## Deep Geometric Knowledge Distillation with Graphs

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## Outline

(1) Define and motivate knowledge distillation;
(2) Introduce the concept of Graph Knowledge Distillation (GKD);
(3) Present empirical evaluation and analysis.

## Motivation



## Motivation



1T FLOPs for one decision


100M parameters to learn

1024 V100 during 1 day for training


4 TPUs during 1 month for training

## Knowledge distillation

## Goal

Neural network compression:

- Teacher transfers knowledge to student;
- Student has less parameters than teacher;
- Student decisions consistent with teacher leads to
- Student's accuracy $\approx$ teacher's accuracy;
- Student mimicks the teacher's output;
- Form of pseudo-labeling;
- Uses teacher understanding of classes;


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## Distilling the Knowledge in a Neural Network, Hinton et al., 2014

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## Distillation

- Modern neural networks tend to be very deep;
- Distilling only the output does not guarantee influencing all layers;
- Solution:
- Enforce [student latent space = teacher latent space];
- Drawback: intermediate representation dimensions may not match.


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## Fitnets, Romero et al., 2015

- Solution: add linear transformations so that dimensions match;
- Drawback: the linear transformations are removed after training, jointly with part of the distilled knowledge.


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- Solution:
- Enforce [student latent space = teacher latent space];
- Drawback: intermediate representation dimensions may not match.


## LIT, Koratana et al., 2019

- Solution: perform the distillation block-wise and ensure that the outputs of each block have the same size;
- Drawback: limits the architecture choice.


## Distillation <br> IKD vs RKD

## Individual Knowledge Distillation (IKD)

- Methods we presented perform IKD;
- Consider each example separately;
- Either need transformations or same size representations.

```
Relational Knowledge Distillation (RKD)
- Formalized in Park et al., 2019.
- Goal: Transfer higher order knowledge to the student, e.g.:
- Distance between pairs of examples;
- Angles between triplets of examples.
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## Distillation

IKD vs RKD

## Training NN with KD

$$
\begin{equation*}
\mathcal{L}=\mathcal{L}_{\text {task }}+\lambda_{\mathrm{KD}} \cdot \mathcal{L}_{\mathrm{KD}} \tag{1}
\end{equation*}
$$

## Individual Knowledge Distillation (IKD)

$$
\mathcal{L}_{\text {IND }}=\sum_{\ell \in \Lambda} \sum_{x \in X} \mathcal{L}_{d}\left(x_{S_{\ell}}, x_{T_{\ell}}\right)
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## Relational Knowledge Distillation (RKD) - distance between

 pairs of examples

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Relational Knowledge Distillation (RKD) - distance between pairs of examples

$$
\begin{equation*}
\mathcal{L}_{\mathrm{RKD}-\mathrm{D}}=\sum_{\ell \in \Lambda} \sum_{\left(\mathbf{x}, \mathbf{x}^{\prime}\right) \in X^{2}} \mathcal{L}_{d}\left(\frac{\left\|\mathbf{x}_{S_{\ell}}-\mathbf{x}^{\prime}{S_{\ell}}\right\|_{2}}{\Delta_{S_{\ell}}}, \frac{\left\|\mathbf{x}_{T_{\ell}}-\mathbf{x}^{\prime}{T_{\ell}}\right\|_{2}}{\Delta_{T_{\ell}}}\right) \tag{3}
\end{equation*}
$$

## Graph Knowledge Distillation

We propose to use graphs to distillate knowledge:

- Use graphs to represent latent spaces;
- Student should mimick the teacher's graphs;
- Introducing a graph formalism opens research directions:
- Graph Signal Processing (GSP) analysis of the results;
- Better normalization $\rightarrow$ easier to compare;
- More meaningful relational distances;
- Graph variations:
(1) Task specific graphs (inter/intra-class graphs);
(2) Localized graphs (k-neighbors graphs);
(3) Smoothed graphs (adjacency matrix to power p).
- Form of RKD.
- Concurrently proposed by Liu et al., 2019; Lee et al., 2019; and this work.


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## Graph representation of latent spaces

## intermediate representations



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\end{equation*}
$$

Graph Knowledge Distillation (GKD)

$$
\begin{gather*}
\mathcal{L}_{G K D}=\sum_{\ell \in \Lambda} \mathcal{L}_{d}\left(\mathcal{G}_{S_{\ell}}(X), \mathcal{G}_{T_{\ell}}(X)\right) .  \tag{5}\\
\mathcal{L}_{G K D}=\sum_{\ell \in \Lambda}\left\|\mathbf{D}_{S_{\ell}}^{-\frac{1}{2}} \mathbf{A}_{S_{\ell}} \mathbf{D}_{S_{\ell}}^{-\frac{1}{2}}-\mathbf{D}_{T_{\ell}}^{-\frac{1}{2}} \mathbf{A}_{T_{\ell}} \mathbf{D}_{T_{\ell}}^{-\frac{1}{2}}\right\|_{2}^{2} . \tag{6}
\end{gather*}
$$

## Empirical experiments and analysis

(1) Error rate comparison against RKD-D in CIFAR-10;
(2) Classification consistency;
(3) Graph signal smoothness analysis;
(0) Effect of using task specific graphs.

## Neural net architectures

Teacher - WideResnet-28-1

$$
\text { Input - }[3,32,32]
$$

Embedding - [16, 32, 32]
Block 1-[16, 32, 32]
Block 2-[32, 16, 16]
Block 3 - [64]
Block 4 / Classification - [10]

## Student - WideResnet-28-0.5

$\approx 4$ times smaller (parameters and FLOPS) than the teacher
Input - [3, 32, 32]

Embedding - [16, 32, 32]


Block 1 - [8, 32, 32]

Block 2 - $[16,16,16]$
Block 3 - [32]
$\downarrow$
Block 4 / Classification - [10]

## Empirical experiments and analysis

## CIFAR-10 error rate

Table: Median error rate and standard deviation on the CIFAR-10 dataset.

| Method | CIFAR-10 | Relative size |
| :---: | :---: | :---: |
| Teacher | $7.27 \%( \pm 0.26)$ | $100 \%$ |
| Student without KD (Baseline) | $10.34 \%( \pm 0.27)$ | $27 \%$ |

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| Teacher | $7.27 \%( \pm 0.26)$ | $100 \%$ |
| Student without KD (baseline) | $10.34 \%( \pm 0.27)$ | $27 \%$ |
| RKD-D | $10.05 \%( \pm 0.28)$ | $27 \%$ |
| GKD | $9.71 \%( \pm 0.27)$ | $27 \%$ |
| GKD (inter-class graph) | $9.31 \%( \pm \mathbf{0 . 2 5 )}$ | $27 \%$ |

## Empirical experiments and analysis

## Classification consistency with teacher



Figure: Analysis of the consistency of classification compared to the teacher, across blocks of RKD-D and GKD students.

## Empirical experiments and analysis

Graph signal smoothness analysis

Teacher's Fiedler vector
Label binary indicator signal



Figure: Analysis of the smoothness evolution across layers of the RKD and GKD students

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Figure: Analysis of the smoothness evolution across layers of the RKD and GKD students

## Empirical experiments and analysis

Task specific graphs


Figure: Analysis of the effect of task specific graphs. A graph of distinct classes has edges only between nodes of different classes, while same class graphs has edges only between nodes of the same class.

## Conclusion

## Wrap up

- Graphs can be used as a proxy to the geometry of latent representations in deep neural networks;
- Using graphs for knowledge distillation allows us to improve the performance of compressed student networks;
- We were able to analyze the intermediate representations of our student networks.
- Small gains, could be combined with other approaches;
- More relevant graph distances, such as spectral distance;
- Train the network block-wise instead of end-to-end.


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- Graphs can be used as a proxy to the geometry of latent representations in deep neural networks;
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## Future work

- Small gains, could be combined with other approaches;
- More relevant graph distances, such as spectral distance;
- Train the network block-wise instead of end-to-end.


## Thank you for watching this presentation.

I will be happy to answer any questions you have via e-mail:
carlos.rosarkoslassance@imt-atlantique.fr.
Code available at github.com/cadurosar/graph_kd

## References

- Hinton et al., 2014, "Distilling the Knowledge in a Neural Network.", NIPS Workshop;
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