Space Filling Curves for MRI Sampling

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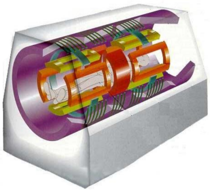
MRI Background

- MRI - Magnetic Resonance Imaging
  - Non - invasive medical imaging technique
- Based on NMR (Nuclear Magnetic Resonance)
- Pros: excellent soft tissue (ligaments, tendons, etc.) contrast and no ionizing radiation.
- Cons: expensive, slow, big.
- Imaging method:
  - Object in a strong, static, homogeneous magnetic field (B_0) of strengths 1.5T, 3T, 7T, 11.7T...
  - An RF magnetic field (B_1) to excite the nuclear spins.
  - Receiving coils to detect signals emitted by the excited spins as they precess within B_0 field.
  - Magnetic linear gradients (G_x, G_y, G_z) to spatially localize the detected signals.



MRI Background

- Received data $\mathbf{Y}$ is frequency encoded: $\mathbf{Y} = \mathcal{F}\mathbf{X}$, where $\mathbf{X}$ is the desired image, $\mathcal{F}$ is the Fourier operator.

$$k_x(t) = \gamma \int_0^t G_x(\tau)d\tau \quad \text{and} \quad k_y(t) = \gamma \int_0^t G_y(\tau)d\tau$$

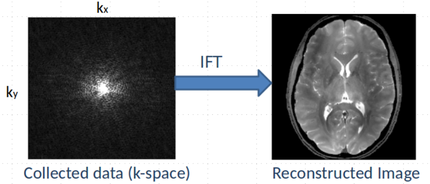
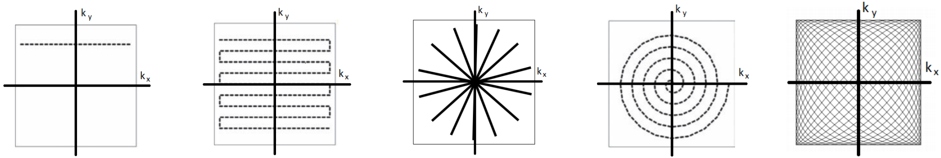


Figure: Image from k-space data.



(a) Cartesian (b) EPI (c) Radial (d) Spiral (e) Lissajous

Figure: Various k-space trajectories in the literature.

Block Diagram

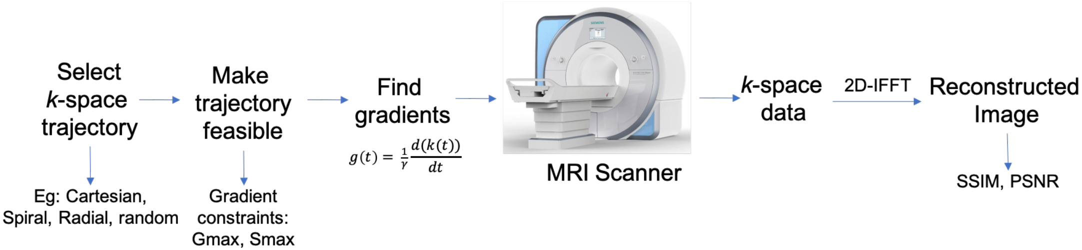


Figure: Block diagram of the steps of getting an MRI image from a selected k-space trajectory.

Motivation

- Fast data acquisition
  - Parallel imaging[1]
  - Compressed sensing[2]
- Design of optimal k-space trajectories
  - Echo-planar imaging (EPI) is a Cartesian-based fast scanning method.
    - Poor image reconstruction quality.
    - Loud acoustic noise.
  - Space filling curves (SFCs) as k-space trajectories[3].
    - Hilbert-Moore SFC trajectory which was observed to be robust to eddy currents.
    - SFCs reduce acoustic noise.

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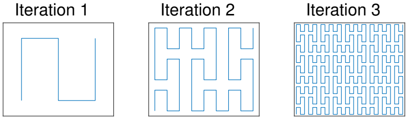
[1] K. P. Pruessmann, *et al.*, "SENSE: sensitivity encoding for fast MRI," MRM, 1999.

[2] M. Lustig *et al.*, "Sparse MRI: The application of compressed sensing for rapid MR imaging," MRM, 2007.

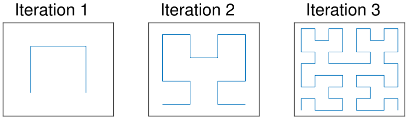
[3] W. D. Blecher, The Hilbert-Moore sequence acoustic noise optimized MR imaging, Ph.D. thesis, 2008.

Space Filling Curves (SFCs) - Introduction

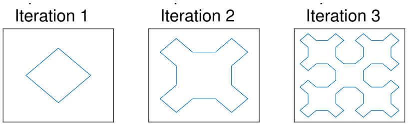
- A continuous mapping from closed unit interval $\mathrm{I} \in [0, 1]$ to a multidimensional unit hyperplane $\Omega \in [0, 1]^{\mathrm{d}}, \mathrm{d} > 1$ such that the curve passes through every point in the space exactly once.
- SFCs are popular in fields of computer science (to linearize multi-dimentional data).
- These curves are constructed iteratively as a sequence of continuous piecewise linear curves.
- Kinds of SFCs depending on the shape of the curve: Peano, Hilbert, Sierpinski, Dragon, Gosper and others.



(a) Peano SFC.



(b) Hilbert SFC.



(c) Sierpinski SFC.

Figure: Various space filling curves up to three iterations.

- Variable Density (VD) sampling is an integral part of designing k-space trajectories.
- VD-SFCs: sample the region near the center of the k-space with an SFC at an iteration higher than the SFC near the boundary region.

Construction of VD-SFCs:

- Divide the k-space into multiple sections.
- The section at the center of the k-space is traversed using an SFC with iteration I_c.
- Divide the boundary of the k-space is divided into 12 smaller sections. Each section is traversed using an SFC with I_b, where $I_b < I_c$.
- SFC with iteration I_b is taken as reference and is rotated and/or flipped such that the ending point of the SFC in one section is nearest to the starting point of the next section.
- The transformations related to the various SFCs are different from others as the starting and ending points vary according to the SFC considered.

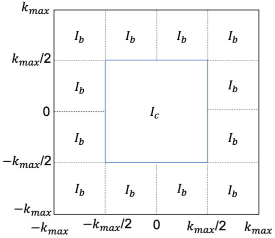


Figure: Construction of single-shot VD SFC trajectories.

- The intensity of the received signal reduces with time during each excitation.
- Hence, for high resolution images, (256×256 or 512×512), the k-space is traversed using multiple RF excitations.

Construction of multi-shot SFC trajectories

- Divide the k-space into four quadrants.
- Each quadrant is to be traversed separately using a VD SFC trajectory.
- Each quadrant is further divided into four sections.
- The section near the center of the k-space is traversed with an SFC of iteration I_c. The remaining three sections are traversed using an SFC with iteration I_b ($I_b < I_c$).

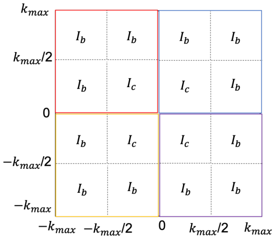


Figure: Construction of 4-shot VD SFC trajectories.

Feasible trajectory

- The hardware and the safety concern in an MRI machine restrict the amount of current through the gradient coils resulting in gradient constraints of maximum amplitude (G_{max}) and slew rate (S_{max}).
- As a result, the traversal of the k-space trajectories will be limited in velocity ($v_{max} = \gamma G_{max}$) and acceleration ($a_{max} = \gamma S_{max}$).
- The trajectories are defined by a few control points and are infeasible.
- The actual points to be sampled along these trajectories such that they satisfy the above constraints are obtained by the optimal control-based method, known as the Time-Optimal Control (TOC) method[1].
  - This method gives the fastest gradients to traverse the given trajectory.

[1] M. Lustig *et al.*, "A fast method for designing time-optimal gradient waveforms for arbitrary k-space trajectories," IEEE TMI, 2008.

Reconstruction Method

- **Sparsity in transform domain**: MR images are sparse in the wavelet, finite differences and DCT domains.
- **Sampling matrix**: By the physics of the system, the sampling is in the frequency domain (k-space).
- Reconstruct a 2D image $\mathbf{X}$ using the undersampled k-space[1]:

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathrm{NFFT}(\mathbf{X}) - \mathbf{Y}\|_2^2 + \lambda_1 \|\mathcal{W}(\mathbf{X})\|_1 + \lambda_2 \|\mathbf{X}\|_{\mathrm{TV}}$$

  where $\mathbf{Y}$ is the observed k-space data, NFFT is the nonuniform fast Fourier transform, $\mathcal{W}(\cdot)$ is the wavelet transform and $\|\cdot\|_{\mathrm{TV}}$ is the total variation (TV) norm.
- This problem is solved using non-linear conjugate gradient with a fast and cheap backtracking line-search[2].

[1] M. Lustig *et al.*, "Sparse MRI: The application of compressed sensing for rapid MR imaging," MRM, 2007.
[2] M. Lustig, "SparseMRI toolbox" downloaded from http://www.eecs.berkeley.edu/mlustig/software.html.

(a) 128 × 128 Shepp-Logan phantom image

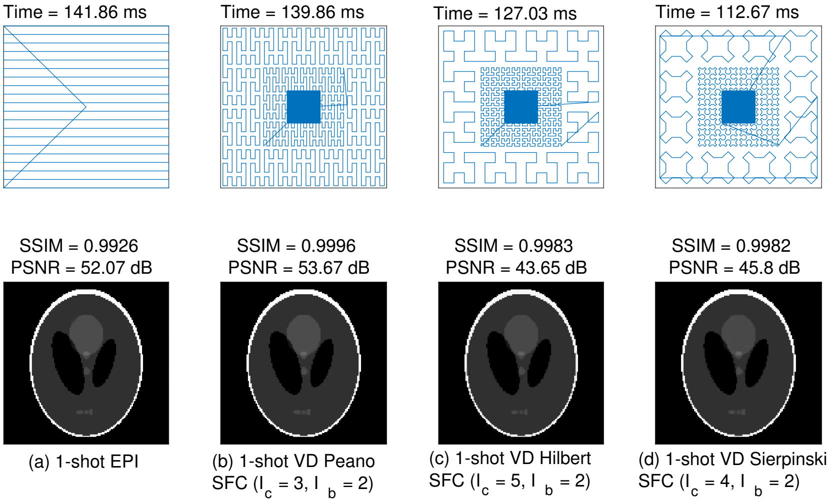


Figure: Performance comparison of single-shot VD SFC trajectories with EPI.

Results

(a) 128×128 Shepp-Logan phantom image

Trajectory	I_c	I_b	Time (ms)	SSIM	PSNR
Peano	2	1	50.34	0.8419	25.41 dB
Peano	3	1	**84.44**	**0.9858**	**35.72 dB**
Peano	3	2	139.86	0.9996	53.21 dB
Hilbert	3	2	63.99	0.9204	31.59 dB
Hilbert	4	2	**80.41**	**0.9829**	**38.75 dB**
Hilbert	4	3	130.55	0.9991	49.90 dB
Hilbert	5	2	127.03	0.9984	43.81 dB
Sier.	3	1	62.15	0.9465	32.22 dB
Sier.	3	2	**77.19**	**0.9820**	**38.22 dB**
Sier.	4	1	97.62	0.9938	38.53 dB
Sier.	4	2	112.67	0.9983	46.08 dB
EPI	-	-	141.86	0.9926	52.07 dB

Table: Performance comparison of single-shot VD SFC trajectories with EPI.

- SFCs provide similar reconstruction performance for similar scan time.
- For little compromise in image quality, SFCs provide reduction in read-out time.

(b) 256 × 256 analytical brain phantom image and brain MRI image.

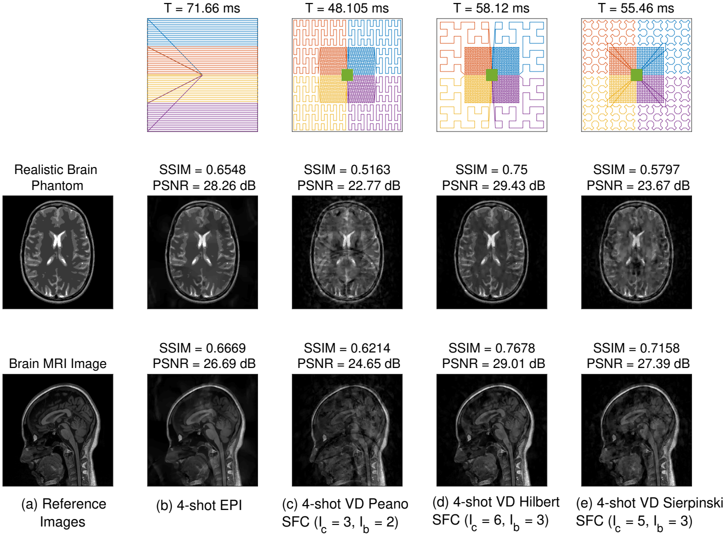


Figure: Performance comparison of 4-shot trajectories.

(b) 256×256 analytical brain phantom image and brain MRI image.

Traj.	I_c	I_b	Time (ms)	Brain Phantom		Brain MRI	
				SSIM	PSNR	SSIM	PSNR
Peano	3	1	34.19	0.5119	22.29 dB	0.6160	24.74 dB
Peano	3	2	48.10	0.5163	22.77 dB	0.6214	24.65 dB
Peano	4	1	159.03	0.9788	33.68 dB	0.9534	34.57 dB
Hilbert	6	3	**58.12**	**0.7500**	**29.43 dB**	**0.7678**	**29.01 dB**
Hilbert	6	4	70.59	0.7830	31.17 dB	0.7871	29.70 dB
Hilbert	6	5	105.32	0.7821	31.32 dB	0.7842	29.64 dB
Sier.	5	3	**55.46**	0.5797	23.67 dB	**0.7158**	**27.39 dB**
Sier.	5	4	81.87	0.6043	23.99 dB	0.7110	27.47 dB
Sier.	6	1	113.53	0.9763	33.26 dB	0.9517	34.43 dB
EPI	-	-	71.66	0.6548	28.26 dB	0.6669	26.69 dB

Table: Performance comparison of 4-shot trajectories.

- For similar read-out time, 4-shot Hibert SFCs provide an improvement of about 3dB PSNR for both images.
- For shorter read-out time, 4-shot Hibert SFCs provide better reconstruction performance for both images.

Conclusion

- Proposed use of variable denisty SFCs for MRI sampling under CS scheme.
- Variable density muti-shot SFCs perform well for reconstruction of high resolution MRI images.
- SFCs with different iterations provide trajectories with different readout time and reconstruction performance.
- The performance of the proposed trajectories is compared with the EPI trajectory.
- Compared to the EPI trajectory, VD Hilbert SFCs are able to improve the reconstruction performance with about 19% shorter readout time.
- For applications such as dynamic cardiac imaging and real-time speech MRI, the proposed Hilbert SFCs will be useful.

Thank you!