Design of a Convergence-aware Based Expectation Propagation Algorithm for Uplink MIMO SCMA Systems

Jih-Yang Lin and Pei-Yun Tsai
Department of Electrical Engineering,
National Central University, Taiwan
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Outline

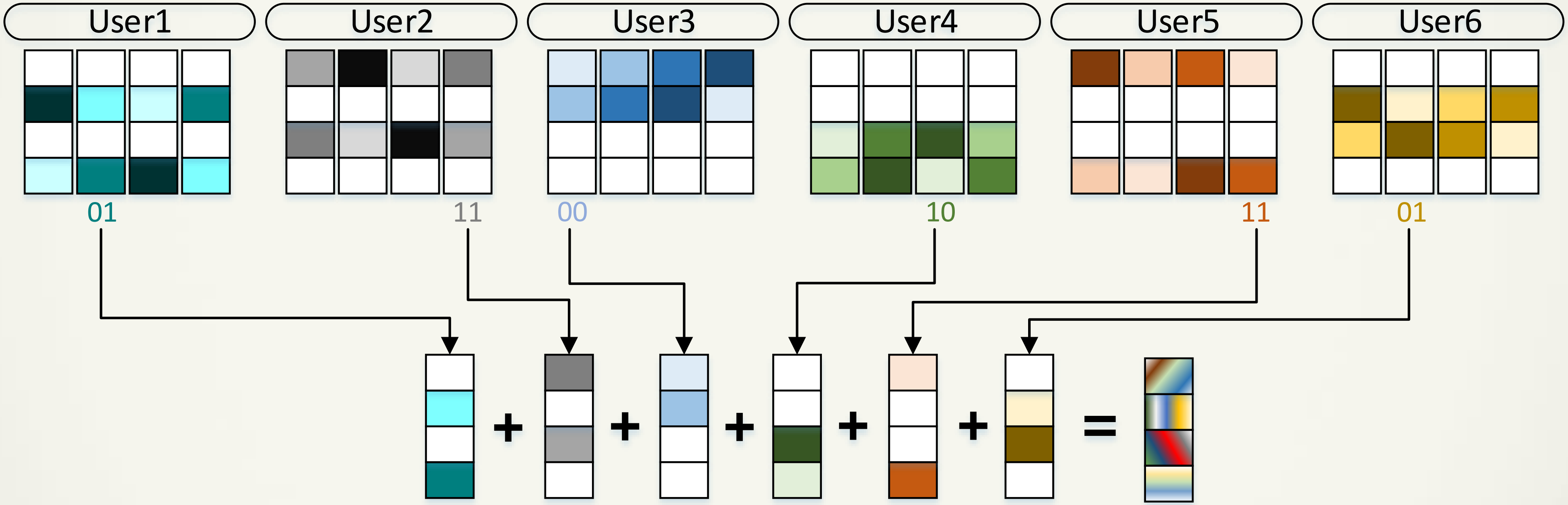
- Introduction
- System model of SCMA
- Proposed convergence-aware based EPA
- Simulation result and comparison
- Conclusion
- Reference

Introduction

- IoT applications in 5G system require massive connections and low latency for wireless communications.
- Non-orthogonal multiple access, or so called NOMA technique, has been proposed as a promising technology to meet the requirement.
- Sparse code multiple access (SCMA) is one of the famous NOMA techniques. Developed detection schemes include
  - Message passing algorithm (MPA)
  - Expectation propagation algorithm (EPA)

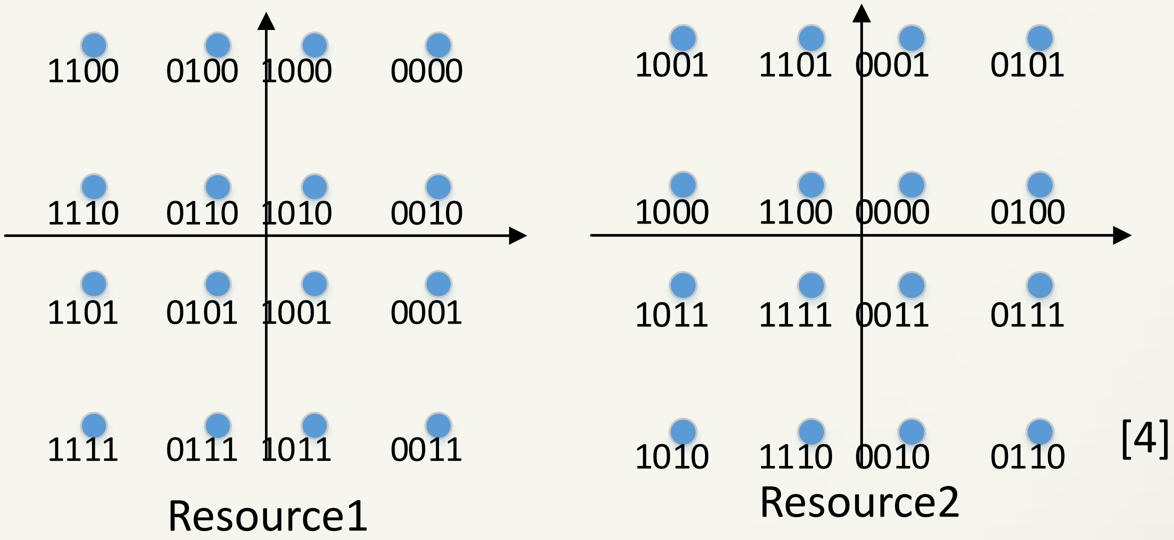
System Model of SCMA

- SCMA



System Model of SCMA

- SCMA
  - Sparse matrix
    - $$\mathbf{S}_{Sparse} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$ [4]
  - Constellation

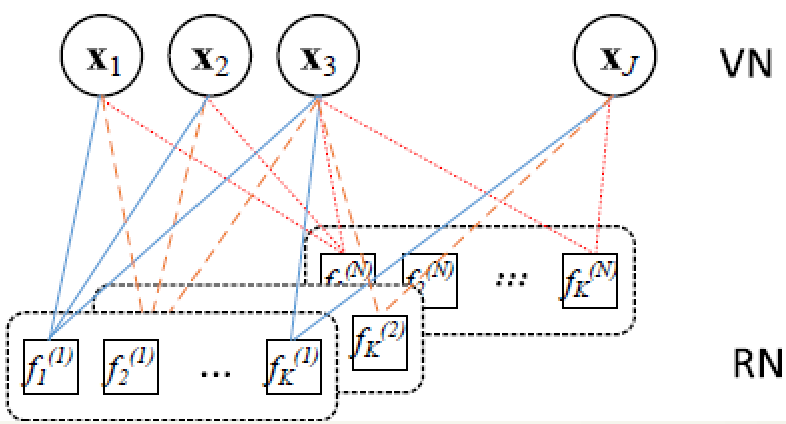


[4]

System Model of SCMA

- Uplink system model:
  - $\mathbf{y}^{(n)} = \sum_{j=1}^{J} diag\left(\mathbf{h}_j^{(n)}\right)\mathbf{x}_j + \mathbf{v}^{(n)}$
    - $\mathbf{y}^{(n)} = \left[y_1^{(n)} \ldots y_K^{(n)}\right]^T$
    - $\mathbf{h}_j^{(n)} = \left[h_{j,1}^{(n)} \ldots h_{j,K}^{(n)}\right]^T$
    - $\mathbf{x}_j = \left[x_{j,1} \ldots x_{j,K}\right]^T$
    - $\mathbf{v}^{(n)} = \left[v_1^{(n)} \ldots v_K^{(n)}\right]^T$
    - j : user index
    - n: antenna index
    - k: resource index

- Factor graph



$$f_k^{(n)} = P\left(y_k^{(n)}\middle|\mathbf{x}\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{|y_k^{(n)} - \sum_{j=1}^{J} h_{j,k}^{(n)} x_{j,k}|^2}{2\sigma^2}}$$

$$SNR = \frac{E\{|y_k^{(n)}|^2\}}{\sigma^2}$$

Conventional EPA

- Conventional algorithm

```
Algorithm : EPA [8]
// Initialization
    $\mu^{(0)}_{(k,n)\to j} = 0, \xi^{(0)}_{(k,n)\to j} = \infty, \mu^{(0)}_{j,k} = 0, \xi^{(0)}_{j,k} = 1, \forall k, n$
for $t$=1:$I$ // Iteration
  for $j$=1:$J$   // VN calculation
    Compute $\mu^{(t)}_{j\to(k,n)}$ and $\xi^{(t)}_{j\to(k,n)}$ by (1)
  endfor
  for $k$=1:$K$, $n$=1:$N$   // RN calculation
    Compute $\mu^{(t)}_{(k,n)\to j}$ and $\xi^{(t)}_{(k,n)\to j}$ by (2)
  endfor
  for $j$=1:$J$   // Posterior probability calculation
    Compute $p^{(t)}(\mathbf{x}_j|\mathbf{y})$ by (3)
    Compute $\mu^{(t)}_{j,k}$ and $\xi^{(t)}_{j,k}$ by (4) for $m \in \chi_j^S$
  endfor
endfor
```

- $R(k)$: users associated with resource k
- $V(j)$: resources associated with user j

- Equations :

Eq. (1)

$$\xi^{(t)}_{j\to(k,n)} = \left(\frac{1}{\xi^{(t-1)}_{j,k}} - \frac{1}{\xi^{(t-1)}_{(k,n)\to j}}\right)^{-1}$$

$$\mu^{(t)}_{j\to(k,n)} = \xi^{(t)}_{j\to(k,n)}\left(\frac{\mu^{(t-1)}_{j,k}}{\xi^{(t-1)}_{j,k}} - \frac{\mu^{(t-1)}_{(k,n)\to j}}{\xi^{(t-1)}_{(k,n)\to j}}\right)$$

Eq. (2)

$$\mu^{(t)}_{(k,n)\to j} = \frac{1}{h^{(n)}_{j,k}}\left(y^{(n)}_k - \sum\nolimits_{l\in R(k), l\neq j} h^{(n)}_{l,k}\,\mu^{(t)}_{l\to(k,n)}\right)$$

$$\xi^{(t)}_{(k,n)\to j} = \frac{1}{|h^{(n)}_{j,k}|^2}\left(\sigma^2 - \sum\nolimits_{l\in R(k), l\neq j} \left|h^{(n)}_{l,k}\right|^2 \xi^{(t)}_{l\to(k,n)}\right)$$

Eq. (3)

$$p^{(t)}(\mathbf{x}_j|\mathbf{y}) \propto \prod\nolimits_{k\in V(j)} I^{(t-1)}_{k,n\to j}(x_{j,k}), \quad I(x|\mu,\xi) = \frac{1}{\sqrt{2\pi\xi}} e^{\frac{-(x-\mu)^2}{2\xi}}$$

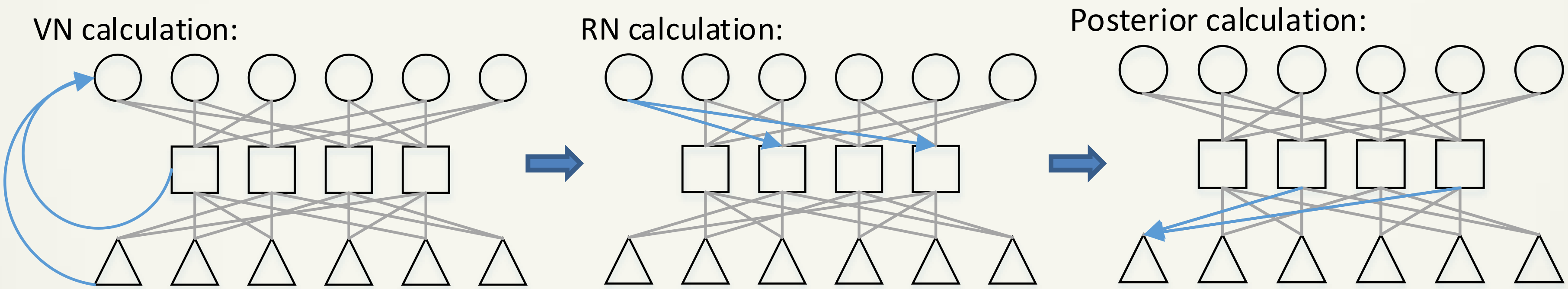
Eq. (4)

$$\mu^{(t)}_{j,k} = \sum\nolimits_m p^{(t)}(\mathbf{x}_j = \mathbf{c}_{j,m}|\mathbf{y})\, c_{j,m,k}$$

$$\xi^{(t)}_{j,k} = \sum\nolimits_m p^{(t)}(\mathbf{x}_j = \mathbf{c}_{j,m}|\mathbf{y}) \left| c_{j,m,k} - \mu^{(t)}_{j,k} \right|^2$$

Conventional EPA

- Conventional EPA flow



VN calculation:

RN calculation:

Posterior calculation:

Proposed Convergence-Aware Based EPA

- Proposed algorithm

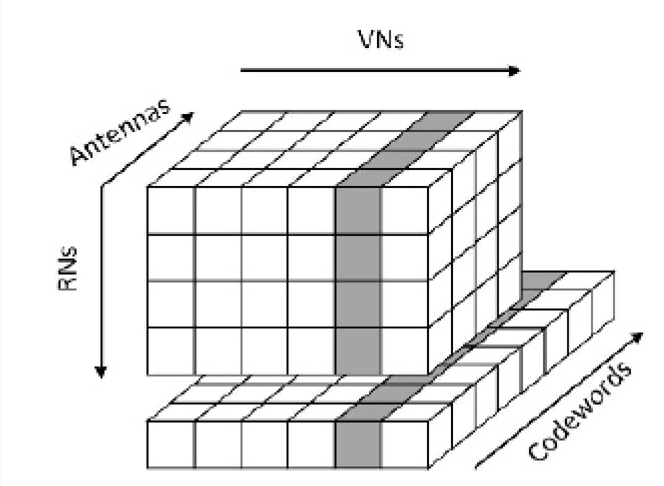
```
Algorithm : Convergence-aware based EPA
// Initialization
    $\mu^{(0)}_{(k,n)\to j} = 0, \xi^{(0)}_{(k,n)\to j} = \infty, \mu^{(0)}_{j,k} = 0, \xi^{(0)}_{j,k} = 1, \forall k, n$
for $t$=1:$I$ // Iteration
  for $j$=1:$J$   // VN calculation
    if (User Termination)||(Antenna Termination)
      $\mu^{(t)}_{j\to(k,n)} = \mu^{(t-1)}_{j\to(k,n)}, \xi^{(t)}_{j\to(k,n)} = \xi^{(t-1)}_{j\to(k,n)}, \forall k, n$
    else
      Compute $\mu^{(t)}_{j\to(k,n)}$ and $\xi^{(t)}_{j\to(k,n)}$ by (1)
    endif
  endfor
  for $k$=1:$K$, $n$=1:$N$   // RN calculation
    if (User Termination)||(Antenna Termination)
      $\mu^{(t)}_{(k,n)\to j} = \mu^{(t-1)}_{(k,n)\to j}, \xi^{(t)}_{(k,n)\to j} = \xi^{(t-1)}_{(k,n)\to j}, \forall j$
    else
      Compute $\mu^{(t)}_{(k,n)\to j}$ and $\xi^{(t)}_{(k,n)\to j}$ by (2)
    endif
  endfor
  for $j$=1:$J$   // Posterior probability calculation
    if (User Termination)
      $p^{(t)}(\mathbf{x}_j|\mathbf{y}) = p^{(t-1)}(\mathbf{x}_j|\mathbf{y})$
    else
      Compute $p^{(t)}(\mathbf{x}_j|\mathbf{y})$ by (3)
    endif
    Check if $(t > I_{User})$ && $(p^{(t)}(\mathbf{x}_j|\mathbf{y}) > \Omega_{User}$ for User Termination
    Check if $\sum_{k\in R(j)} \xi^{(t)}_{j\to(k.n)} < \Omega_{Ant}$ for Antenna Termination
    Perform Codebook Reduction and obtain $\chi_j^S$
    Compute $\mu^{(t)}_{j,k}$ and $\xi^{(t)}_{j,k}$ by (4) for $m \in \chi_j^S$
  endfor
endfor
```

Parameter :

- Ω_{User} : Threshold for user termination
- Ω_{Ant} : Threshold for antenna termination
- Ω_{Acc} : Threshold for codebook reduction

Proposed Convergence-Aware Based EPA

- User termination



```
23:     Check if $(t > I_{User})$ && $(p^{(t)}(\mathbf{x}_j|\mathbf{y}) > \Omega_{User}$ for User Termination
```

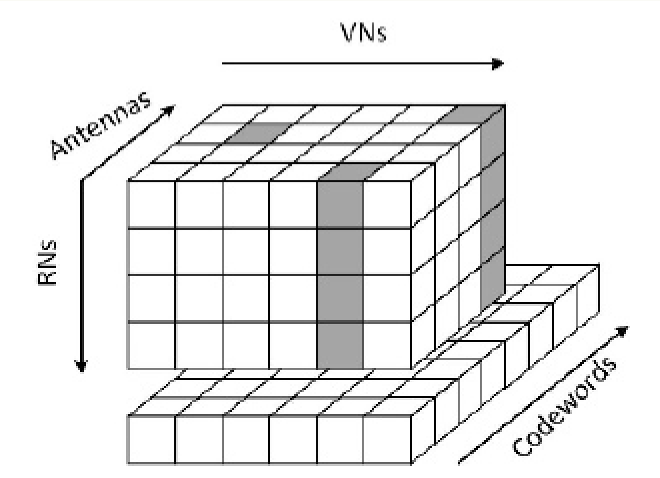
```
3:      for $j$=1:$J$   // VN calculation
4:        if (User Termination)||(Antenna Termination)
```

```
10:     for $k$=1:$K$, $n$=1:$N$   // RN calculation
11:       if (User Termination)||(Antenna Termination)
```

```
17:     for $j$=1:$J$   // Posterior probability calculation
18:       if (User Termination)
```

Proposed Convergence-Aware Based EPA

- Antenna termination



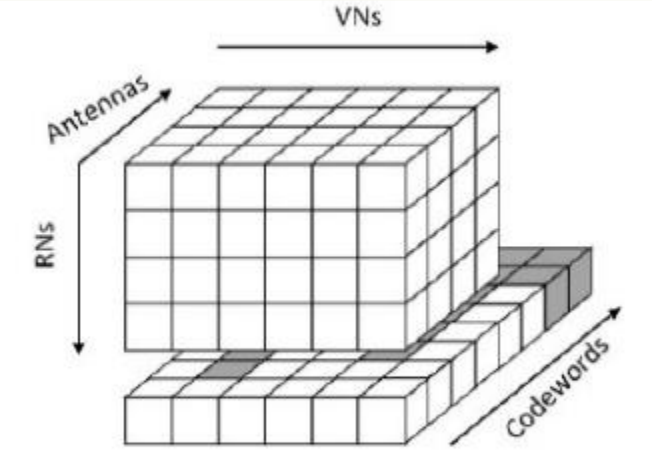
```
Check if $\sum_{k\in R(j)} \xi^{(t)}_{j\to(k,n)} < \Omega_{Ant}$ for Antenna Termination
```

```
for j=1:J   // VN calculation
  if (User Termination)||(Antenna Termination)
```

```
for k=1:K, n=1:N   // RN calculation
  if (User Termination)||(Antenna Termination)
```

Proposed Convergence-Aware Based EPA

- Codebook reduction

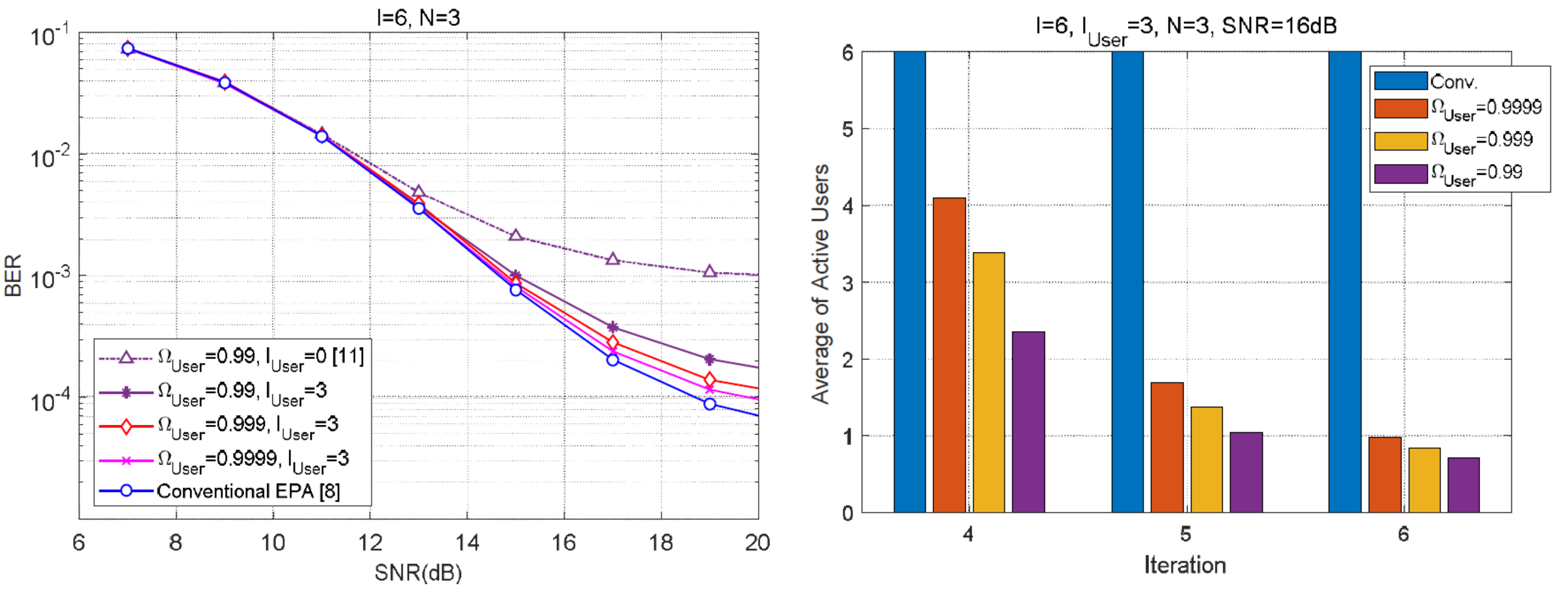


```
Perform *Codebook Reduction* and obtain $\chi_j^S$
```

```
Compute $\mu_{j,k}^{(t)}$ and $\xi_{j,k}^{(t)}$ by (4) for $m \in \chi_j^S$
```

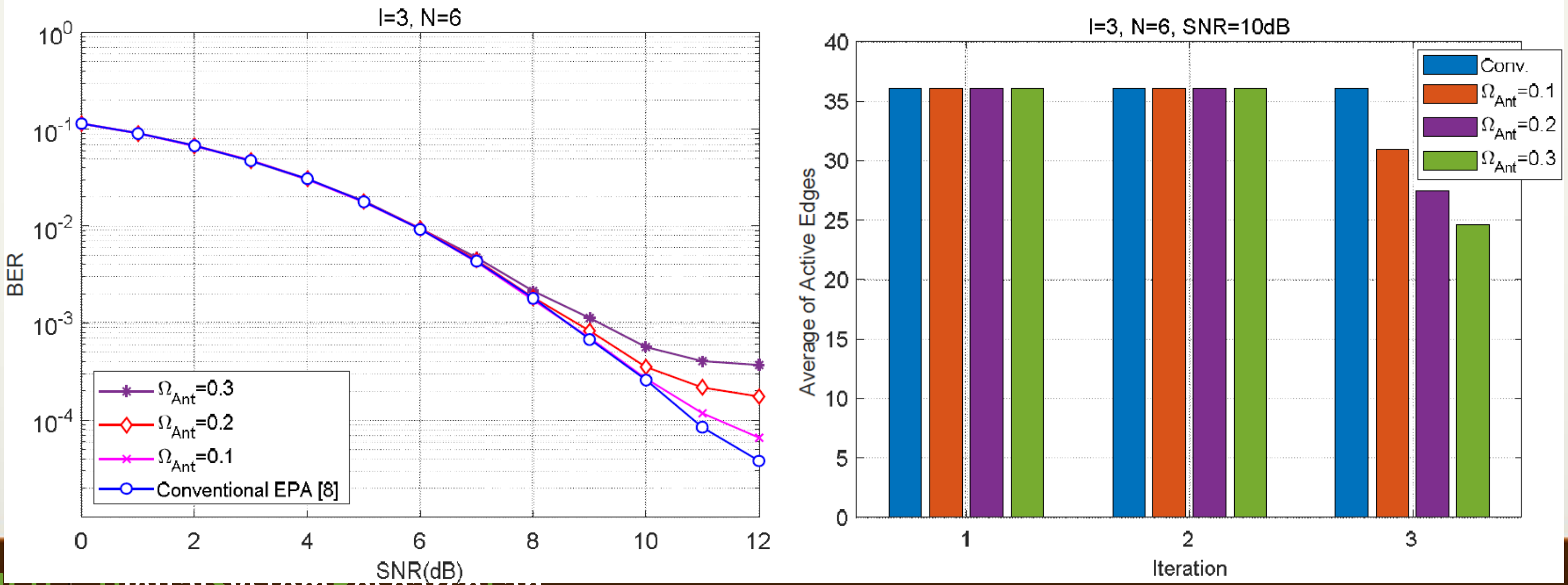
Simulation Result and Comparison

- User termination
  - Parameter :
    - I : iteration time
    - I_{User} : iteration constraint
    - N : number of received antennas
    - Ω_{User} : threshold for user termination



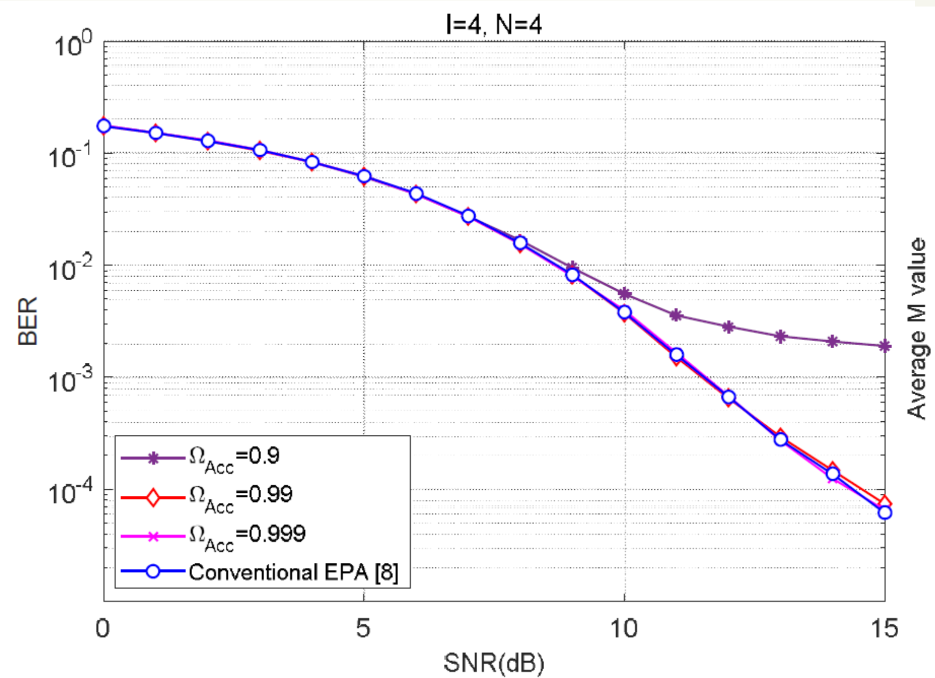
Simulation Result and Comparison

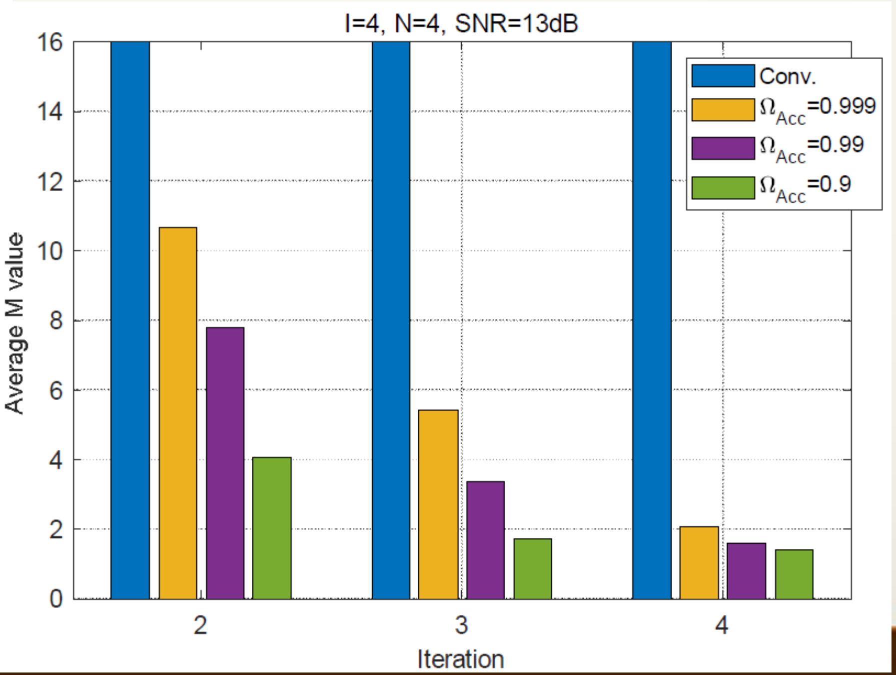
- Antenna termination
  - Parameter :
    - I : iteration time
    - N : number of received antennas
    - Ω_{Ant} : threshold for antenna termination



Simulation Result and Comparison

- Codebook reduction
  - Parameter :
    - I : iteration time
    - N : number of received antennas
    - Ω_{ACC} : threshold for codebook reduction

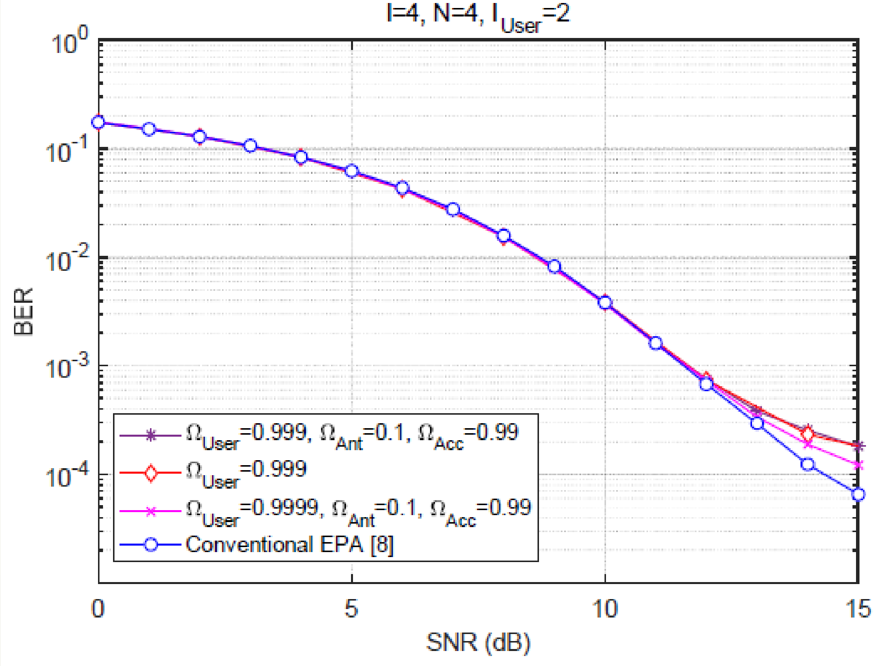


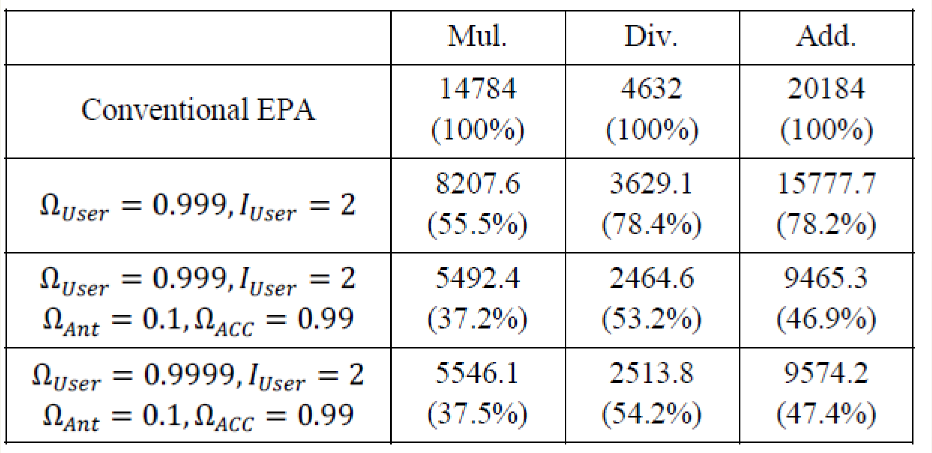


Simulation Result and Comparison

Algorithm

- Proposed convergence-aware based EPA





	Mul.	Div.	Add.
Conventional EPA	14784 (100%)	4632 (100%)	20184 (100%)
$\Omega_{User} = 0.999, I_{User} = 2$	8207.6 (55.5%)	3629.1 (78.4%)	15777.7 (78.2%)
$\Omega_{User} = 0.999, I_{User} = 2$ $\Omega_{Ant} = 0.1, \Omega_{ACC} = 0.99$	5492.4 (37.2%)	2464.6 (53.2%)	9465.3 (46.9%)
$\Omega_{User} = 0.9999, I_{User} = 2$ $\Omega_{Ant} = 0.1, \Omega_{ACC} = 0.99$	5546.1 (37.5%)	2513.8 (54.2%)	9574.2 (47.4%)

Conclusion

- The proposed convergence-aware EPA contains three termination schemes.
  - User termination
  - Antenna termination
  - Codebook reduction

- Complexity reduction is achieved by stopping unnecessary computations for information update. And the simulation result shows that the algorithm only needs 37%, 54%, and 47% of the computation complexity in multiplication, divisions, and additions compared to the conventional one.

Reference

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