Regularized partial phase synchrony index applied to dynamical functional connectivity estimation

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Context

Data from epileptic patient

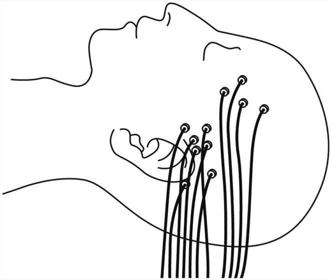


Figure 1: The patient stays one week at the hospital to record several seizures[1].

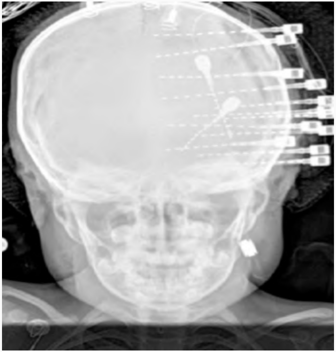


Figure 2: intracranial EEG electrodes implemented in the brain[2].

[1] Iida, K., & Otsubo, H. (2017). Stereoelectroencephalography: indication and efficacy. Neurologia medico-chirurgica, 57(8), 375-385..

[2] https://consultqd.clevelandclinic.org/turning-to-seeg-for-pediatric-patients-with-refractory-epilepsy/

General problem of my thesis

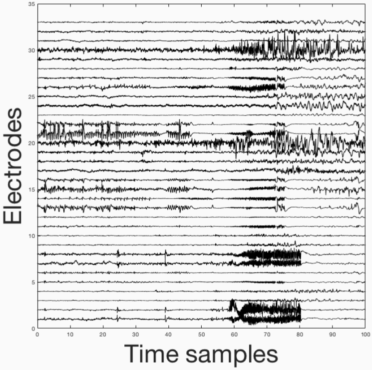


Figure 3: Example of a multivariate signal from the recording of a seizure

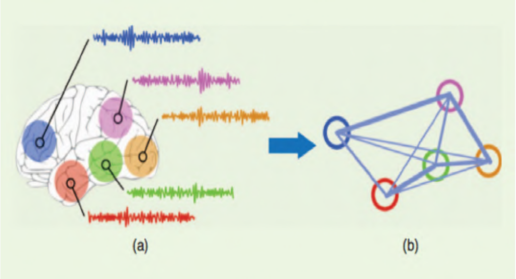
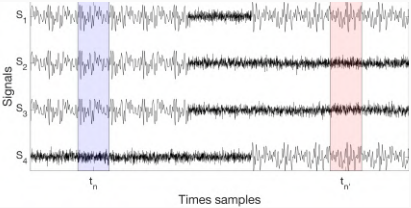


Figure 4: Converting multivariate signal to a graph[1], [2].

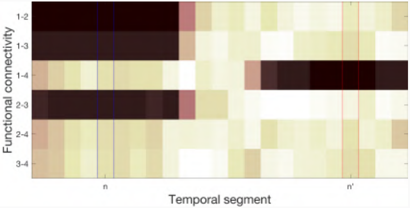
[1] Dong et Al. (2019). Learning graphs from data: A signal representation perspective. IEEE Signal Processing Magazine, 36(3), 44-63..

[2] Richiardi et Al.(2009). Machine learning with brain graphs: predictive modeling approaches for functional imaging in systems neuroscience. IEEE Signal Processing Magazine, 30(3), 58-70.

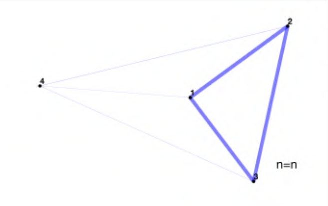
What is a connection ? Functional connectivity

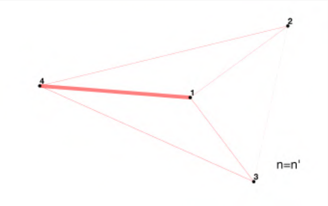


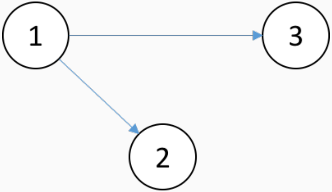
Model of 4 signals with temporal synchrony



Corresponding functional connectivity as a function of time



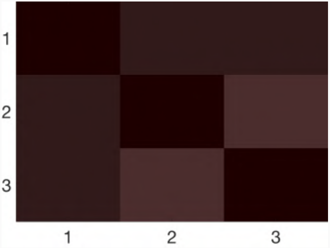




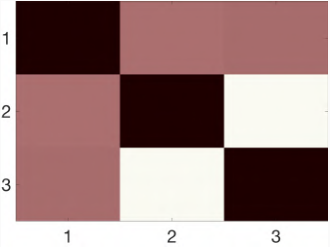
Example of graphical model

$$s_1(t) = \eta_1(t),$$
$$s_2(t) = 0.8s_1(t) + 0.2\eta_2(t),$$
$$s_3(t) = 0.8s_1(t) + 0.2\eta_3(t),$$

$\eta(t)$ is a realisation from a Gaussian distribution

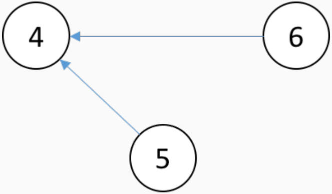


Correlation



Partial correlation

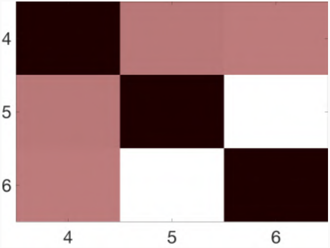
Correlation and partial correlation 2/2



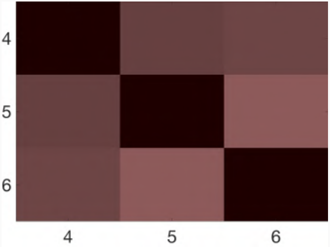
Example of graphical model

$$
\begin{aligned}
s_4(t) =& 0.4 s_5(t) + 0.4 s_6(t) + 0.2 \eta_4(t), \\
s_5(t) =& \eta_5(t), \\
s_6(t) =& \eta_6(t),
\end{aligned}
$$

$\eta(t)$ is a realisation from a Gaussian distribution



Correlation



Partial correlation

Precision matrix

Considering L real centred signals $s_l(t)$, $l \in 1, ..., L$, the empirical covariance between the signal l and l' is noted :

$$c_{ll'} = \sum_{t=1}^{T} s_l(t) s_l'(t)$$

$\mathbf{C}$ is the empirical covariance matrix and the correlation index:

$$\rho_{ll'} = \frac{c_{ll'}}{\sqrt{c_{ll} c_{l'l'}}}$$

We call $\Theta = \mathbf{C}^{-1}$ the precision matrix, it can be shown that the partial correlation index, noted $\check{\rho}$, between different signal l and l' is[1] :

$$\check{\rho}_{ll'} = -\frac{\theta_{ll'}}{\sqrt{\theta_{ll}\theta_{l'l'}}}$$

[1] Whittaker, J. (2009). Graphical models in applied multivariate statistics. Wiley Publishing.

Graphical lasso

Graphical lasso aims to add regularizations on the precision matrix assuming signal follows a multivariate Gaussian distribution:

$$p(\mathbf{s}, \Theta) = \frac{1}{2\pi\sqrt{det(\Theta^{-1})}} e^{-\frac{1}{2}\mathbf{s}^t \Theta \mathbf{s}} \frac{e^{-\lambda Reg(\Theta)}}{\int e^{-\lambda Reg(\Theta)} \, \mathrm{d}\Theta}$$

minimizing the regularized negative log-likelihood function gives [1] :

$$\underset{\Theta \in \mathbb{S}^p_{++}}{\operatorname{argmin}} \; -ln(det(\Theta)) + Tr\left(\Theta \mathbf{C}\right) + \lambda Reg(\Theta)$$

The usually performed regularisation corresponds to impose sparsity on the precision matrix $Reg(\Theta) = || \; \Theta \; ||_{od,1}$

[1] Friedman et Al. (2008). Sparse inverse covariance estimation with the graphical lasso. Biostatistics, 9(3), 432-441.

The phase locking value measure

- We note the analytic representation of the signal $s_l(t)$:

$$z_l(t) = a_l(t)e^{i\phi_l(t)}$$

- $a_l(t)$ the instantaneous amplitude of the signal $s_l(t)$.
- $\phi_l(t)$ the instantaneous phase of the signal $s_l(t)$.
- The Phase Locking Value (PLV)[1] between signals l and l' is defined as:

$$P_{ll'} = \frac{1}{T}\left|\sum_{t=1}^{T} e^{i\phi_l(t)}(e^{i\phi_{l'}(t)})^*\right| = \frac{1}{T}\left|\sum_{t=1}^{T} e^{i(\phi_l(t)-\phi_{l'}(t))}\right|$$

- We propose a regularized and partial extension of the Phase locking value index.

[1] JLachaux, J. P., Rodriguez, E., Martinerie, J., & Varela, F. J. (1999). Measuring phase synchrony in brain signals. Human brain mapping, 8(4), 194-208.

Regularized partial Phase Locking Value

Non parametric partial PLV

A non parametric estimate of the PLV can be computed, first we consider the pairwise phase synchrony interraction:

$$R_{ll'} = \frac{1}{T} \sum_{t=1}^{T} e^{i(\phi_l(t) - \phi_{l'}(t))}$$

R corresponds to the covariance matrix **C** between phase signals $e^{i\phi_l(t)}$, $\forall l \in 1, ..., L$. PLV index is :

$$P_{ll'} = \mid R_{ll'} \mid \quad (1)$$

Calling $\Omega = \mathbf{R}^{-1}$ the inverse of the pairwise phase synchrony matrix. a partial PLV index (pPLV), noted Q, can be introduced[1]:

$$Q_{ll'} = \frac{\mid \Omega_{ll'} \mid}{\sqrt{\Omega_{ll} \Omega_{l'l'}}}$$

[1] Schelter et Al. (2006). Partial phase synchronization for multivariate synchronizing systems. Physical review letters, 96(20), 208103.

Regularized pPLV

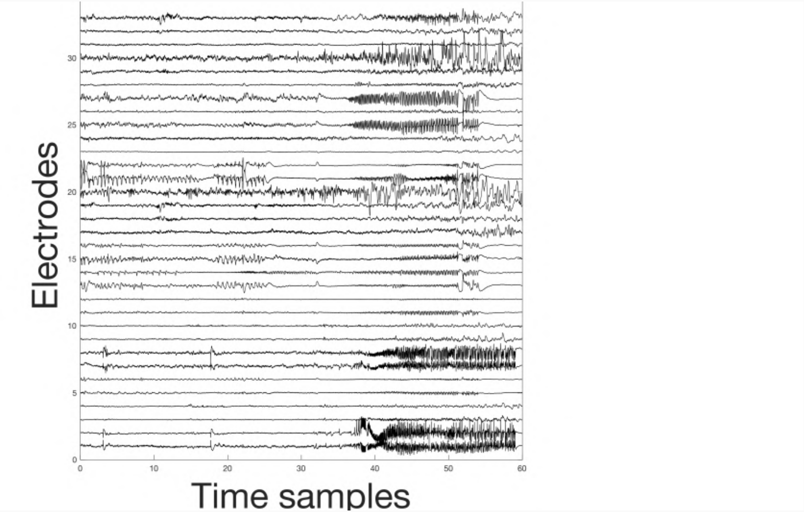
We add regularization on the pPLV function minimizing :

$$\underset{\Omega \in \mathbb{S}^{p}_{++}}{\operatorname{argmin}} \; -ln(det(\Omega)) + Tr\,(\Omega \mathbf{R}) + \lambda Reg(\Omega)$$

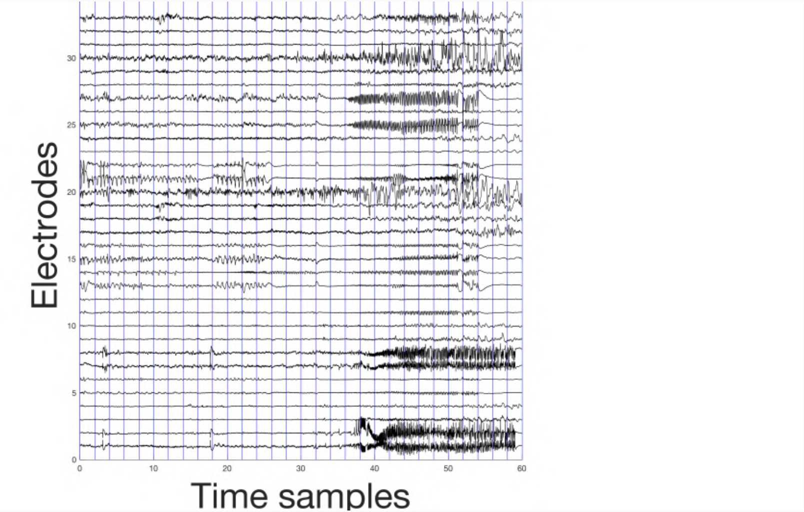
Even if the phase signal doesn't follow a multivariate gaussian distribution, this cost function is relevant since :

- The pairwise synchony matrix $\mathbf{R}$ is positive definite, thus the cost function is convex.
- If $\lambda = 0$, the minimizer will coincide with $\Omega = \mathbf{R}^{-1}$.

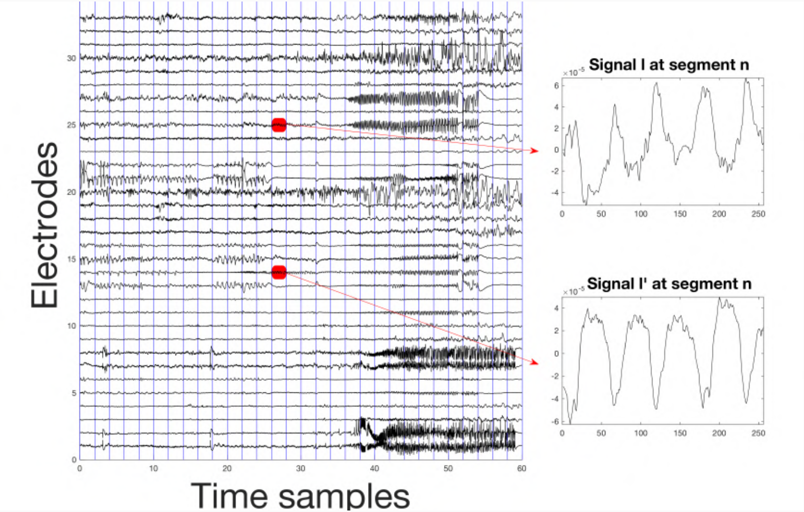
Time series 1/4



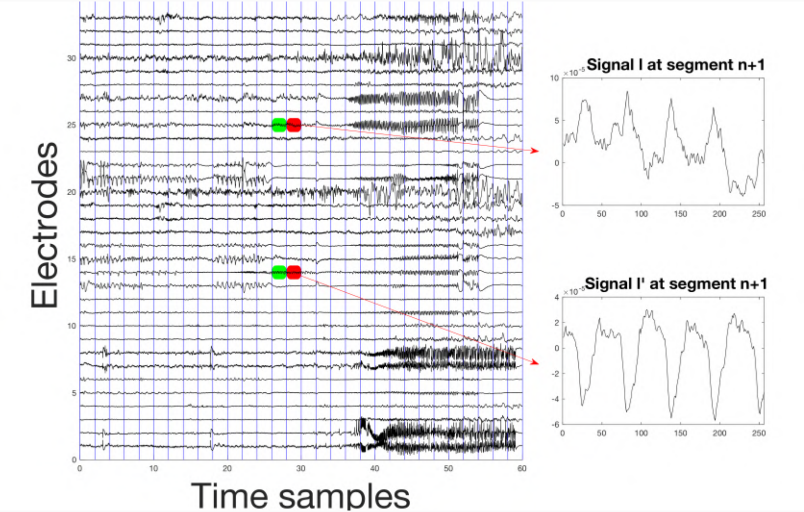
Time series 2/4



Time series 3/4



Time series 3/4



Time varying graphical lasso (TVGL)

The Sparse and the temporal regularization are added to the Graphical lasso criteria:

$$\underset{\{\Omega^{(1)},\ldots,\Omega^{(N)}\}\in\mathbb{S}^p_{++}}{\operatorname{argmin}} \sum_{n=1}^{N} l\left(\Omega^{(n)}, \mathbf{R}^{(n)}\right) + \lambda \mid\mid \Omega^{(n)} \mid\mid_{od,1} + \gamma \sum_{m=2}^{N} g(\Omega^{(m)} - \Omega^{(m-1)}),$$

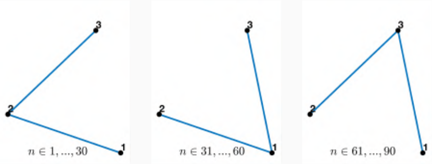
Where $l\left(\Omega^{(n)}, \mathbf{R}^{(n)}\right) = -ln(det(\Omega^{(n)})) + Tr\left(\Omega^{(n)}\mathbf{R}^{(n)}\right)$

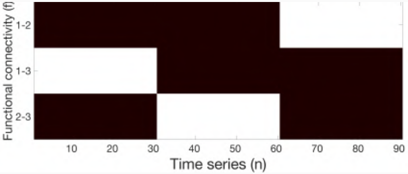
$g(\mathbf{X}) = \sum_{i=1}^{N} \mid\mid x_i(t) \mid\mid_F^2$

TVGL[1] is convex, can be implemented with ADMM[1], forward-backward [2].

[1] Hallac et Al. (2017, August). Network inference via the time-varying graphical lasso. In Proceedings of the 23rd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (pp. 205-213). ACM. [2] Tomasi et Al. (2018, August). Forward-Backward Splitting for Time-Varying Graphical Models. In International Conference on Probabilistic Graphical Models (pp. 475-486).

Application

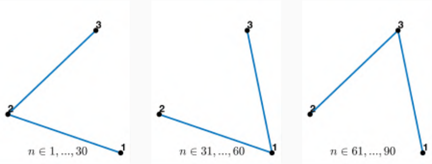


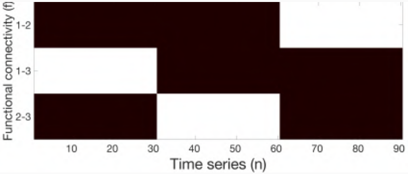


Ideal functional connectivity matrix

$$\begin{pmatrix} \frac{d}{dt}(s_l^{(n)}(t)) \\ \frac{d}{dt}(x_l(t)) \\ \frac{d}{dt}(y_l(t)) \end{pmatrix} = \begin{pmatrix} -w_l y_l(t) + \sigma \mathcal{N}(0,1) + \sum_{l' \neq l} \epsilon_{ll'}^{(n)} (s_{l'}^{(n)}(t) - s_l^{(n)}(t)) \\ w_{l'} s_l^{(n)}(t) + \alpha x_l(t) \\ b + (s_l^{(n)}(t) - c) y_l(t) \end{pmatrix}$$

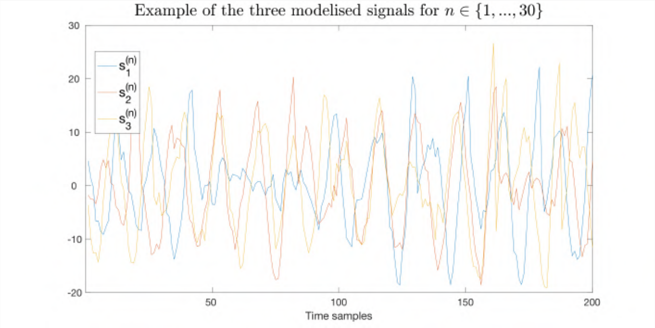
For $n \in \{1, ..., 30\}$ we have $\epsilon_{31}^{(n)} = 0$,
For $n \in \{31, ..., 60\}$ we have $\epsilon_{23}^{(n)} = 0$,
For $n \in \{61, ..., 90\}$ we have $\epsilon_{21}^{(n)} = 0$,
Else $\epsilon_{ll'}^{(n)}$ takes values from a uniform distribution $[0.05, 0.3]$.





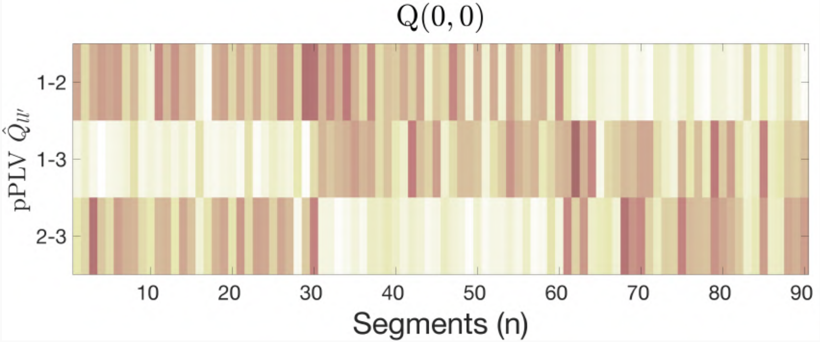
Roessler Model

Ideal functional connectivity matrix



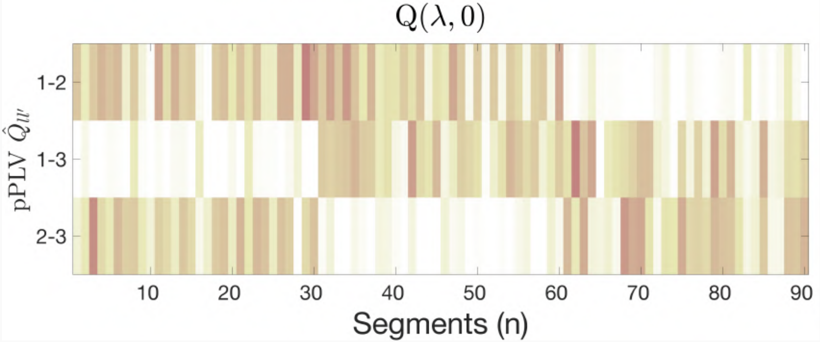
pPLV on the Roessler model

$$\underset{\{\Omega^{(1)},\ldots,\Omega^{(N)}\}\in\mathbb{S}^p_{++}}{\text{argmin}} \quad \sum_{n=1}^{N} l\left(\Omega^{(n)}, \mathbf{R}^{(n)}\right), \qquad Q_{ll'}(0,0) = \frac{|\ \Omega_{ll'}\ |}{\sqrt{\Omega_{ll}\Omega_{l'l'}}}$$



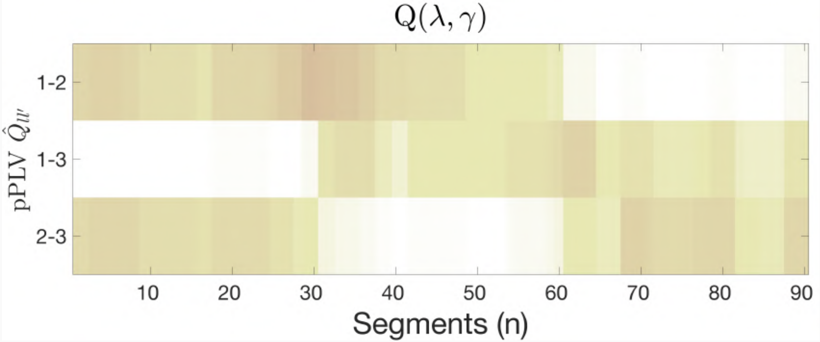
pPLV on the Roessler model

$$\underset{\{\Omega^{(1)},\ldots,\Omega^{(N)}\}\in\mathbb{S}^p_{++}}{\operatorname{argmin}} \sum_{n=1}^{N} l\left(\Omega^{(n)}, \mathbf{R}^{(n)}\right) + \lambda \mid\mid \Omega^{(n)} \mid\mid_{od,1},$$



pPLV on the Roessler model

$$\underset{\{\Omega^{(1)},\ldots,\Omega^{(N)}\}\in\mathbb{S}^p_{++}}{\operatorname{argmin}} \sum_{n=1}^{N} l\left(\Omega^{(n)}, \mathbf{R}^{(n)}\right) + \lambda \mid\mid \Omega^{(n)} \mid\mid_{od,1} + \gamma \sum_{m=2}^{N} g(\Omega^{(m)} - \Omega^{(m-1)}),$$



pPLV vs partial correlation

- The regularized pPLV corresponds to minimize this criteria :

$$\underset{\{\Omega^{(1)},\ldots,\Omega^{(N)}\}\in\mathbb{S}^p_{++}}{\operatorname{argmin}} \sum_{n=1}^{N} l\left(\Omega^{(n)}, \mathbf{R}^{(n)}\right) + \lambda \mid\mid \Omega^{(n)} \mid\mid_{od,1} + \gamma \sum_{m=2}^{N} g(\Omega^{(m)} - \Omega^{(m-1)}),$$

Then :

$$Q_{ll'} = \frac{\mid \Omega_{ll'} \mid}{\sqrt{\Omega_{ll}\Omega_{l'l'}}}$$

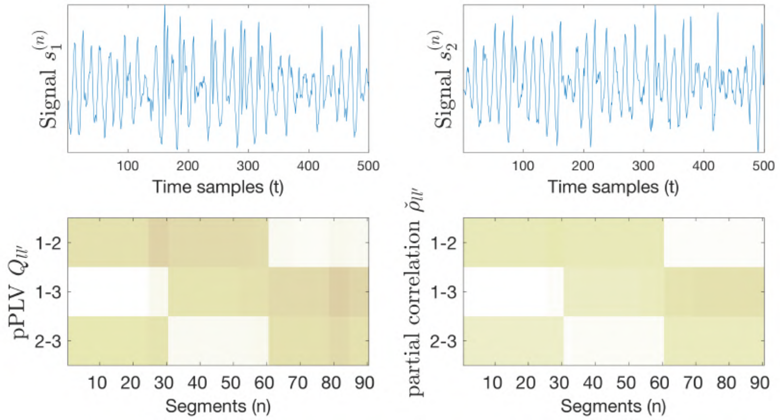
- The regularized partial correlation corresponds to minimize this criteria :

$$\underset{\{\Theta^{(1)},\ldots,\Theta^{(N)}\}\in\mathbb{S}^p_{++}}{\operatorname{argmin}} \sum_{n=1}^{N} l\left(\Theta^{(n)}, \mathbf{C}^{(n)}\right) + \lambda' \mid\mid \Theta^{(n)} \mid\mid_{od,1} + \gamma' \sum_{m=2}^{N} g(\Theta^{(m)} - \Theta^{(m-1)})$$

Then :

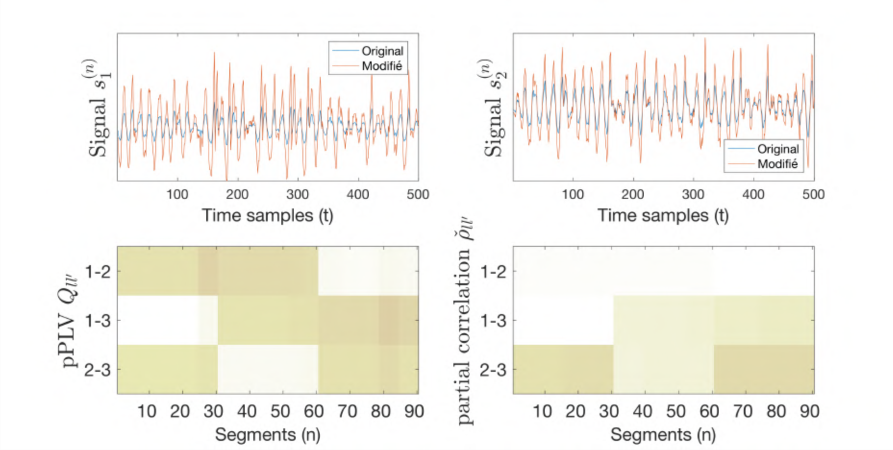
$$\check{\rho}_{ll'} = \frac{\mid \Theta_{ll'} \mid}{\sqrt{\Theta_{ll}\Theta_{l'l'}}}$$

Comparison of different methods



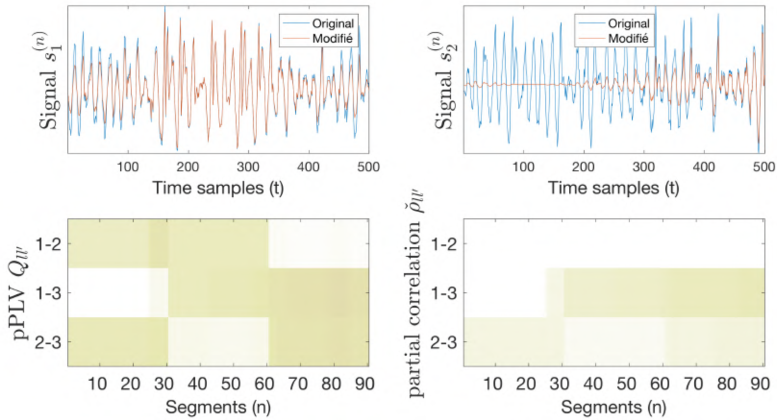
Effect of the amplitude

A multiplication with different constant amplitudes modifies the signals



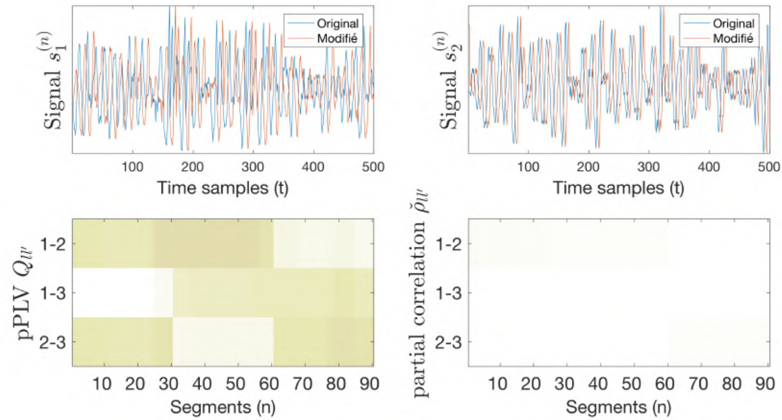
Effect of the amplitude modulation

Amplitude modulations are performed to modify the signals

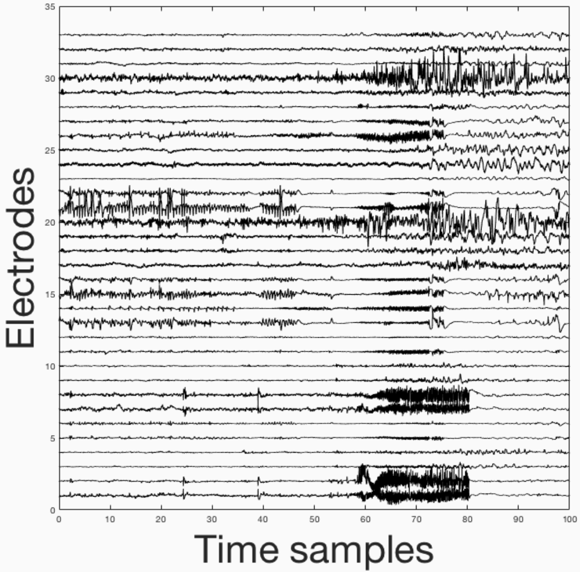


Effect of the lag

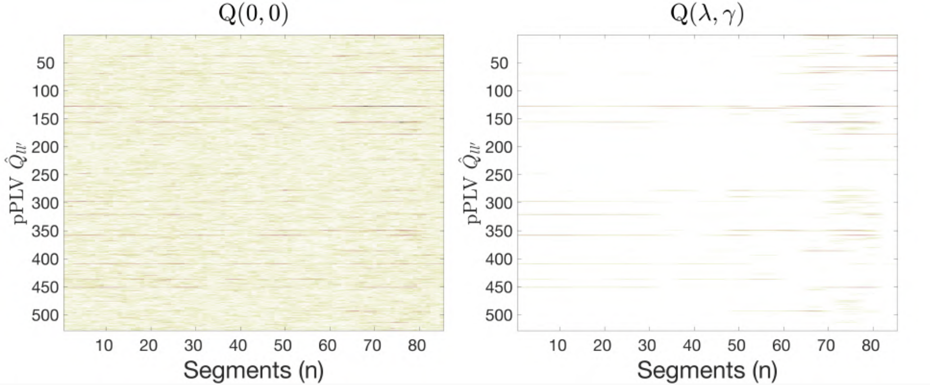
Different lags are added to modify the signals



Real functional connectivity matrix



Real functional connectivity matrix



Conclusion

During this this talk we considered :

- Inference of conditionally independent dynamical graph from pPLV
- Selection of relevant regularizations to process iEEG datasets
- Application on a model and a real iEEG multivariate signal.

Follow-up of this work :

- Investegating parametric regularized pPLV assuming the signals follow a multivariate Gaussian distribution
- Automatic selection of parameters

Thank you for your attention !