# Spatially Guided Independent Vector Analysis

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## Introduction

- Independent Vector Analysis (IVA) faces **outer permutation ambiguity**
- Fast-converging optimization algorithm for IVA based on Majorize-Minimize (MM) algorithm is available

**Contribution:**

- Bayesian framework for incorporation of prior knowledge on demixing vectors that guides the optimization to a desired solution
- Fast-converging MM-based optimization algorithm

## Algorithm

### Bayesian Model

- Frame index $n \in \{1,\dots,N\}$, frequency bin index $f \in \{1,\dots,F\}$ and channel index $k \in \{1,\dots,K\}$
- **Separation model** (recorded signals $\mathbf{x}_{f,n}$, separated signals $\mathbf{y}_{f,n}$)

$$\mathbf{y}_{f,n} = [y_{1,f,n},\dots,y_{K,f,n}]^{\mathsf{T}} = \mathbf{W}_f\mathbf{x}_{f,n}, \quad \text{with } \mathbf{W}_f = [\mathbf{w}_{1,f},\dots,\mathbf{w}_{K,f}]^{\mathsf{H}} \in \mathbb{C}^{K\times K},$$

and $\mathbf{x}_{f,n} = [x_{1,f,n},\dots,x_{K,f,n}]^{\mathsf{T}} \in \mathbb{C}^K$, $\underline{\mathbf{y}}_{k,n} = [y_{k,1,n},\dots,y_{k,F,n}]^{\mathsf{T}} \in \mathbb{C}^F$

- **Posterior density**

$$p(\mathcal{W}|\mathcal{X}) \propto \int p(\mathcal{W})p(\mathcal{Y}|\mathcal{W})p(\mathcal{X}|\mathcal{W},\mathcal{Y})\mathrm{d}\underline{\mathbf{y}}_{1,1}\dots\mathrm{d}\underline{\mathbf{y}}_{K,N},$$

where $\mathcal{W}$, $\mathcal{Y}$, $\mathcal{X}$ contain all $\mathbf{W}_f$, $\mathbf{y}_{f,n}$ and $\mathbf{x}_{f,n}$, respectively

- Noiseless **likelihood** (i.i.d. over frames & frequencies)

$$p(\mathcal{X}|\mathcal{W},\mathcal{Y}) = \prod_{n=1}^{N}\prod_{f=1}^{F}\delta\left(\mathbf{x}_{f,n} - \mathbf{W}_f^{-1}\mathbf{y}_{f,n}\right)$$

- Broadband **source model** (i.i.d. over frames and channels)

$$p(\mathcal{Y}|\mathcal{W}) = \prod_{n=1}^{N}\prod_{k=1}^{K} p\left(\underline{\mathbf{y}}_{k,n}\right)$$

- **Incorporation of prior knowledge:** here free-field steering vector $\mathbf{h}_f$

$$p(\mathcal{W}) = \prod_{f=1}^{F} p(\mathbf{W}_f) = \prod_{f=1}^{F}\prod_{k=1}^{K}\frac{\exp\left(-\frac{N}{\sigma_f^2}\mathbf{w}_{k,f}^{\mathsf{H}}\left(\lambda_{\mathrm{E}}\mathbf{I} + \mathbf{h}_f\mathbf{h}_f^{\mathsf{H}}\right)\mathbf{w}_{k,f}\right)}{\sqrt{\left(\pi\frac{\sigma_f^2}{N}\right)^K \det\left(\lambda_{\mathrm{E}}\mathbf{I} + \mathbf{h}_f\mathbf{h}_f^{\mathsf{H}}\right)}}$$

- **Cost function** derived from MAP estimation problem

$$\mathbf{W}_f = \underset{\mathbf{W}_f\in\mathbb{C}^{K\times K}}{\arg\min} -\frac{\log p(\mathcal{W})}{N} - 2\sum_{f=1}^{F}\log|\det\mathbf{W}_f| + \sum_{k=1}^{K}\hat{\mathbb{E}}\left\{G\left(\underline{\mathbf{y}}_{k,n}\right)\right\},$$

with $G(\underline{\mathbf{y}}_{k,n}) = -\log p(\underline{\mathbf{y}}_{k,n})$ and $\hat{\mathbb{E}}\{\cdot\} = \frac{1}{N}\sum_{n=1}^{N}(\cdot)$

### Update Rules

- Minimize **cost function** $J(\mathcal{W})$
- **MM principle:** Design easy-to-optimize upper bound $Q$ fulfilling majorization and tangency to the cost function $J$

$$J(\mathcal{W}) \leq Q\left(\mathcal{W}|\mathcal{W}^{(l)}\right), \quad J\left(\mathcal{W}^{(l)}\right) = Q\left(\mathcal{W}^{(l)}|\mathcal{W}^{(l)}\right)$$

- Inequality for supergaussian PDFs [1]

$$\hat{\mathbb{E}}\left\{G\left(\underline{\mathbf{y}}_{k,n}\right)\right\} \leq \frac{1}{2}\sum_{f=1}^{F}\left(\mathbf{w}_{k,f}^{\mathsf{H}}\mathbf{V}_{k,f}^{(l)}\mathbf{w}_{k,f}\right) + \text{const.},$$

with weighted microphone signal covariance matrix

$$\mathbf{V}_{k,f}^{(l)} = \hat{\mathbb{E}}\left\{\frac{G'(r_{k,n}^{(l)})}{r_{k,n}^{(l)}}\mathbf{x}_{f,n}\mathbf{x}_{f,n}^{\mathsf{H}}\right\}, \quad \text{where } r_{k,n}^{(l)} = \left\|\underline{\mathbf{y}}_{k,n}^{(l)}\right\|_2$$

- **Upper bound** of $J(\mathcal{W})$

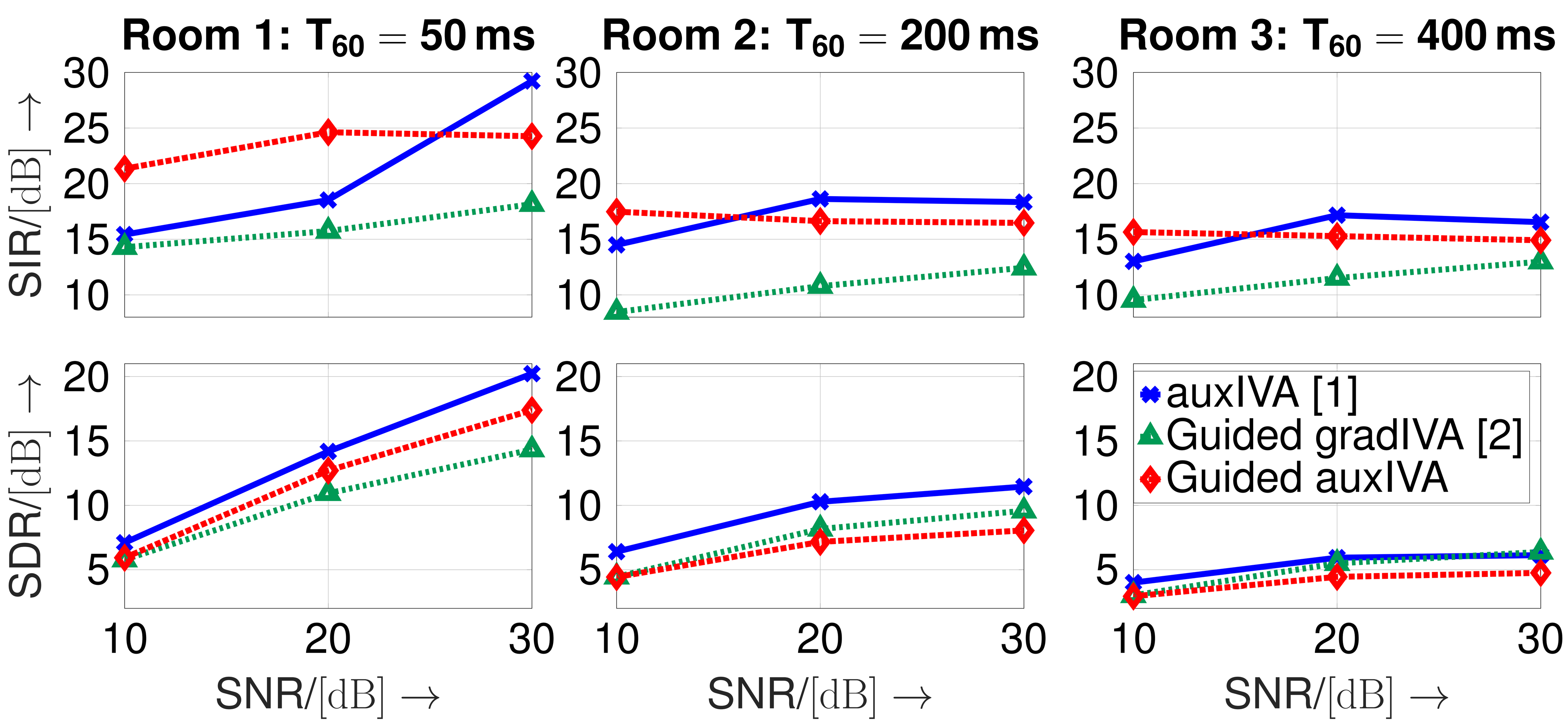
$$Q\left(\mathcal{W}|\mathcal{W}^{(l)}\right) = \sum_{f=1}^{F}\Bigg[-2\log|\det\mathbf{W}_f|\dots + \sum_{k=1}^{K}\left(\mathbf{w}_{k,f}^{\mathsf{H}}\left(\mathbf{V}_{k,f}^{(l)} + \frac{\lambda_{\mathrm{E}}\mathbf{I} + \mathbf{h}_f\mathbf{h}_f^{\mathsf{H}}}{\sigma_f^2}\right)\mathbf{w}_{k,f}\right)\Bigg] + \text{const.}$$

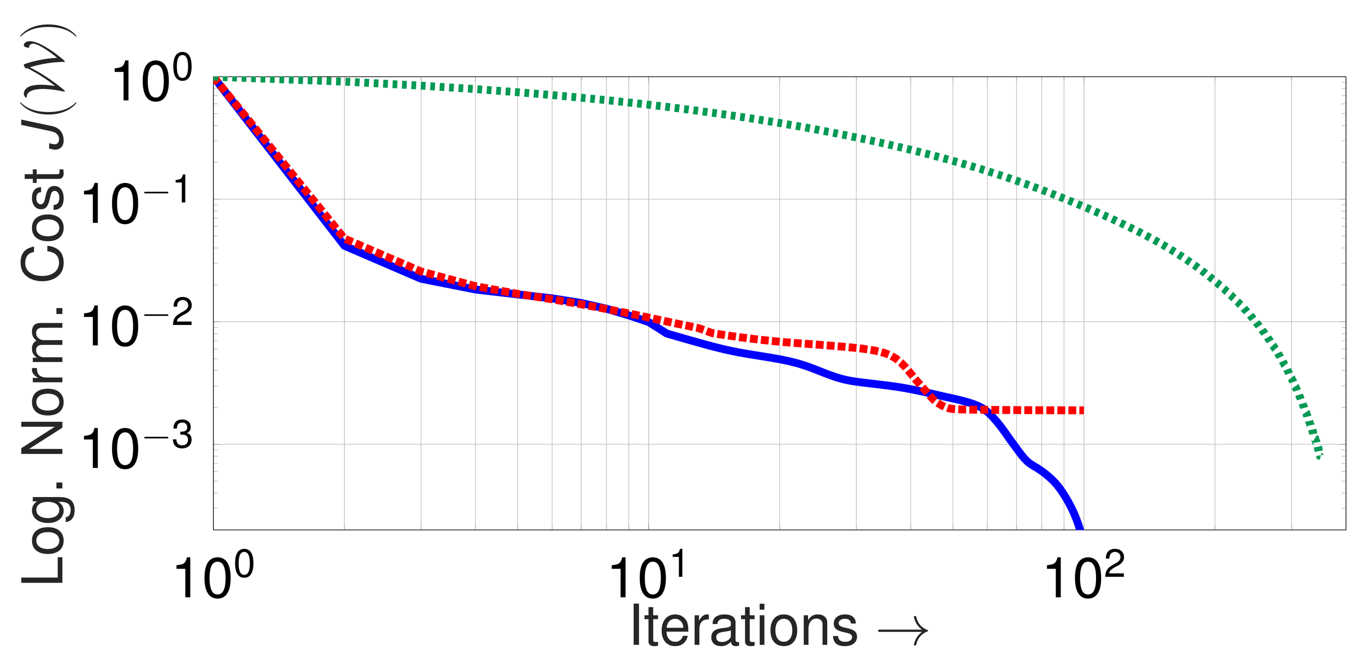
- Optimization of $Q$ by **update rules** based on iterative projection

$$\tilde{\mathbf{w}}_{k,f}^{(l+1)} = \left(\mathbf{W}_f^{(l)}\left[\mathbf{V}_{k,f}^{(l)} + \frac{\lambda_{\mathrm{E}}\mathbf{I} + \mathbf{h}_f\mathbf{h}_f^{\mathsf{H}}}{\sigma_f^2}\right]\right)^{-1}\mathbf{e}_k,$$

$$\mathbf{w}_{k,f}^{(l+1)} = \frac{\tilde{\mathbf{w}}_{k,f}^{(l+1)}}{\sqrt{\left(\tilde{\mathbf{w}}_{k,f}^{(l+1)}\right)^{\mathsf{H}}\left[\mathbf{V}_{k,f}^{(l)} + \frac{\lambda_{\mathrm{E}}\mathbf{I}+\mathbf{h}_f\mathbf{h}_f^{\mathsf{H}}}{\sigma_f^2}\right]\tilde{\mathbf{w}}_{k,f}^{(l+1)}}}$$

## Experiments




- RIRs measured at three different configurations
- Laplacian source model, determined $2\times 2$ scenario
- Guided auxIVA: $\lambda_{\mathrm{E}} = 10^{-3}$, $\sigma_f^2 = \sigma^2 = 40$
- Guided gradIVA [2]: step size $= 0.05$, weighting $= 0.5$

## Conclusion

- Demixing filters are equipped with **prior knowledge**
- **Optimization scheme as fast as auxIVA**
- Adaptation **guided to desired solution** ⇒ outer permutation ambiguity solved

## References

1 N. Ono, *Stable and Fast Update Rules for Independent Vector Analysis Based on Auxiliary Function Technique* WASPAA, 2011.
2 A. Khan et al. *A Geometrically Constrained Independent Vector Analysis Algorithm for Online Source Extraction* LVA/ICA, 2015.
3 A. Brendel et al. *A unified Bayesian view on spatially informed source separation and extraction based on independent vector analysis*, https://arxiv.org/abs/2001.05958



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