PACO and PACO-DCT

PAtch Consensus and its Application to Inpainting

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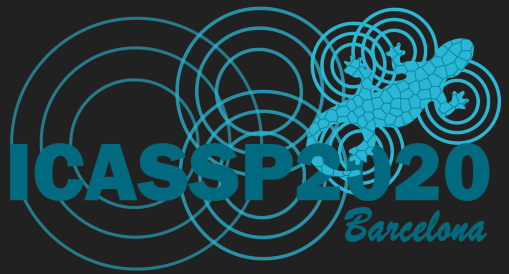
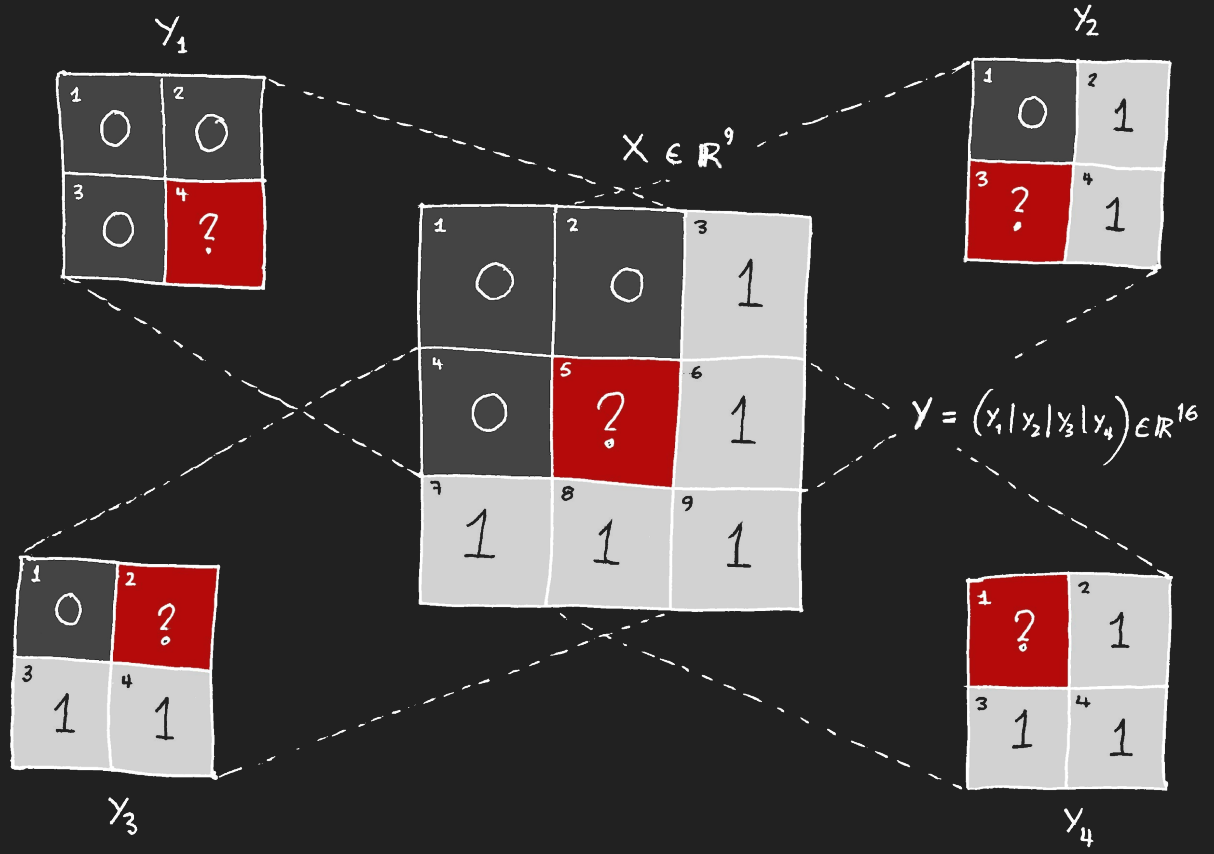


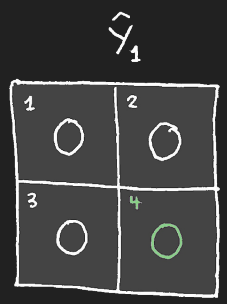
Summary

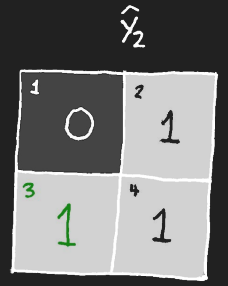
- Patch-based methods and their potential shortcomings
- Patch consensus: what is it and what is it good for?
- The PACO problem
- General solution
- The PACO-DCT Inpainting problem
- The PACO-DCT Inpainting algorithm
- Results
- Concluding remarks / future work

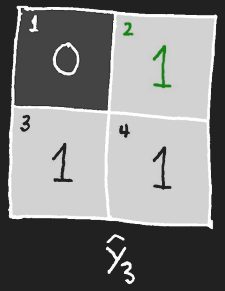
Patch-based restoration: patch extraction

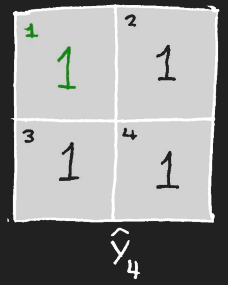


Patch-based restoration: patch estimation (median)

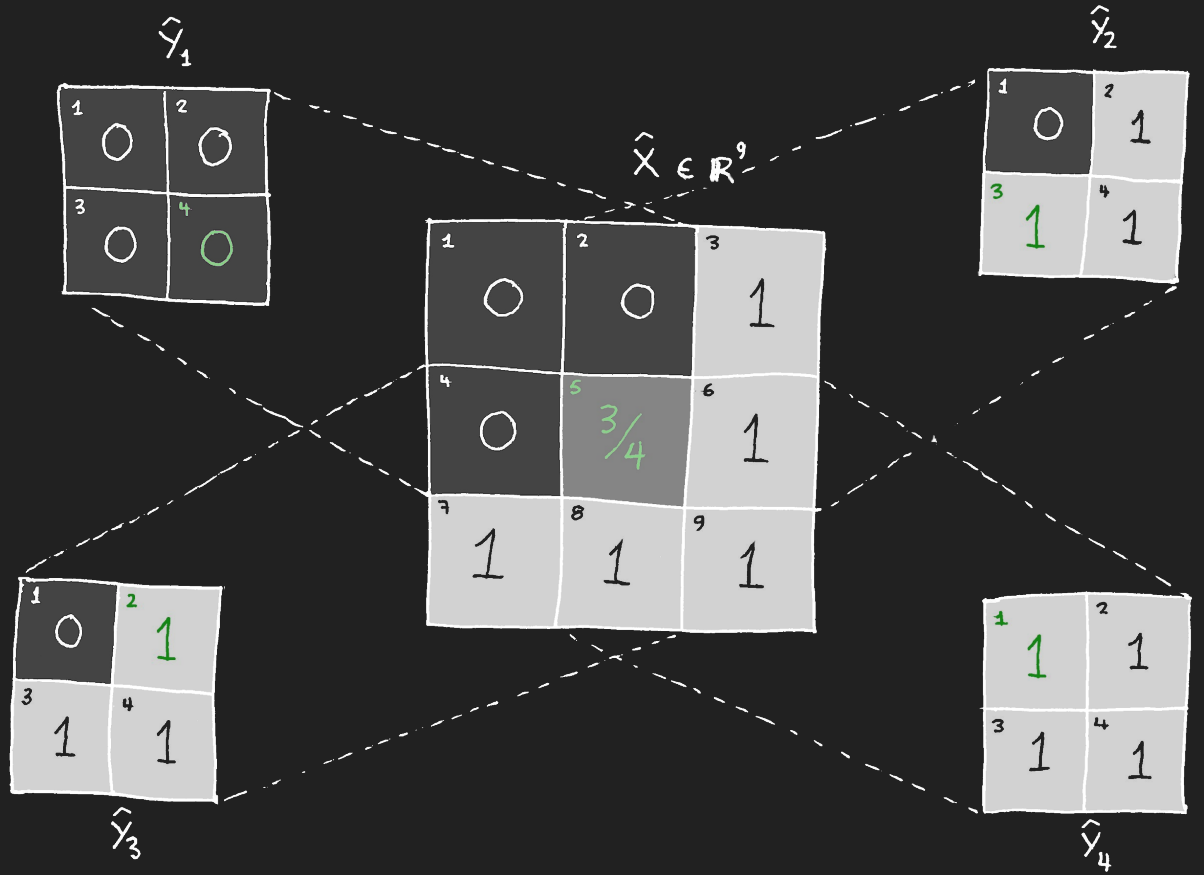




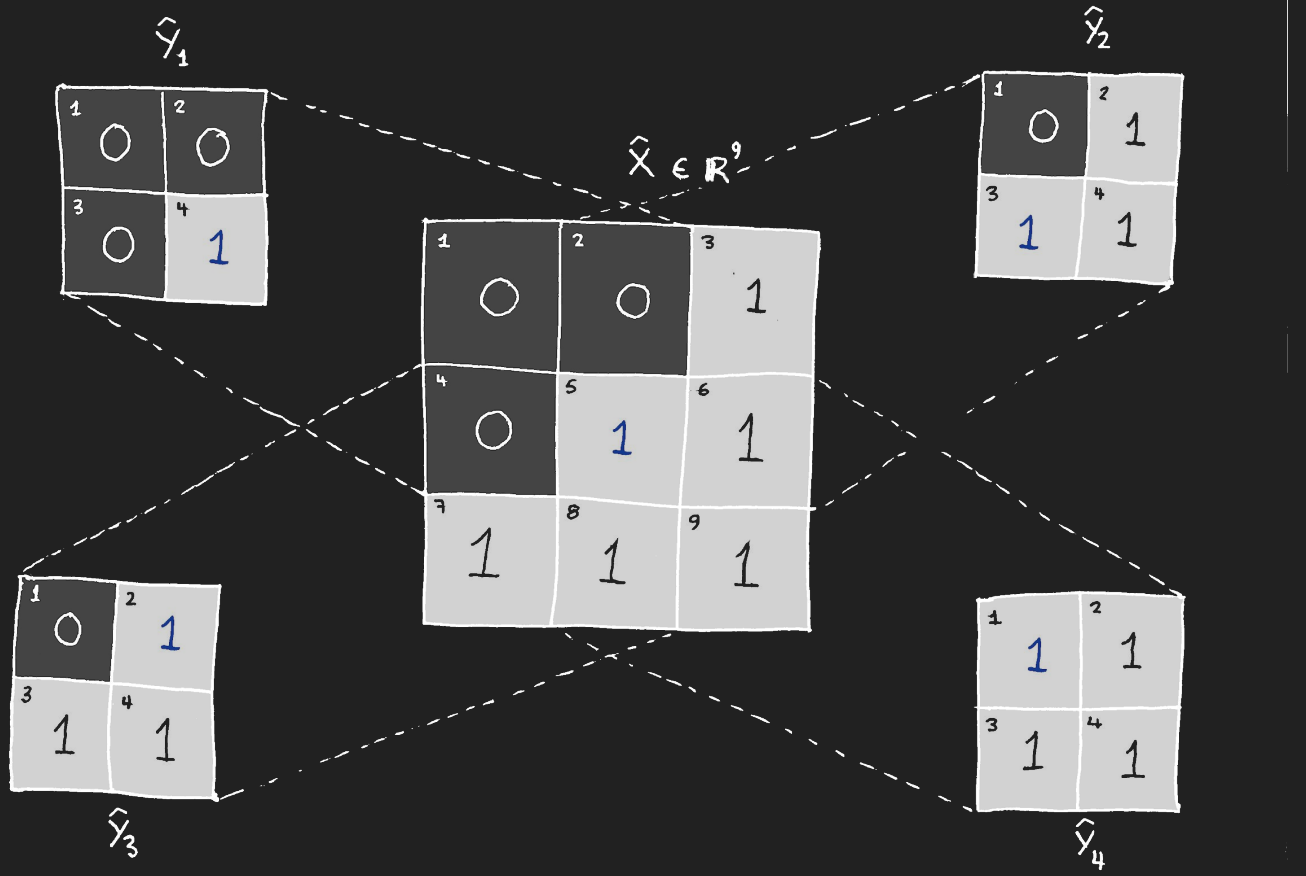




Patch-based restoration: stitching (average)



Patch Consensus



General PACO problem:

$$\hat{\mathbf{Y}} = \arg\min_{\mathbf{Y}\in\Omega} f(\mathbf{Y}) \text{ s.t. } \mathbf{Y} \in \mathbb{C}$$

Equivalent problem using indicator function g(.):

$$\hat{\mathbf{Y}} = \arg\min_{\mathbf{Y}} f(\mathbf{Y}) + g(\mathbf{Y}),$$

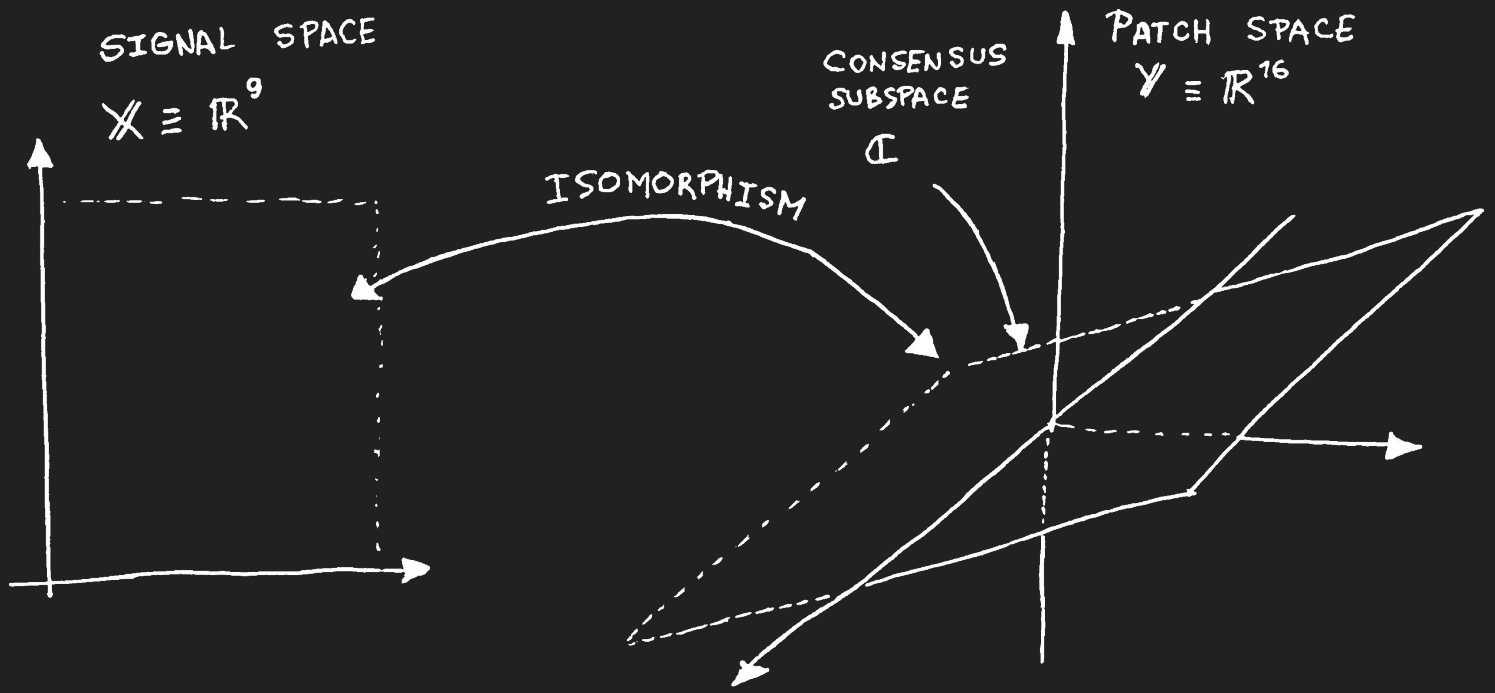
$$g(\mathbf{Y}) = 0 \text{ if } \mathbf{Y} \in \mathbb{C} \cap \Omega \text{ and } +\infty \text{ otherwise.}$$

PACO: Splitting and solution using ADMM

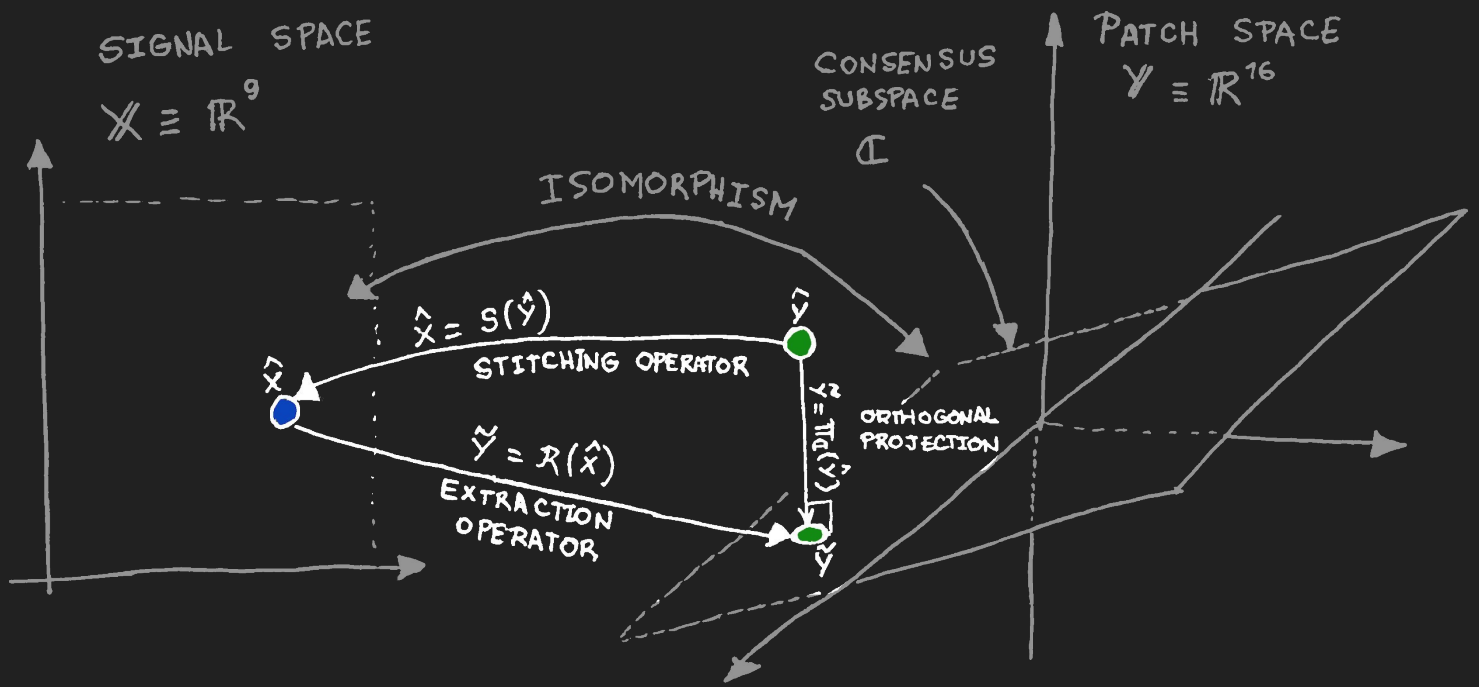
$$(\hat{\mathbf{Y}}, \hat{\mathbf{Z}}) = \arg\min_{\mathbf{Y},\mathbf{Z}} f(\mathbf{Y}) + g(\mathbf{Z}) + \frac{1}{2\lambda}\|\mathbf{Y}-\mathbf{Z}\|_F^2 \text{ s.t. } \mathbf{Y}=\mathbf{Z}.$$

$$\begin{aligned}
\mathbf{Y}^{(t+1)} &\leftarrow \operatorname{prox}_{\lambda f}\left(\mathbf{Z}^{(t)} - \mathbf{U}^{(t)}\right), \\
\mathbf{Z}^{(t+1)} &\leftarrow \Pi_{C\cap\Omega}(\mathbf{Y}^{(t+1)} + \mathbf{U}^{(t)}), \\
\mathbf{U}^{(t+1)} &\leftarrow \mathbf{U}^{(t)} + \mathbf{Y}^{(t+1)} - \mathbf{Z}^{(t+1)}
\end{aligned}$$

Signal Space, Patch Space, Consensus Subspace



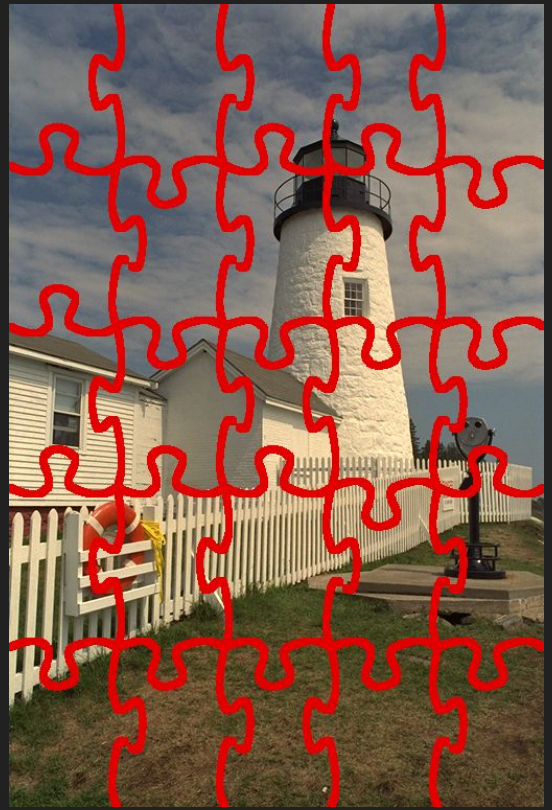
Efficient projection onto C: the stitching trick



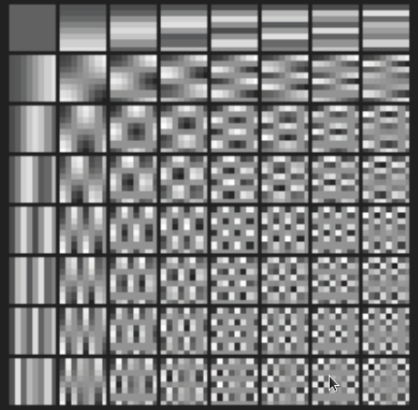
The Inpainting Problem

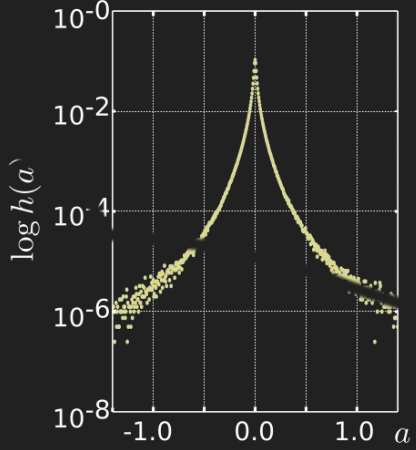
O: index set of observed samples

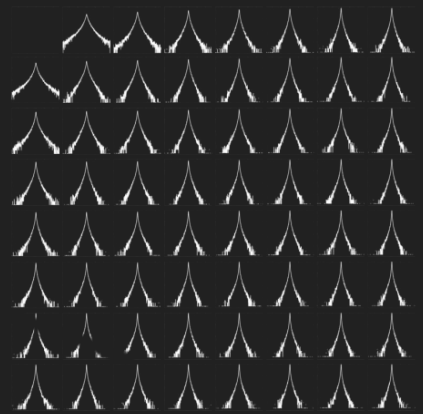
O^c : unknown samples (red pixels)

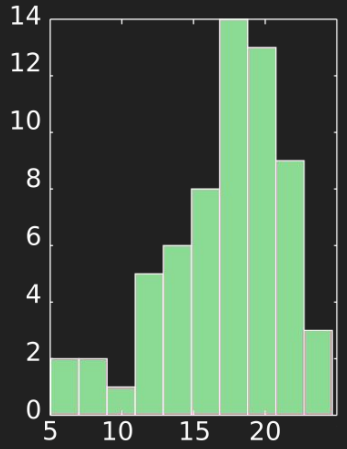


PACO-DCT Inpainting: Weighted L1 DCT prior







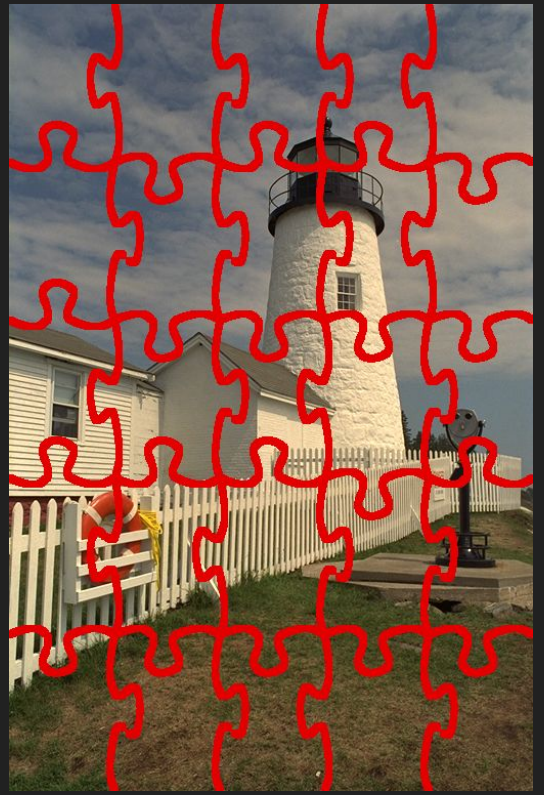


$$f(\mathbf{D}^{-1}\mathbf{Y}) = f(\mathbf{A}) = \sum_{i,j} w_i |a_{i,j}|$$

PACO-DCT Inpainting: feasible set

$$\Omega = \{\mathbf{z} : z_i = x_i, i \in O\}$$

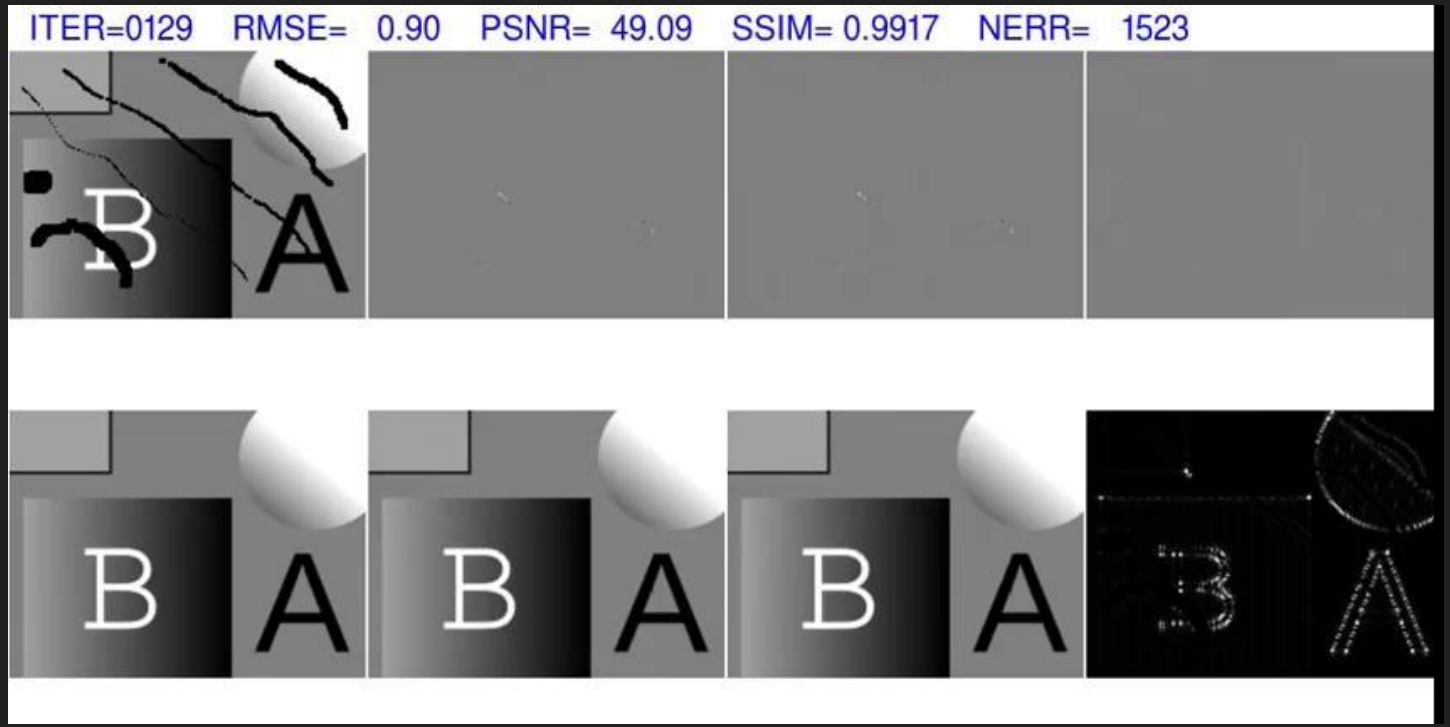
- Feasible solutions Z are those whose pixels coincide with the known pixels of the input X.
- Ω a linear subspace of X
- As C is a linear isomorphism of X, the mapping of Ω is also a linear proper subspace of C!
- Projection onto Ω is trivial



PACO-DCT Inpainting: complete ADMM algorithm

$$
\begin{aligned}
a_{i,j}^{(t+1)} &\leftarrow \mathcal{S}_{\lambda w_{i,j}}(z_{i,j}^{(t)} - u_{i,j}^{(t)}),\ \forall\, i,j \\
\hat{\mathbf{Y}}^{(t+1)} &\leftarrow \mathbf{D}(\,\mathbf{A}^{(t+1)} + \mathbf{U}^{(t)}\,) \\
\hat{\mathbf{x}}^{(t+1)} &\leftarrow \mathbf{R}^{\intercal}\mathrm{vec}(\hat{\mathbf{Y}}^{(t+1)}) \\
\hat{x}_i^{(t+1)} &\leftarrow x_i^{(t+1)},\ \forall\, i \in O \\
\mathbf{Z}^{(t+1)} &\leftarrow \mathbf{D}^{\intercal}\mathrm{mat}(\mathbf{R}\hat{\mathbf{x}}^{(t+1)}) \\
\mathbf{U}^{(t+1)} &\leftarrow \mathbf{U}^{(t)} + \mathbf{A}^{(t+1)} - \mathbf{Z}^{(t+1)}.
\end{aligned}
$$

PACO-DCT Inpainting: Images (test)



PACO-DT Inpainting: Images (Kodak Dataset)

PACO-DCT Inpainting: Color video

PACO-DCT Inpainting: Audio

Original:

Erased:

Restored:

Concluding Remarks / Future Work

- PACO: framework for solving patch-based restoration problems with consensus constraints
- Simple ADMM-based algorithm
- Efficient projection onto Consensus set: stitching trick
- Encouraging results on Audio, Video and Image inpainting

Future work

- Apply PACO to other problems
- Upcoming: better priors (e.g., GMM), deblocking, block compressive sensing
- Many more possibilities to explore later

Thank you