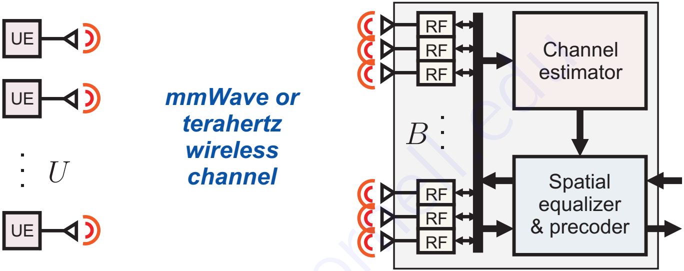
Sparse Beamspace Equalization for Massive MU-MIMO mmWave Systems

Seyed Hadi Mirfarshbafan and Christoph Studer

CORNELL TECH

ICASSP2020 Barcelona

Massive MU-MIMO at millimeter wave (mmWave)



- Equip basestation (BS) with B-antenna array
- Enables high array gain and fine-grained beamforming
- Serve U user equipments (UEs) in same time-frequency resource

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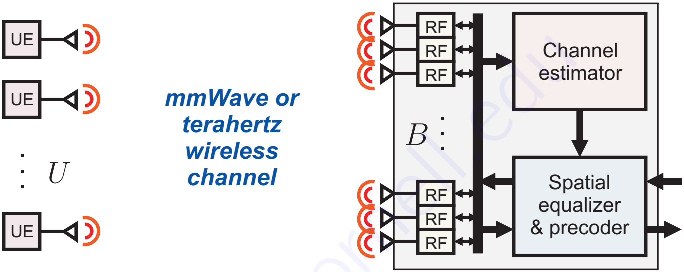

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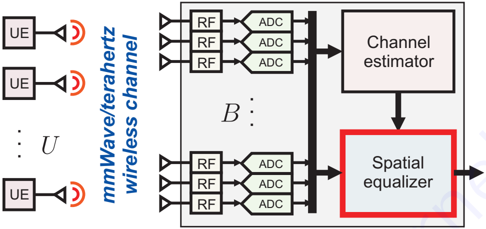
Combining mmWave communication with massive multi-user (MU) MIMO enables extremely high data rates

mmWave massive MU-MIMO uplink



Uplink system model: $\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}$

- $\mathbf{y} \in \mathbb{C}^B$ BS-receive signals; $\mathbf{y} = [y_1, \ldots, y_B]^T$
- $\mathbf{H} \in \mathbb{C}^{B \times U}$ MIMO channel matrix with $B \geq U$
- $\mathbf{s} \in \mathcal{S}^U$ UE-transmit vector; expected power E_s
- $\mathbf{n} \in \mathbb{C}^B$ noise; i.i.d. zero-mean Gaussian with variance N_0

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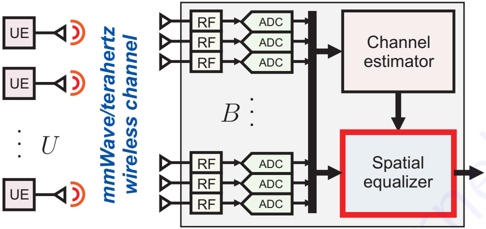

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Goal: **Estimate** transmit data $\mathbf{s}$ for each $\mathbf{y}$, given knowledge of $\mathbf{H}$

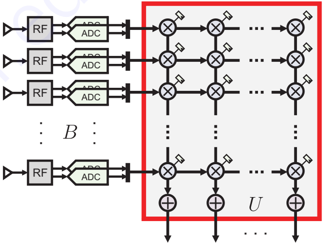
Standard approach: Antenna-domain spatial equalization

- Spatial equalization removes inter-UE interference

- In massive MU-MIMO systems, **linear spatial equalization** is preferable:

$$\hat{\mathbf{s}} = \mathbf{W}\mathbf{y} \quad \text{with L-MMSE matrix:}$$

$$\mathbf{W} = \left(\mathbf{H}^H\mathbf{H} + \frac{N_0}{E_s}\mathbf{I}\right)^{-1}\mathbf{H}^H$$



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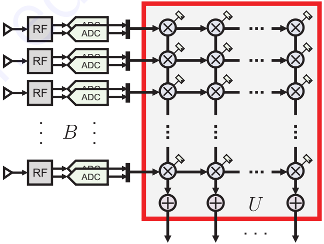
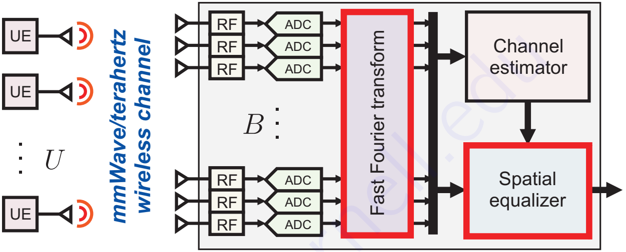


- **Larger bandwidth → higher ADC sampling rate → high power consumption and processing complexity**

Beamspace processing is an alternative



- Convert received vectors $\mathbf{y}$ into **beamspace domain** using a fast Fourier transform (FFT) across antennas $\hat{\mathbf{y}} = \mathbf{F}\mathbf{y}$

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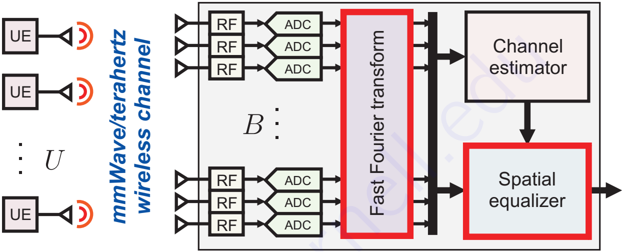

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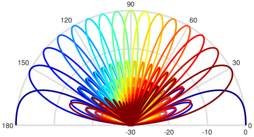

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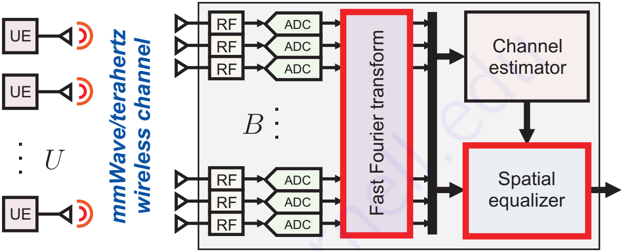

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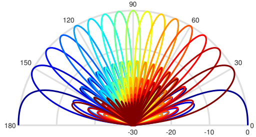

- Perform spatial equalization in beamspace domain $\hat{\mathbf{s}} = \hat{\mathbf{W}}\hat{\mathbf{y}}$

mmWave channels are sparse in beamspace

- Wave propagation at mmWave frequencies is **directional**
- Model for channel vector in antenna space:

  $\mathbf{h} = \sum_{\ell=0}^{L-1} \alpha_\ell \mathbf{a}(\omega_\ell)$ with $\mathbf{a}(\omega) = [e^{j0\omega}, e^{j1\omega}, \ldots, e^{j(B-1)\omega}]^T$

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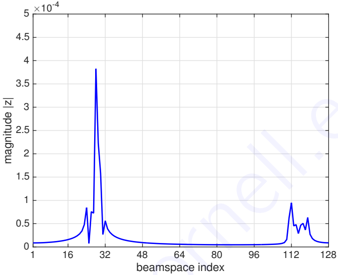

Line-of-sight (LoS) beamspace channel snapshots at 60 GHz with mmMAGIC UMi model

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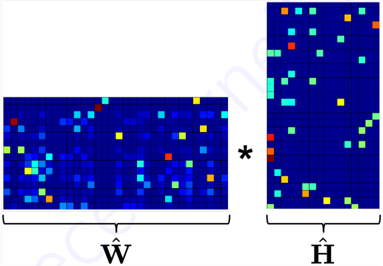
Channel vectors are **sparse in beamspace** $\hat{\mathbf{h}}_u = \mathbf{F}\mathbf{h}_u,\ u = 1, \ldots, U$

Beamspace sparsity can reduce equalization complexity

- **Beamspace channel matrix** $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, \ldots, \hat{\mathbf{h}}_U]$ is **sparse**
- To avoid multiplications with zeros, **beamspace equalization matrix** $\hat{\mathbf{W}}$ can be **sparse** as well $\rightarrow$ **reduces complexity**



- **Density coefficient** $\delta \in (0, 1]$ determines fraction of nonzero entries we want in beamspace equalization matrix: $\delta = \frac{\|\hat{\mathbf{W}}\|_0}{BU}$

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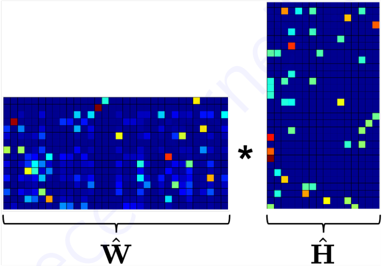

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Spatial equalization $\hat{\mathbf{s}} = \hat{\mathbf{W}}\hat{\mathbf{y}}$ requires only δBU multiplications

When is beamspace-domain equalization useful?

Assume that complexity of computing matrix $\hat{\mathbf{W}}$ is negligible

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$$2B\log_2(B) + 4U\delta B < 4UB \Longrightarrow \delta < 1 - \frac{\log_2(B)}{2U}$$

- Since $\delta > 0$, number of UEs must satisfy $U > \frac{1}{2}\log_2(B)$

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What is the complexity of preprocessing?

Two architectures that exploit beamspace sparsity

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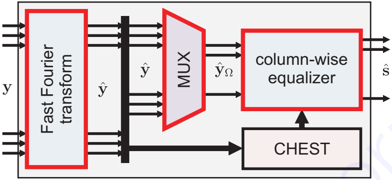

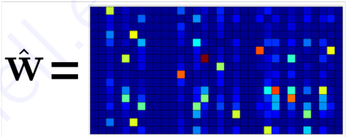

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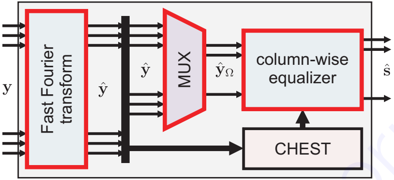
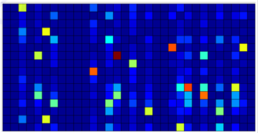
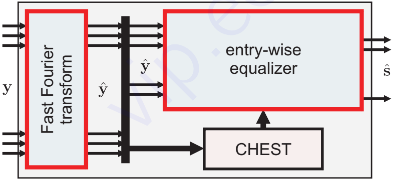
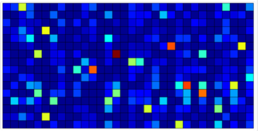


$\hat{\mathbf{W}} =$ 

- **Entry-wise equalizer**: Matrix $\hat{\mathbf{W}}$ is sparse $\rightarrow$ each row of $\hat{\mathbf{W}}$ is sparse with support set Ω_u so that $|\Omega_u| = \delta B$



$\hat{\mathbf{W}} =$ 

Column-wise orthogonal matching pursuit (COMP)

- Goal: determine optimal support set Ω and equalization matrix $\hat{\mathbf{W}}_\Omega$ that minimizes post-equalization MSE: $\mathbb{E}\left[\|\hat{\mathbf{W}}_\Omega\hat{\mathbf{y}}_\Omega - \mathbf{s}\|^2\right]$
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Initialize $\mathbf{A}^{(1)} = \mathbf{I}$ and $\Omega^{(1)} = \varnothing$ and repeat for $k = 1, \ldots, \delta B$:

1. Determine index $b^{(k+1)}$ of best column of equalization matrix

$$b^{(k+1)} = \underset{b' \in \{1,\ldots,B\} \setminus \Omega^{(k)}}{\arg\max} \frac{\|\mathbf{A}^{(k)} \mathbf{h}^r_{b'}\|^2}{\|\mathbf{h}^r_{b'}\|^2 + N_0/E_s}$$

2. Update support set $\Omega^{(k+1)} = \Omega^{(k)} \cup b^{(k+1)}$ and compute

$$\hat{\mathbf{W}}^{(k+1)} = (\mathbf{H}^{\mathrm{H}}_{\Omega^{(k+1)}} \mathbf{H}_{\Omega^{(k+1)}} + \rho \mathbf{I}_U)^{-1} \mathbf{H}^{\mathrm{H}}_{\Omega^{(k+1)}}$$

3. Update residual $\mathbf{A}^{(k)} = \mathbf{I} - \hat{\mathbf{W}}^{(k)} \mathbf{H}_{\Omega^{(k)}}$

Entry-wise orthogonal matching pursuit (EOMP)

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For each row $\hat{\mathbf{w}}_u^r$ of $\hat{\mathbf{W}}$ perform the following algorithm:

Initialize $\mathbf{z}^{(1)} = \mathbf{e}_u$ and $\Omega_u^{(1)} = \varnothing$ and repeat for $k = 1, \ldots, \delta B$:

1. Determine index $b^{(k+1)}$ of best entry of equalization matrix row

$$b^{(k+1)} = \underset{b' \in \{1,\ldots,B\} \setminus \Omega_u^{(k)}}{\arg\max} \frac{|(\mathbf{z}^{(k)})^{\mathrm{H}} \mathbf{h}_{b'}^r|^2}{\|\mathbf{h}_{b'}^r\|^2 + \rho}$$

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$$\hat{\mathbf{w}}_u^{r(k+1)} = \mathbf{H}_{\Omega^{(k+1)}} (\mathbf{H}_{\Omega^{(k+1)}}^{\mathrm{H}} \mathbf{H}_{\Omega^{(k+1)}} + \rho \mathbf{I}_U)^{-1} \mathbf{e}_u$$

3. Update residual $\mathbf{z}^{(k)} = \mathbf{e}_u - \mathbf{H}_{\Omega^{(k)}}^T \hat{\mathbf{w}}_u^{r(k)}$

Alternative beamspace equalization algorithms

- For column-wise methods, we can also use the selection criterion of COMP of first iteration for all columns:

$$b = \underset{b' \in \{1,\ldots,B\}}{\arg\max} \frac{\|\mathbf{h}^r_{b'}\|^2}{\|\mathbf{h}^r_{b'}\|^2 + N_0/E_s},$$

which leads to the **largest column (LC)** approximation from [1]

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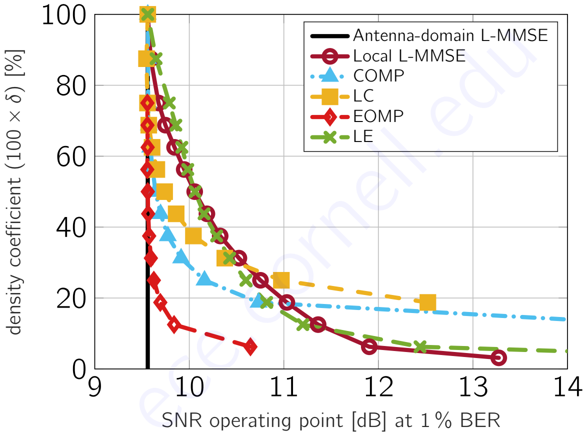
- We can also use the **local MMSE** algorithm from [2]

[1] M. Mahdavi, O. Edfors, V. Öwall, and L. Liu, "A low complexity massive MIMO detection scheme using angular-domain processing," GlobalSIP 2018

[2] M. Abdelghany, U. Madhow, and A. Tölli, "Beamspace Local LMMSE: An efficient digital backend for mmWave massive MIMO," IEEE SPAWC 2019

SNR operating point comparison



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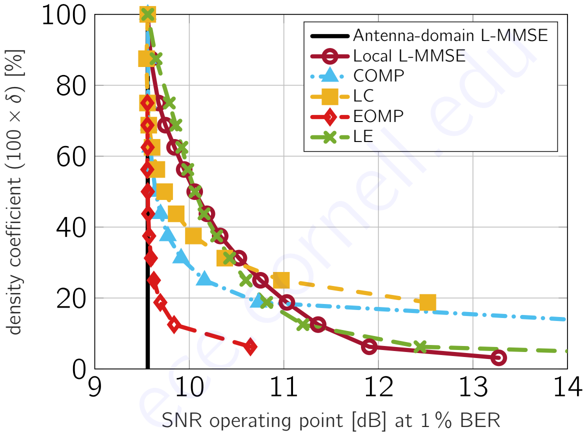

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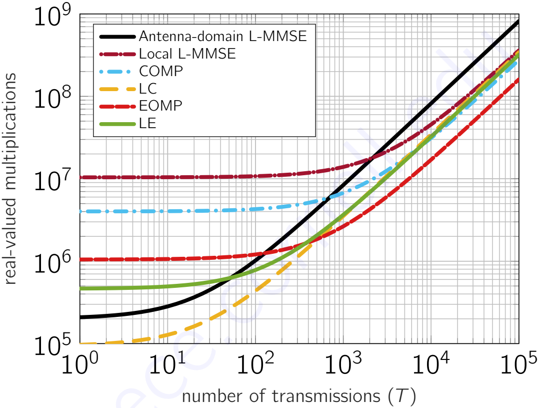
EOMP supports smallest density coefficient δ

Complexity comparison (including preprocessing)



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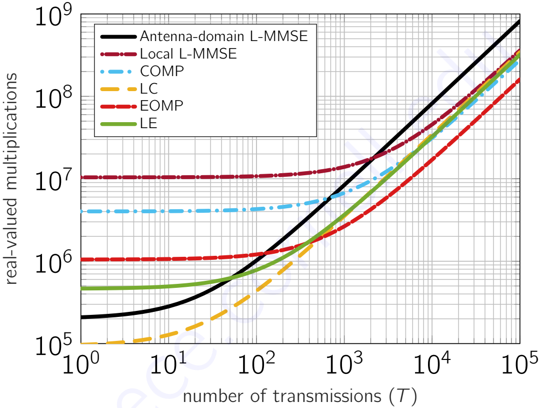

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EOMP suitable for systems with coherence times $T > 1000$ symbols

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More information $\rightarrow$ **vip.ece.cornell.edu**

[1] S. H. Mirfarhsbafan and CS, "Sparse Beamspace Equalization for Massive MU-MIMO mmWave Systems," IEEE ICASSP, May 2020