



Stony Brook University

# Indoor Altitude Estimation of Unmanned Aerial Vehicles Using a Bank of Kalman Filters

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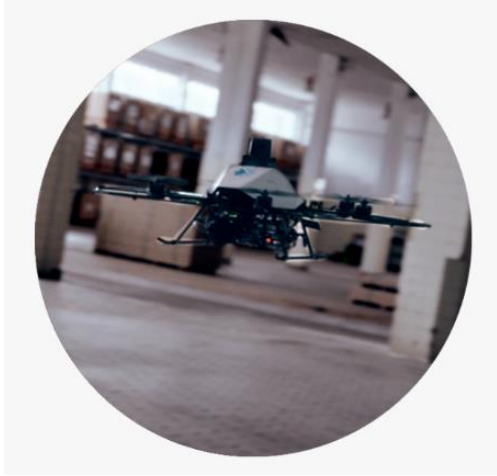
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**FAR  
BEYOND**

# Overview

- ❖ Motivation
- ❖ Problem Description
- ❖ State-space Formulation
- ❖ Multiple Model Adaptive Estimation
- ❖ Experiments
- ❖ Conclusion

# Motivation



Unmanned Aerial Vehicles (UAVs)

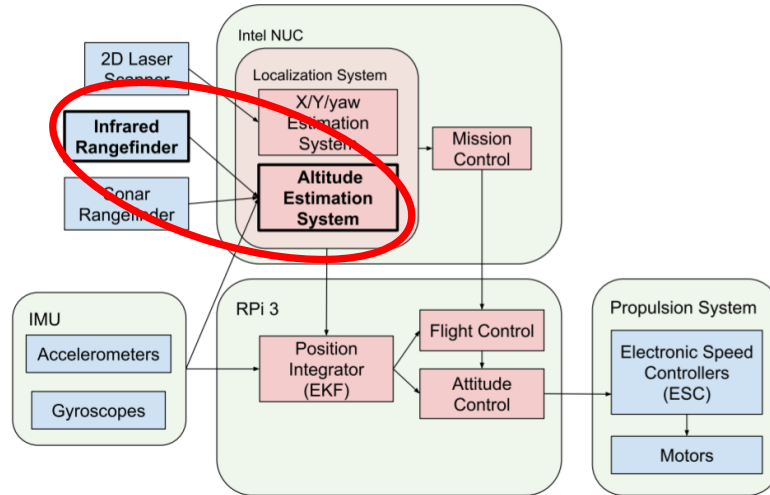
No GPS  
signals

Tradeoff  
between  
accuracy,  
weight and  
cost

Irregular  
obstructions

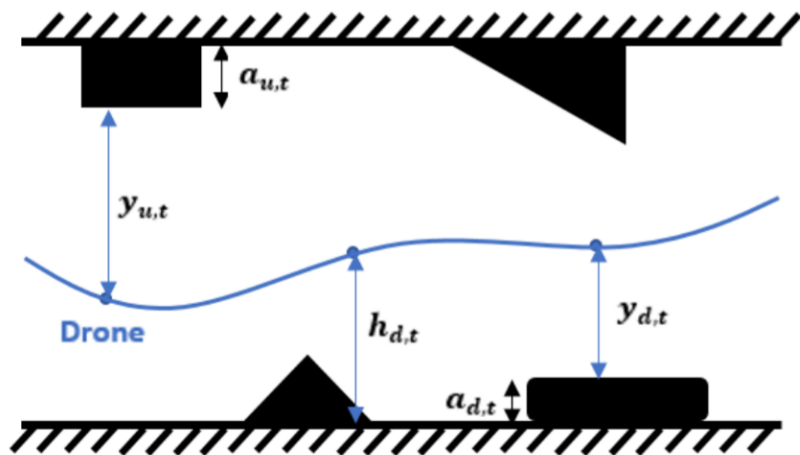
# Problem Description

The UAV architecture



# Problem Description

The altitude estimation scenario



Assumption:

1. Only access the data from two IR sensors
2. Two IR sensors are aligned in the same position.
3. The angle of tilt can be ignored.
4. The roof is level.

Goal:

Estimate the true altitude of the UAV  $h_{d,t}$

by measuring the biased ranges from the upward and downward IR sensors

# State-space Formulation

- The range measurements from two IR sensors:

$$y_{u,t} = R - h_{d,t} - a_{u,t} + \epsilon_{u,t},$$

$$y_{d,t} = h_{d,t} - a_{d,t} + \epsilon_{d,t},$$

- Independent measurement errors:

$$\epsilon_{u,t}, \epsilon_{d,t} \sim \mathcal{N}(0, \sigma_y^2)$$

- The observation equation in matrix form:

$$\mathbf{y}_t = \mathbf{H}\mathbf{x}_t + \boldsymbol{\epsilon}_t,$$

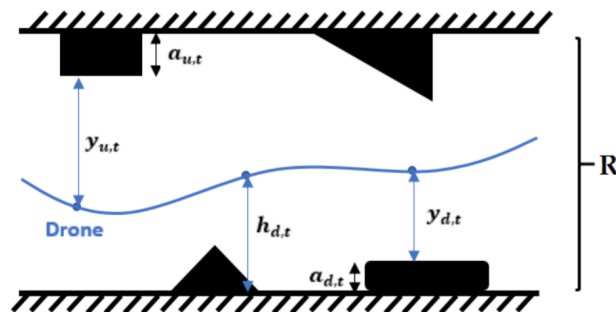
Observation vector:  $\mathbf{y}_t = [y_{u,t} - R, y_{d,t}]^\top$

State vector:  $\mathbf{x}_t = [h_{d,t}, v_t, a_{u,t}, a_{d,t}]^\top$

Error vector:  $\boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \sigma_y^2 \mathbf{I}_2)$

The matrix  $\mathbf{H}$  is

$$\mathbf{H} = \begin{bmatrix} -1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$



# State-space Formulation

- State transition equations:

$$h_{d,t} = h_{d,t-1} + T_s v_{t-1} + 0.5T_s^2 u_{v,t-1},$$

$$v_t = v_{t-1} + T_s u_{v,t-1},$$

$$a_{u,t} \sim \mathcal{N}(0, \sigma_a^2),$$

$$a_{d,t} \sim \mathcal{N}(0, \sigma_a^2),$$

Vertical velocity:  $v_t$

Sampling interval:  $T_s$

Vertical acceleration:  $u_{v,t-1} \sim \mathcal{N}(0, \sigma_v^2)$

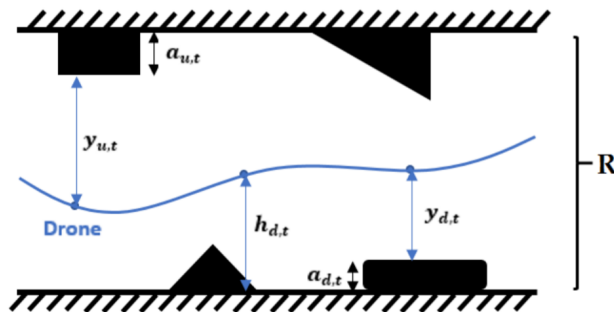
- The transition equation in matrix form:

$$\mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-1} + \mathbf{u}_{t-1}$$

The state error:  $\mathbf{u}_{t-1} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$

The covariance matrix is  $\mathbf{Q} = \begin{bmatrix} 0.25T_s^4\sigma_v^2 & 0.5T_s^4\sigma_v^2 & 0 & 0 \\ 0.5T_s^4\sigma_v^2 & T_s^4\sigma_v^2 & 0 & 0 \\ 0 & 0 & \sigma_a^2 & 0 \\ 0 & 0 & 0 & \sigma_a^2 \end{bmatrix}$

The matrix  $\mathbf{F}$  is  $\begin{bmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ .



# Multiple Model Adaptive Estimation

- Candidate models:

No obstacles  $\mathbf{H}_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Obstacle above  $\mathbf{H}_2 = \begin{bmatrix} -1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Obstacles below  $\mathbf{H}_3 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$

Obstacles above and below  $\mathbf{H}_4 = \begin{bmatrix} -1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$

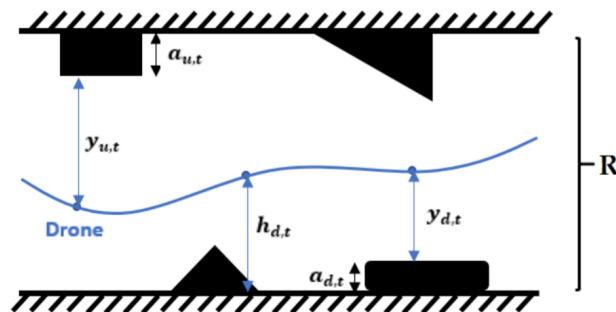
- We use the same state variable  $\mathbf{x}_t$  and transition equation in all candidate models.

$$y_{u,t} = R - h_{d,t} - a_{u,t} + \epsilon_{u,t},$$

$$y_{d,t} = h_{d,t} - a_{d,t} + \epsilon_{d,t},$$

$$\mathbf{y}_t = [y_{u,t} - R, y_{d,t}]^\top$$

$$\mathbf{x}_t = [h_{d,t}, v_t, a_{u,t}, a_{d,t}]^\top$$





# Multiple Model Adaptive Estimation

- MMAE for UAV altitude estimation:

Initialize  
 $\mathbf{x}_0, \mathbf{P}_0$

Marginal likelihood

$$\begin{aligned} p(\mathbf{y}_{1:t}) &= p(\mathbf{y}_{1:t-1})p(\mathbf{y}_t|\mathbf{y}_{1:t-1}) \\ &= p(\mathbf{y}_{1:t-1}) \int p(\mathbf{y}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{y}_{1:t-1})d\mathbf{x}_t \\ &= p(\mathbf{y}_{1:t-1})\mathcal{N}(\mathbf{y}_t|\mathbf{H}\mathbf{x}_{t|t-1}, \mathbf{S}) \end{aligned}$$

a. Predict the prior estimate  $\mathbf{x}_{t|t-1}^{(m)}$  and its covariance  $\mathbf{P}_{t|t-1}^{(m)}$

b. Calculate the innovation  $\mathbf{z}^{(m)}$  and its covariance  $\mathbf{S}^{(m)}$

c. Compute the optimal Kalman gain  $\mathbf{K}^{(m)}$

d. Update the posterior estimate  $\mathbf{x}_{t|t}^{(m)}$  and covariance  $\mathbf{P}_{t|t}^{(m)}$

**Kalman Filter**

Forgetting factor [0,1]

e. Update the likelihood of model

$$l_{\log}^{(m)} = \alpha \times l_{\log}^{(m)} - \frac{1}{2} \left[ \mathbf{z}^{(m)T} \{\mathbf{S}^{(m)}\}^{-1} \mathbf{z}^{(m)} + \log(|\mathbf{S}^{(m)}|) + 2\log(2\pi) \right]$$

Convert the log-likelihood to likelihood and normalize it as  $w^{(m)}$

Weighted sum the estimate  $\mathbf{x}_t$  and covariance  $\mathbf{P}_t$

$$\mathbf{x}_t = \sum_{m=1}^M w^{(m)} \mathbf{x}_{t|t}^{(m)}$$

$$\mathbf{P}_t = \sum_{m=1}^M w^{(m)} \left[ \mathbf{P}_{t|t}^{(m)} + (\mathbf{x}_{t|t}^{(m)} - \mathbf{x}_t)(\mathbf{x}_{t|t}^{(m)} - \mathbf{x}_t)^T \right]$$

# Experiments (Synthetic Data)

- Scenario

Height of room  $R = 3\text{m}$

Sampling interval  $T_s = 0.02\text{s}$

Process noise  $\sigma_v^2 = 0.001\text{m}^2/\text{s}^4$  Observation noise  $\sigma_y^2 = 0.001\text{m}^2$

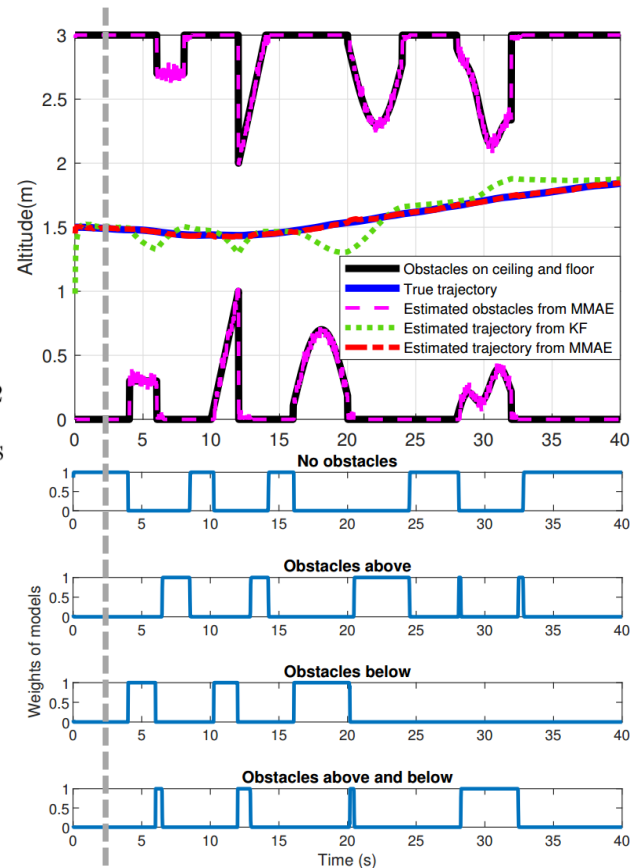
Time length  $T = 2000$

Initial states  $h_{d,0} = 1.5\text{m}$   $v_0 = 0\text{m/s}$

- Mean squared errors (MSEs) of altitude estimation

in 1000 Monte Carlo simulations

$\sigma_y^2 \backslash \sigma_v^2$	0.001			0.01		
$\sigma_a^2 \backslash \sigma_v^2$	0.01	0.1	1	0.01	0.1	1
0.01	2.8e-3	4.8e-3	1.3e-2	4.6e-3	6.6e-3	1.4e-2
0.1	1.4e-3	2.4e-4	5.1e-4	1.9e-3	7.3e-4	7.7e-4
1	1.4e-3	<b>2.3e-4</b>	2.9e-4	2.1e-3	1.4e-3	2.9e-2



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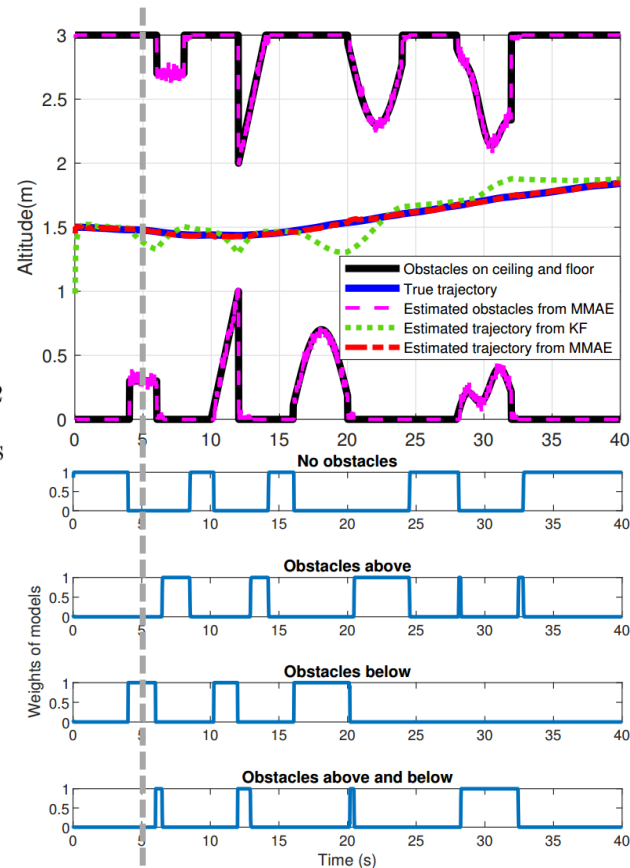
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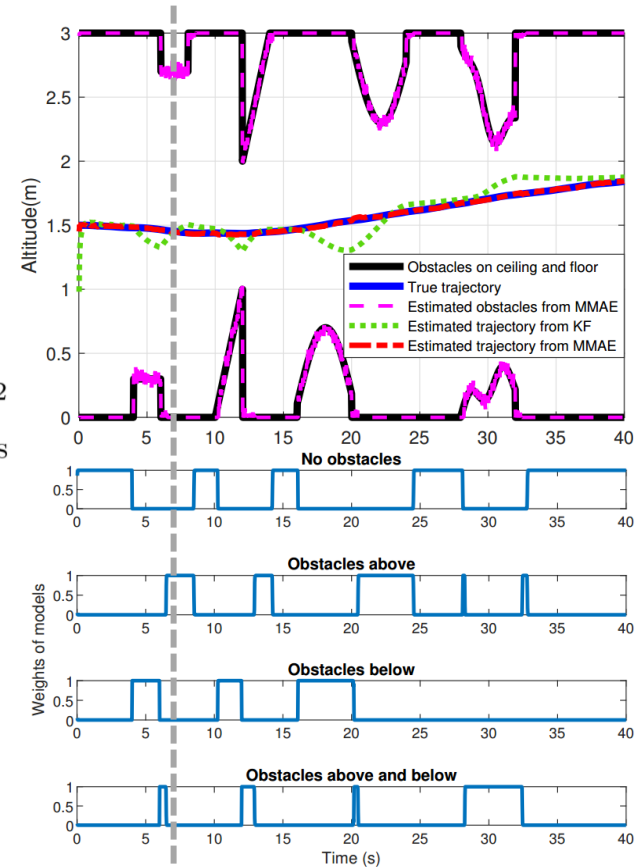
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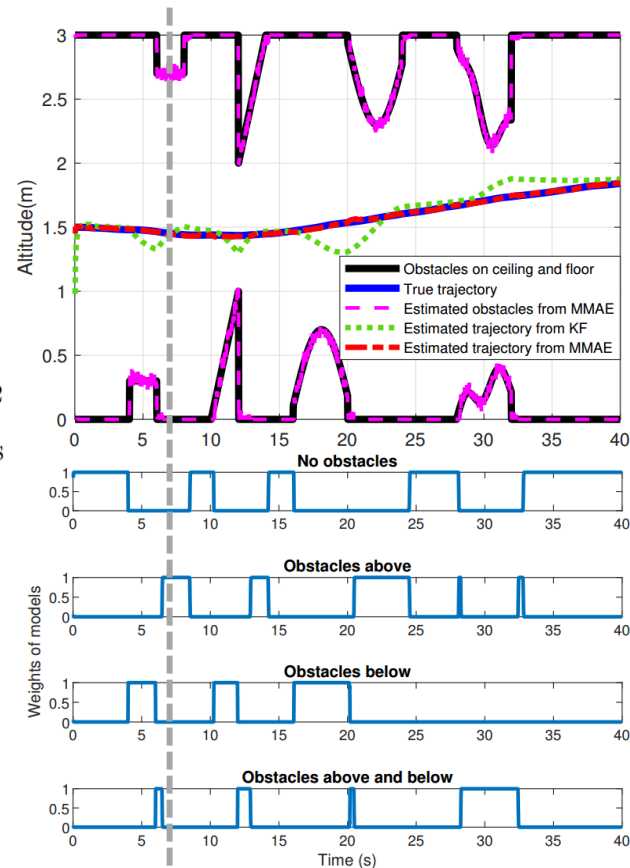
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1	1.4e-3	<b>2.3e-4</b>	2.9e-4	2.1e-3	1.4e-3	2.9e-2



# Experiments (Real Data\*)

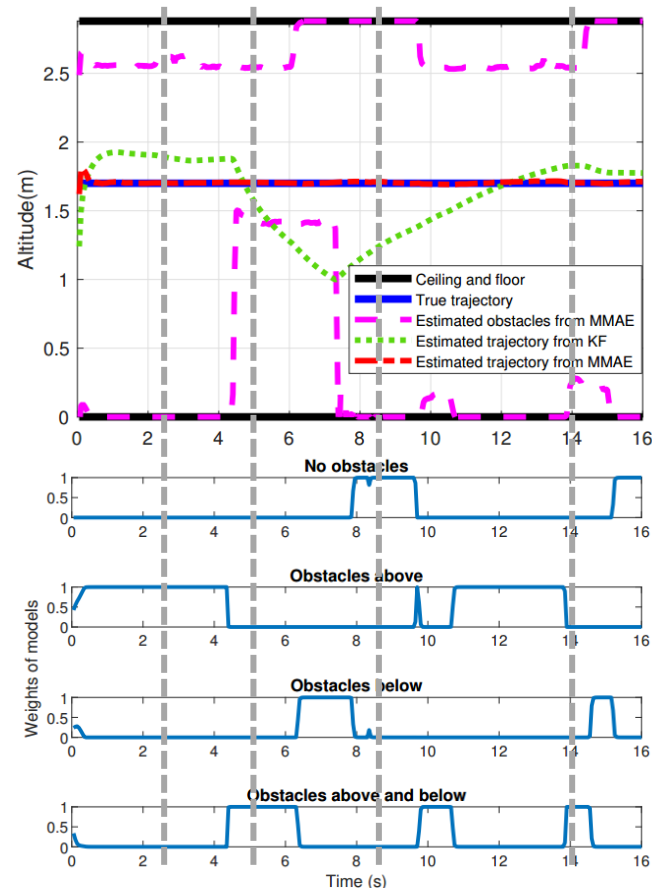
- Scenario

Height of room  $R = 2.88\text{m}$   
 Process noise  $\sigma_v^2 = 0.2\text{m}^2/\text{s}^4$   
 Real altitude  $h_{d,t} = 1.7\text{m}$

Sampling interval  $T_s = 0.05\text{s}$   
 Observation noise  $\sigma_y^2 = 0.001\text{m}^2$   
 Uncertainty of obstacles  $\sigma_a^2 = 1\text{m}^2$

- MSEs of altitude estimation in different arrangements of the obstacles

Ceiling \ Floor	None	Regular	Regular	Irregular
	None	Parallel	Inclined	
None	1.36e-4	3.57e-5	9.42e-5	1.49e-4
Regular	2.53e-4	3.61e-4	3.58e-4	4.25e-4
Irregular	4.51e-4	5.93e-4	9.50e-4	5.14e-4



# Conclusion

- Addressed the problem of altitude estimation for UAVs in indoor setting only using infrared sensor data
- Tackled the problem by formulating four candidate state-space models and applying multiple model adaptive estimation with a bank of Kalman filters
- Experiments using both synthetic data and real data show the promise

# Thank you very much for your attention!

## Contacts

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