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SIGNAL RECONSTRUCTION IN THE PRESENCE OF SIDE INFORMATION: THE IMPACT OF PROJECTION KERNEL DESIGN

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Introduction

• Research Background

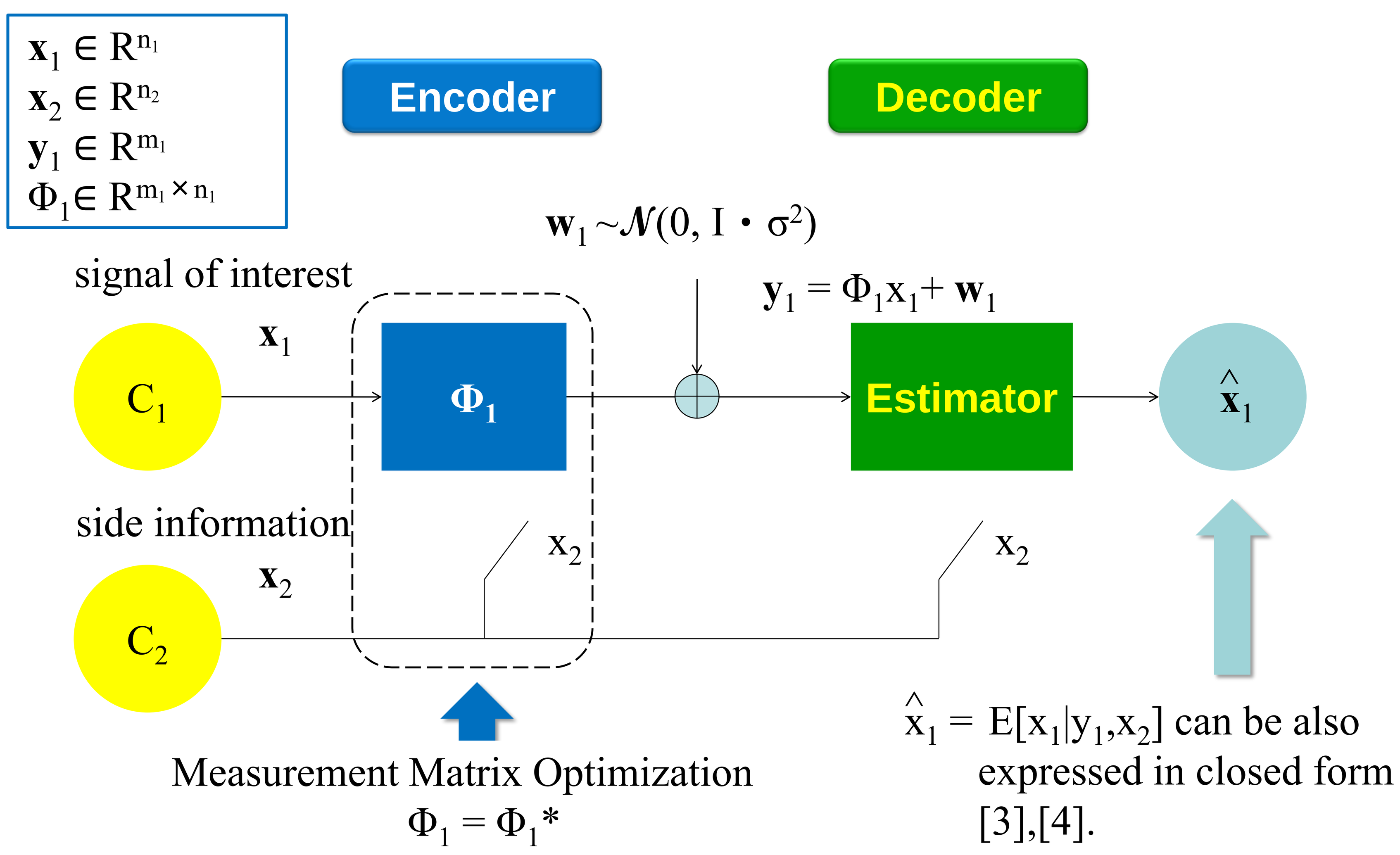
- Compressive sensing (CS) [1],[2].
- CS with Gaussian mixture model (GMM) [3].
- CS with side information [4].

• Research Question

What is the impact of projection kernel design on the reconstruction performance of CS of GMM signals with side information?

Model

• Compressive Sensing Model with Side Information



We will be assuming that $\mathbf{x}_1$ and $\mathbf{x}_2$ are drawn from a joint GMM, characterized by underlying class labels $C_1 \in \{1,\dots,K_1\}$ and $C_2 \in \{1,\dots,K_2\}$, obey the joint probability density function (pdf):

$$p(x_1,x_2 \mid C_1=i, C_2=k) \sim \mathcal{N}(\mu_x^{(i,k)}, \Sigma_x^{(i,k)}),$$

where mean $\mu_x^{(i,k)} = \begin{bmatrix} \mu_{x_1}^{(i,k)} \\ \mu_{x_2}^{(i,k)} \end{bmatrix}$, and the covariance matrix $\Sigma_x^{(i,k)} = \begin{bmatrix} \Sigma_{x_1}^{(i,k)} & \Sigma_{x_{12}}^{(i,k)} \\ \Sigma_{x_{21}}^{(i,k)} & \Sigma_{x_2}^{(i,k)} \end{bmatrix}$.

The covariance matrices are assumed to be possibly low-rank, and we denote such ranks by $r_x^{(i,k)} = \mathrm{rank}(\Sigma_x^{(i,k)})$, $r_{x_1}^{(i,k)} = \mathrm{rank}(\Sigma_{x_1}^{(i,k)})$ and $r_{x_2}^{(i,k)} = \mathrm{rank}(\Sigma_{x_2}^{(i,k)})$.

Projection Kernel Design

- Performance metric: MMSE

$$\mathrm{MMSE}(\sigma^2,\Phi_1) = E[\| x_1 - \hat{x}_1(y_1,x_2) \|^2]$$

- Side information is available at decoder only

$$\Phi_1^* = \underset{\Phi_1}{\text{minimize}}\ \mathrm{MMSE}(\sigma^2,\Phi_1) \quad \text{subject to } \mathrm{tr}(\Phi_1\Phi_1^T) \le m_1 \quad \Longrightarrow \quad \mathrm{MMSE}_d^{opt}(\sigma^2) = \mathrm{MMSE}(\sigma^2, \Phi_1^*)$$

- Side information is available at both encoder and decoder

$$\mathrm{MMSE}(\sigma^2,\Phi_1,x_2) = E[\| x_1 - \hat{x}_1(y_1,x_2) \|^2 \mid x_2]$$

$$\Phi_1^*(x_2) = \underset{\Phi_1}{\text{minimize}}\ \mathrm{MMSE}(\sigma^2,\Phi_1,x_2) \quad \text{subject to } \mathrm{tr}(\Phi_1\Phi_1^T) \le m_1 \quad \Longrightarrow \quad \mathrm{MMSE}_{ed}^{opt}(\sigma^2) = E[\mathrm{MMSE}(\sigma^2, \Phi_1^*(x_2), x_2)]$$

Analysis

• Necessary and Sufficient Conditions for Reliable Reconstruction

i) Side information is available at decoder only

$$\lim_{\sigma^2 \to 0} \mathrm{MMSE}_d^{opt}(\sigma^2) = 0 \quad \Longrightarrow \quad m_1 \ge \max_{i,k} r_x^{(i,k)} - r_{x_2}^{(i,k)}$$

$$m_1 > \max_{i,k} r_x^{(i,k)} - r_{x_2}^{(i,k)} \quad \Longrightarrow \quad \lim_{\sigma^2 \to 0} \mathrm{MMSE}_d^{opt}(\sigma^2) = 0$$

ii) Side information is available at both encoder and decoder

$$\lim_{\sigma^2 \to 0} \mathrm{MMSE}_{ed}^{opt}(\sigma^2) = 0 \quad \Longrightarrow \quad m_1 \ge \max_{i,k} r_x^{(i,k)} - r_{x_2}^{(i,k)}$$

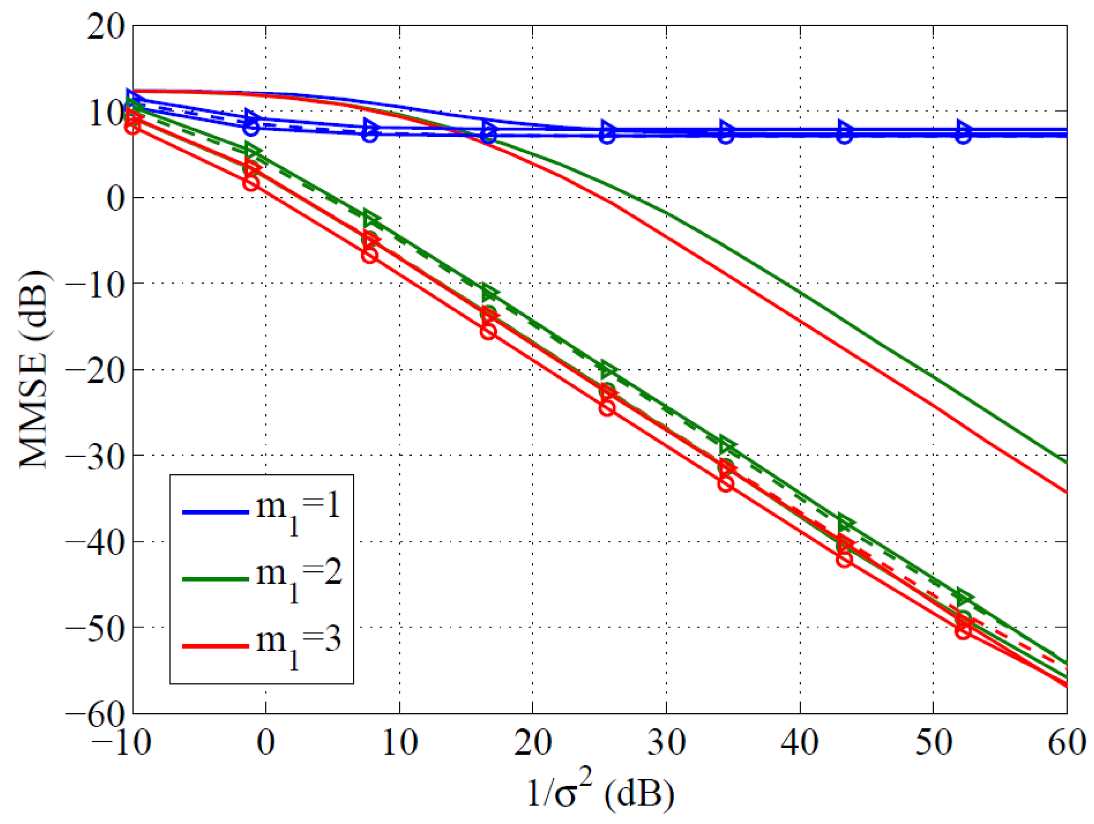
$$m_1 > \max_{i,k} r_x^{(i,k)} - r_{x_2}^{(i,k)} \quad \Longrightarrow \quad \lim_{\sigma^2 \to 0} \mathrm{MMSE}_{ed}^{opt}(\sigma^2) = 0$$

- Conditions for reliable reconstruction with designed projection kernels are the same as those for random kernels, as provided in [3], [4].

Numerical Results

• Synthetic Data

Setup:
$n_1 = 10$, $n_2 = 6$, $K_1 = K_2 = 2$, all the means are zero where $\mu_x^{(i,k)} = 0$, the covariance matrices are randomly generated such that $r_x^{(i,k)} = 5$, $r_{x_1}^{(i,k)} = 3$, $r_{x_2}^{(i,k)} = 3$, for $i = 1,2$ and $k = 1,2$.



- Note that suboptimal design of measurement Φ_1^* is obtained by considering classification of x_2 (at both decoder and encoder) we denote $\mathrm{MMSE}_{ed}^{subopt}(\sigma^2)$ as suboptimal case.

Solid Lines : $\mathrm{MMSE}(\sigma^2,\Phi_1)$
Dashed Lines : $\mathrm{MMSE}_d^{opt}(\sigma^2)$
Circle Lines : $\mathrm{MMSE}_{ed}^{opt}(\sigma^2)$
Triangle Lines : $\mathrm{MMSE}_{ed}^{subopt}(\sigma^2)$

• Real Data

We consider signal of interest x_1 is 512*512 image "Lena" and side information x_2 is 128*128 image of the same image. A 20-classes joint GMM distribution describing x_1 and x_2 is trained via EM algorithm. Below figures report reconstruction results with $m_1 = 15$ from each patch and with noise level $\sigma^2 = -60$ dB. In particular, result (c) is produced by using suboptimal design measurement.

PSNR = 30.7 dB (a) Random kernel, side information at decoder

PSNR = 36.3 dB (b) Designed kernel, side information at decoder

PSNR = 36.1 dB (c) Designed kernel, side information at encoder and decoder

Conclusions

- Sharp necessary and sufficient conditions for reliable reconstruction.
- Conditions for reliable reconstruction with designed projection kernels are the same as those for random projection kernels.
- Projection design guarantees significant reconstruction error reduction at finite noise levels.
- Side information at both encoder and decoder does not reduce significantly the reconstruction error with respect to side information at the decoder only.

• Reference

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[3] F. Renna, L. Wang, X. Yuan, J. Yang, G. Reeves, R. Calderbank, L. Carin, and M.R.D. Rodrigues, "Classification and reconstruction of compressed GMM signals with side information," in IEEE Int. Symp Information Theory (ISIT), Jun. 2015.
[4] F. Renna, L. Wang, X. Yuan, J. Yang, G. Reeves, R. Calderbank, L. Carin, and M.R.D. Rodrigues, "Classification and reconstruction of high-dimensional signals from low-dimensional noisy features in the presence of side information," arXiv preprint arXiv:1412.0614, 2014.