kappa}(\boldsymbol{r})$$

$$\boldsymbol{K} = \begin{bmatrix} \kappa(\boldsymbol{r}_1, \boldsymbol{r}_1) & \cdots & \kappa(\boldsymbol{r}_1, \boldsymbol{r}_M) \\ \vdots & \ddots & \vdots \\ \kappa(\boldsymbol{r}_M, \boldsymbol{r}_1) & \cdots & \kappa(\boldsymbol{r}_M, \boldsymbol{r}_M) \end{bmatrix}$$

$$\boldsymbol{\kappa}(\boldsymbol{r}) = \left[\kappa(\boldsymbol{r}, \boldsymbol{r}_1) \quad \cdots \quad \kappa(\boldsymbol{r}, \boldsymbol{r}_M)\right]^{\mathsf{T}}$$

Cost function based on regional noise power

- Cost function based on regional noise power

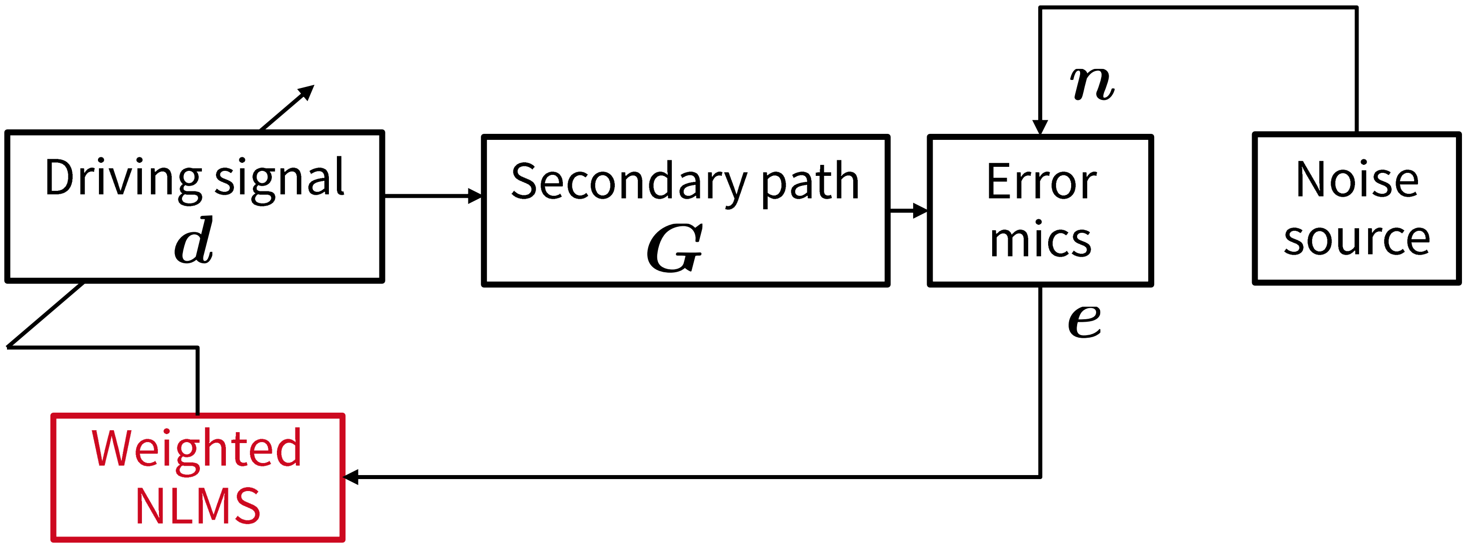
$$\mathcal{L} = \int_{\Omega} |\hat{u}(\boldsymbol{r})|^2 \mathrm{d}\boldsymbol{r} = \boldsymbol{e}^{\mathsf{H}} \boldsymbol{A} \boldsymbol{e}$$

$$\begin{cases} \boldsymbol{A} = \boldsymbol{P}^{\mathsf{H}} \left(\int_{\Omega} \boldsymbol{\kappa}(\boldsymbol{r}^*) \boldsymbol{\kappa}(\boldsymbol{r})^{\mathsf{T}} \mathrm{d}\boldsymbol{r} \right) \boldsymbol{P} \\ \boldsymbol{P} = (\boldsymbol{K} + \lambda \boldsymbol{I})^{-1} \end{cases}$$

  - Cost function becomes signal power of error mics with weighting matrix $\boldsymbol{A}$ that can be calcuated in advance.
  - Difference from the cost function of MPC is only in $\boldsymbol{A}$, which means various types of adaptive filtering algorithms can be applied with the same computational cost.

Feedback spatial ANC

- Focusing on simple feedback ANC for stationary noise



- Weighted NLMS for updating driving signals

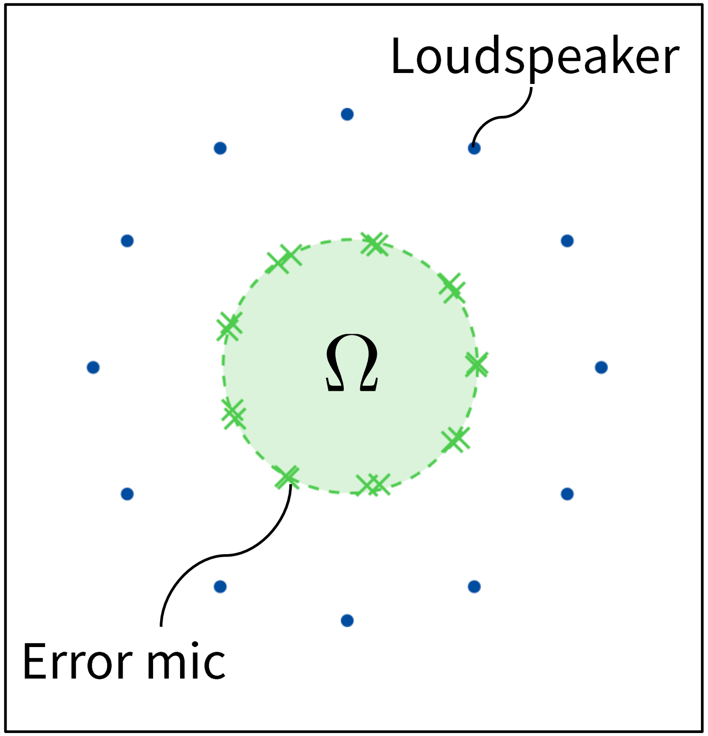
$$\boldsymbol{d}(n+1) = \boldsymbol{d}(n) - \frac{\mu_0}{\|\boldsymbol{G}^{\mathsf{H}}\boldsymbol{A}\boldsymbol{G}\| + \epsilon}\boldsymbol{G}^{\mathsf{H}}\boldsymbol{A}\boldsymbol{e}(n)$$

Normalized stepsize parameter (μ_0)

Regularization parameter (ϵ)

Experimental setting

- Performace of feedback spatial ANC
  - 2D free-field simulation in freq domain
  - Comparing proposed method and MPC
  - Target region is inside circle with radius of 0.5 m
- Secondary loudspeakers:
  - 12 point sources on circle with radius of 1.0 m
- Error mics:
  - 18 omni-directional mics distributed around circle with radius of 0.5 m
- Primary noise source:
  - Single point source
- Parameter setting:
  - Parameter of weighting function is set as β=0 or 2
  - β=0 corresponds to uniform weighting



Performance measure

➢ Regional noise power reduction

Total pressure field at i th evaluation point $\boldsymbol{r}_i$ for n th iteration

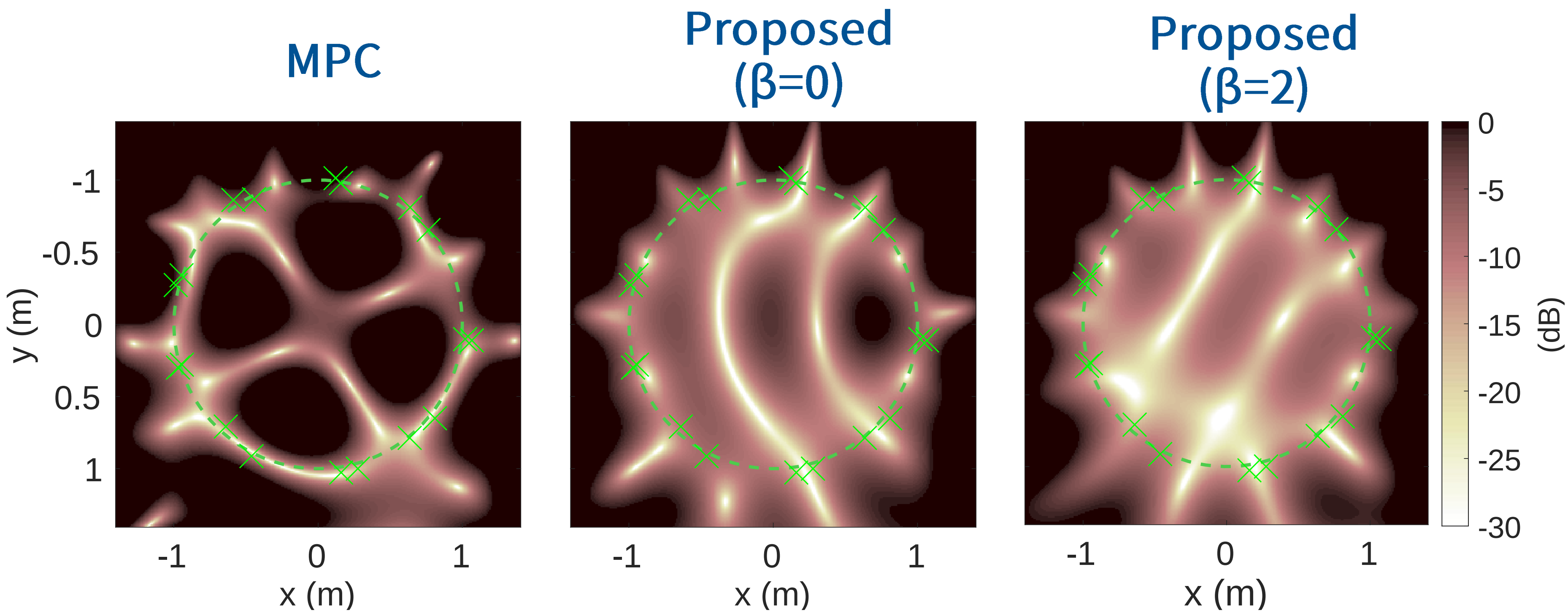
$$P_{\mathrm{red}} = 10\log_{10}\frac{\sum_i |u^{(n)}(\boldsymbol{r}_i)|^2}{\sum_i |u^{(0)}(\boldsymbol{r}_i)|^2}$$

Original pressure field generated by primary noise source

- Evaluation points were obtained by discretizing Ω at intervals of 0.02 m.
- Smaller P_{red} means better performance.

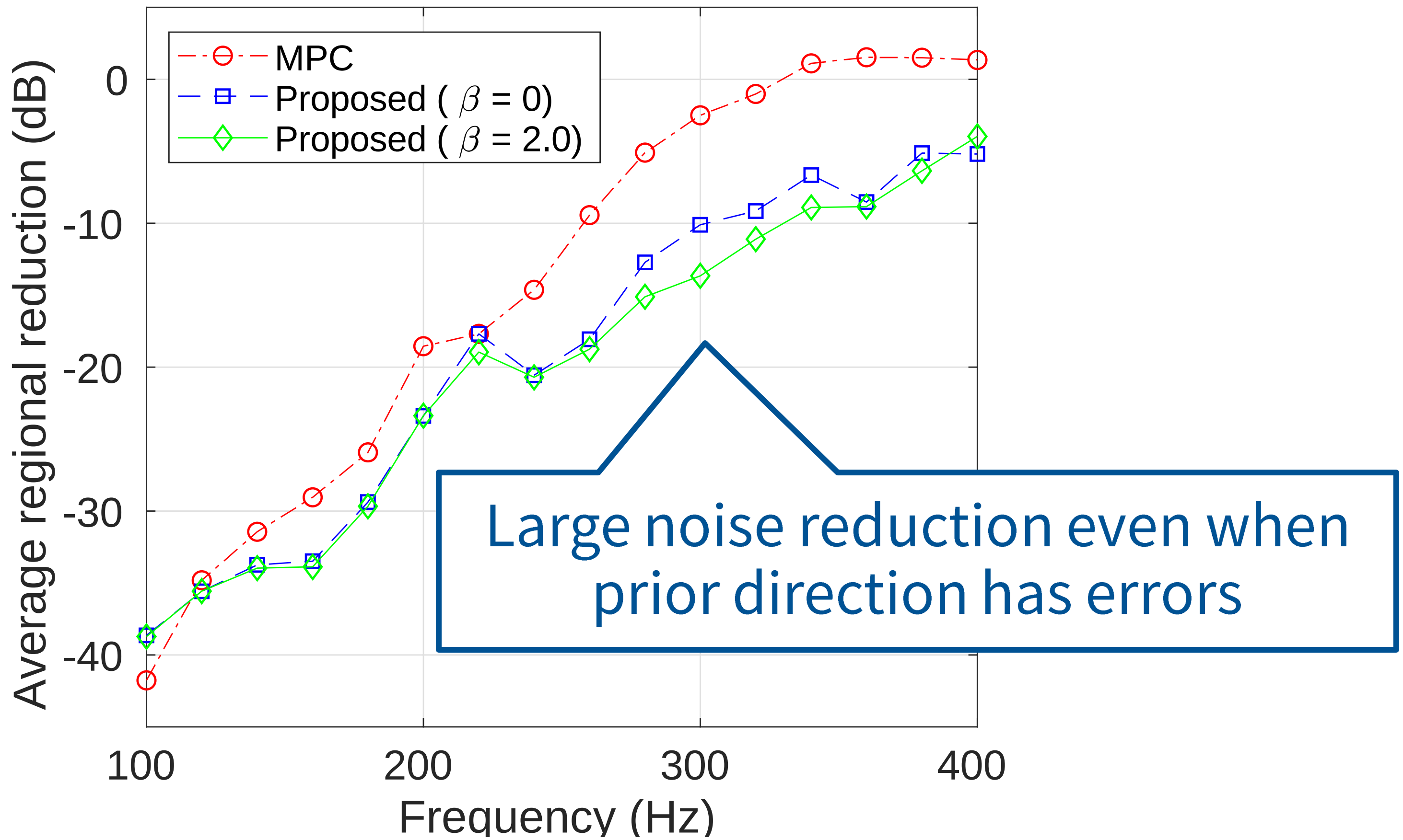
Results

- Power distribution after 500 iterations at 280Hz
  - Primary noise source at (10.0 m, 180 deg)



$P_{\mathrm{red}} = -8.0$ dB $P_{\mathrm{red}} = -16.1$ dB $P_{\mathrm{red}} = -20.3$ dB

- Average P_{red} of 100 trials (after 500 iterations) when position of noise source was randomly shifted
  - Standard deviation in radial and angular directions are 0.04 m and 3.0 deg



Conclusion

- Spatial ANC based on kernel interpolation with directional weighting
  - Cost function formulated with regional error power
  - Kernel interpolation of sound field with error mic signals
  - Approximate direction of primary noise source is used as prior knowledge
  - Computational cost of proposed weighted NLMS is equivalent to that of conventional MPC
  - Better performance even when prior noise source direction is slightly different from true direction