$\Bbbk$-Space Trajectory Design for Reduced MRI Scan Time

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MRI Background

Magnetic Resonance Imaging (MRI) is a non-invasive medical imaging technique that provides excellent soft tissue contrast and no ionizing radiation.

- Object in a strong, static, homogeneous magnetic field (B_0) of strengths 1.5T, 3T, 7T, 11.7T...
- An RF magnetic field (B_1) to excite the nuclear spins.
- Coils to detect signals emitted by the excited spins as they precess within the magnetic field (B_0).
- Magnetic linear gradients (G_x, G_y, G_z) to spatially localize the detected signals.
- The measured data matrix $\mathbf{Y}$ corresponds to

$$\mathbf{Y} = \mathcal{F}\mathbf{X}$$

where $\mathcal{F}$ is the 2D FT operator, $\mathbf{X}$ is the image.

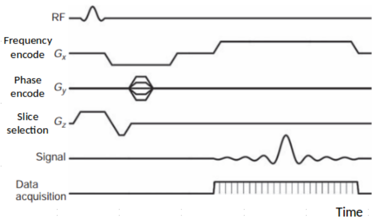

Figure: Gradient recalled echo (GRE) pulse sequence[1].

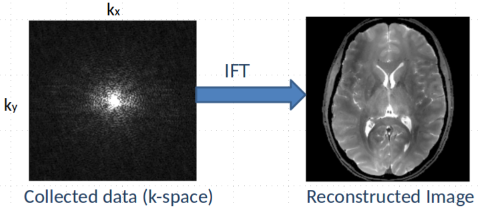

Figure: Image from k-space data.

- 2D k-space (Fourier domain):

$$k_x(t) = \gamma \int_0^t G_x(\tau) d\tau$$

$$k_y(t) = \gamma \int_0^t G_y(\tau) d\tau$$

- Physical constraints on gradients (due to physical limitations and safety concerns)
  - maximum gradient magnitude (G_{max})
  - maximum slew rate (S_{max})
- Various k-space trajectories can be used to scan the k-space.

[1] R. B. Buxton, Introduction to Functional Magnetic Resonance Imaging: Principles and Techniques, Cambridge University Press, 2002.

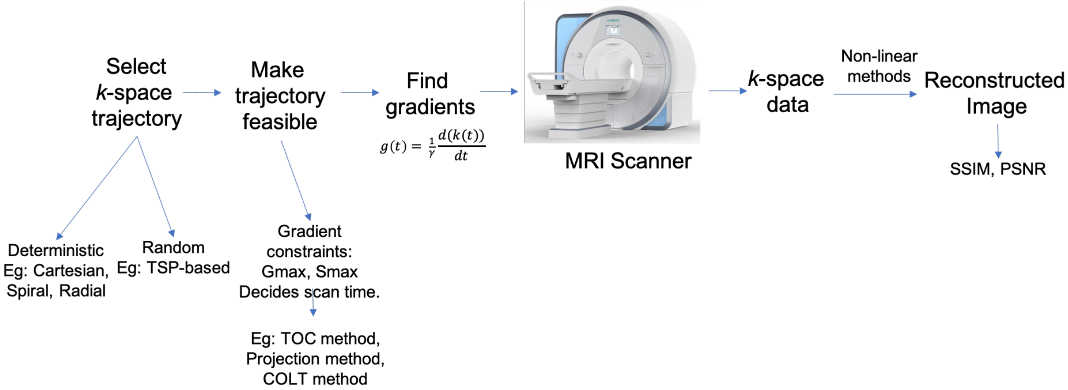

Figure: Block diagram of the steps of getting an MRI image from a selected k-space trajectory.

Motivation

- Long scan time
  - Parallel imaging[1]
  - Use compressed sensing (CS)[2] techniques.
- Use of variable density (VD)[2] sampling
  - The center of the k-space (low-frequency region) should be more densely sampled as compared to the boundaries (high-frequency region).
- TSP-based trajectories[3]
  - Finds a continuous trajectory through the randomly sampled points.
  - Ensures a short readout time.
  - Infeasible due to sharp turns.
- Need for a smooth and continuous trajectory
  - Gradients are limited in magnitude (G_{max}) and slew rate (S_{max}) due to physical and safety constraints.
- *Problem considered*: Design of feasible trajectories under the CS framework with the aim to reduce readout time without compromising the reconstruction performance.

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[1] K. P. Pruessmann, *et al.*, "SENSE: sensitivity encoding for fast MRI," MRM, 1999.

[2] Michael Lustig *et al.*, "Sparse MRI: The application of compressed sensing for rapid MR imaging," MRM, 2007.

[3] N. Chauffert *et al.*, "Travelling salesman-based variable density sampling," SampTA, 2013.

Feasible Trajectories

- The time-optimal control (TOC) method[1]
  - Uses optimal control theory to provide the fastest way to traverse the trajectory satisfying the gradient constraints.
  - For trajectories like the TSP trajectory need more samples at the corners which increases the total readout time.
- The projection-based method[2]
  - Parameterizes and projects given trajectory on the set of feasible trajectories.
  - Allows the trajectory to deviate from the original trajectory, hence smoothing out the sharp turns in a TSP trajectory.
  - This is used as a basis to build the proposed method.
- Proposed method is an alternative method to the projection method
  - Includes the lengths of the segments of the trajectory in the optimization problem.
  - This gives the designer the flexibility to alter the readout time by choosing a different weighting parameter.
- Assumptions: The effects of field inhomogeneity, T2* decay and other irregularities including gradient errors due to eddy currents are negligible and can be ignored.

[1] M. Lustig *et al.*, "A fast method for designing time-optimal gradient waveforms for arbitrary k-space trajectories," IEEE TMI, 2008.
[2] N. Chauffert *et al.*, "A projection algorithm for gradient waveforms design in magnetic resonance imaging," IEEE TMI, 2016.

Trajectory Design

- Set of feasible trajectories with k_x- and k_y-coordinates ($\mathbf{s}_x \in \mathbb{R}^m$ and $\mathbf{s}_y \in \mathbb{R}^m$, respectively) stacked in a vector $\mathbf{s} \in \mathbb{R}^{2m} = [\mathbf{s}_x^T \;\; \mathbf{s}_y^T]^T$ is given by

$$\mathcal{S}^m = \{\mathbf{s} \in \mathbb{R}^{2m} : \|\mathbf{D}_1^{(2)}\mathbf{s}\|_\infty \leqslant T_s\alpha, \|\mathbf{D}_2^{(2)}\mathbf{s}\|_\infty \leqslant T_s^2\beta\}$$

  where
  $\alpha = \gamma G_{\text{max}}$, $\beta = \gamma S_{\text{max}}$, T_s is the sampling time,
  $\mathbf{D}_1^{(2)}$ is the block diagonal matrix constructed from the first order difference matrix $\mathbf{D}_1$ and
  $\mathbf{D}_2^{(2)}$ is the block diagonal matrix constructed from the second order difference matrix $\mathbf{D}_2$,
  $\mathbf{D}_2 = -\mathbf{D}_1^T\mathbf{D}_1$.

- The feasible trajectory from the projection method[1] is given by

$$\mathbf{s}_P = \underset{\mathbf{s}\in\mathcal{S}^m}{\arg\min} \quad \frac{1}{2}\|\mathbf{s} - \mathbf{c}_{\text{par}}\|_2^2$$

  where $\mathbf{c}_{\text{par}} \in \mathbb{R}^{2m}$ is the trajectory obtained after constant velocity parameterization (CVP) of the original trajectory $\mathbf{c} \in \mathbb{R}^{2n}$, $n < m$.

[1] N. Chauffert *et al.*, "A projection algorithm for gradient waveforms design in magnetic resonance imaging," IEEE TMI, 2016.

Proposed Method - COLT

Constrained length trajectory (COLT) Method:

$$\mathrm{s}_{\mathrm{COLT}} = \underset{\mathrm{s}\in\mathcal{S}^n}{\arg\min} \quad \frac{1}{2}\|\mathbf{s}-\mathbf{c}\|_2^2 + \frac{\lambda}{2}\|\mathbf{D}_1^{(2)}\mathbf{s}\|_2^2$$

where $\lambda \in \mathbb{R}^+$ is a weighting parameter and $\|\mathbf{D}_1^{(2)}\mathbf{s}\|_2^2$ is the sum of squares of the Euclidean distances between consecutive points of s.

- The second term in the cost function imposes a cost on the segments of the trajectory s which in turn decides the overall length of the trajectory.
- To reduce the velocity variation in consecutive points, $\mathrm{s}_{\mathrm{COLT}}$ is parameterized using a constant velocity v which is a factor of $v_{\mathrm{max}}(=\gamma G_{\mathrm{max}})$, denoted $\mathrm{s}'_{\mathrm{COLT}}$.
- Higher value of $\lambda \implies$ points on trajectory closer together $\implies$ smaller number of samples points after CVP $\implies$ shorter readout time.
- Can be easily extended to 3D.

Single-shot and Multi-shot TSP Trajectories

- Long readout time of about 140 ms is not practical for MRI due to subsequent loss in signal strength, off-resonance effects and other acquisition errors.
- Multi-shot acquisition: Acquire different regions of k-space in multiple RF excitations.

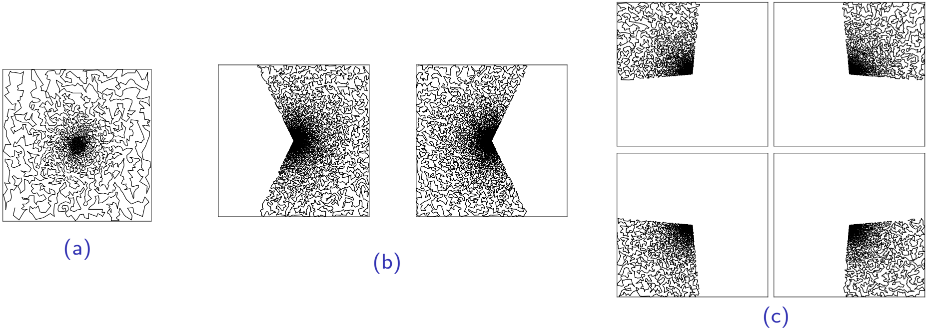

Figure: (a) Single-shot TSP trajectory. Multi-shot TSP trajectories: (b) 2-shot, and (c) 4-shot.

Image Reconstruction

- The MRI images are sparse in the finite difference domain and wavelet domain[1].
- With incoherent sampling, images can be reconstructed using:

$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\operatorname{argmin}} \; \|\mathsf{NFT}(\mathbf{X}) - \mathbf{Y}\|_2^2 + \lambda_1 \|\mathcal{W}(\mathbf{X})\|_1 + \lambda_2 \|\mathbf{X}\|_{\mathsf{TV}}$$

  where $\mathbf{Y}$ is the observed k-space data,
  $\mathcal{W}(\cdot)$ is the wavelet transform and
  $\|\cdot\|_{\mathsf{TV}}$ is the total variation (TV) norm.
- This is solved using non-linear conjugate gradient with a fast and cheap backtracking line-search[2].
- λ_1 and λ_2 are taken as 0.01 for the reconstruction of the phantom and the MRI images.
- A Daubechies-4 wavelet is used as the sparsifying basis.

[1] M. Lustig *et al.*, "Sparse MRI: The application of compressed sensing for rapid MR imaging," MRM, 2007.
[2] M Lustig, "SparseMRI toolbox downloaded from http://www.eecs.berkeley.edu/mlustig/software.html," 2014.

Simulation setup

- Test images: of size 256×256 realistic analytical brain phantom image[1] and a 256×256 T1-weighted sagittal brain MRI image (obtained using Cartesian imaging).
- The maximum gradient magnitude $\mathrm{G}_{\mathrm{max}}$ and slew rate S_{max} are taken as 40mT/m and 200mT/m/ms, respectively.
- The sampling frequency is taken to be 250MHz.
- Two types of trajectories are created using the COLT method:
  (a) TSP-based, and (b) random-like trajectories.
- Density function: $\propto 1/|\mathrm{k}|^2$.
- Comparisons of the methods are done based on the readout time, SSIM and PSNR for the reconstructed images.
- For TOC: TSP with 2500 initial points is used.
- For projection and COLT methods: $\mathbf{c}$ with 16384 ($= 25\%$ of $256\times$ 256) initial sample points is used.
- COLT is solved using proximal gradient descent in the dual domain (details in the paper).

[1] M. Guerquin-Kern *et al.*, "Realistic analytical phantoms for parallel magnetic resonance imaging," IEEE TMI, 2012.

Comparison of Performance

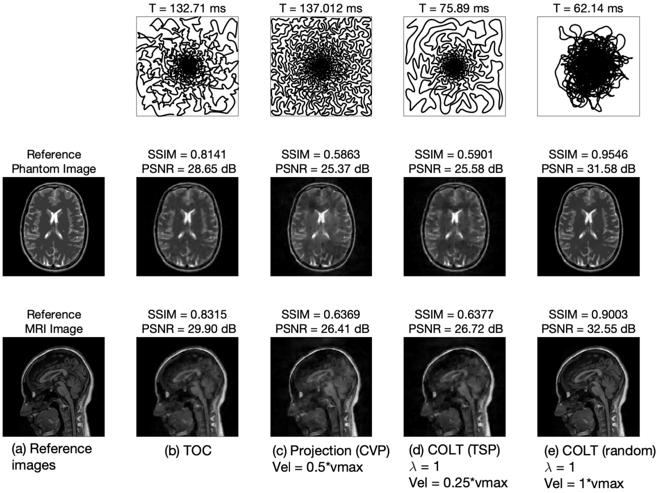

Figure: (a) Reference images used for simulations (brain phantom image and MRI image). Trajectories and reconstructed reference images using (b) TOC method, (c) projection method, (d) COLT with TSP method, (e) COLT with random method.

Method	# shots	Readout time (ms)		Phantom image				MRI image			
				SSIM		PSNR (dB)		SSIM		PSNR (dB)	
		Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
TOC	1	133.35	1.41	0.8131	0.0008	28.59	0.30	0.8325	0.0011	29.94	0.38
	2	89.95	0.91	0.8628	0.0108	29.89	0.23	0.8566	0.0045	30.72	0.13
	4	54.00	0.61	0.8918	0.0114	30.89	0.29	0.8699	0.0043	31.17	0.14
Projection	1	137.13	0.60	0.5352	0.0840	23.53	3.55	0.6105	0.0747	25.82	2.75
	2	86.51	0.40	0.6492	0.1135	26.30	3.41	0.6892	0.0759	27.15	1.60
	4	52.31	0.18	0.4692	0.0784	19.55	2.26	0.5382	0.0794	23.27	2.21
COLT TSP	1	**75.21**	1.06	0.5742	0.0786	24.53	3.09	0.6557	0.0790	26.85	3.22
	2	**48.73**	0.56	0.7295	0.0686	27.32	1.36	0.7433	0.0462	27.48	0.90
	4	**29.26**	0.31	0.6100	0.1260	23.84	3.68	0.6604	0.0940	26.56	2.43
COLT random	1	**61.59**	0.36	**0.8893**	0.0760	**30.11**	2.09	**0.8728**	0.0877	**29.55**	2.79

Table: Mean and standard deviation (S.D.) of readout time, SSIM and PSNR over 100 trials for the brain phantom and MRI image reconstructions using different methods under single and multi-shot schemes to obtain feasible trajectories. For Projection method: $\nu = 0.5\nu_{\max}$, For COLT TSP: $\nu = 0.25\nu_{\max}$, $\lambda = 1$. For COLT random: $\nu = \nu_{\max}$, $\lambda = 1$.

*Since there is a lot of randomness involved, for practical purposes, a trajectory that leads to a higher SSIM during simulations is proposed to be used.

Conclusion

- An alternative method for obtaining faster and feasible k-space sampling trajectories in MRI has been discussed.
- The effectiveness of interpolating the trajectory post-projection in reducing the readout time for TSP-based and random-like trajectories is shown.
- The proposed method provides an acceleration of about 50% which is significant.
- The proposed method also gives the designer the freedom to choose the weighting parameter in order to tune the trade-off between readout time and reconstruction performance.

Thank you!