Graph Neural Net Using Analytical Graph Filter and Topology Optimizing for Image Denoising

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Introduction

- Image Denoising based on Deep Learning [2]
  - Use Convolutional Neural Networks (CNNs)
  - State-of-the-art Performance

- What are the limits of deep learning ?
  - A Large CNN models (Million of network parameters)
  - Large Collections of Labelled Data
  - Mismatch for training and testing data

[2] K. Zhang, W. Zuo, Y. Chen, D. Meng, and L. Zhang, "Beyond a Gaussian denoiser: Residual learning of deep CNN for image denoising," IEEE Transactions on Image Processing, vol. 26, no. 7, pp. 3142–3155, 2017.

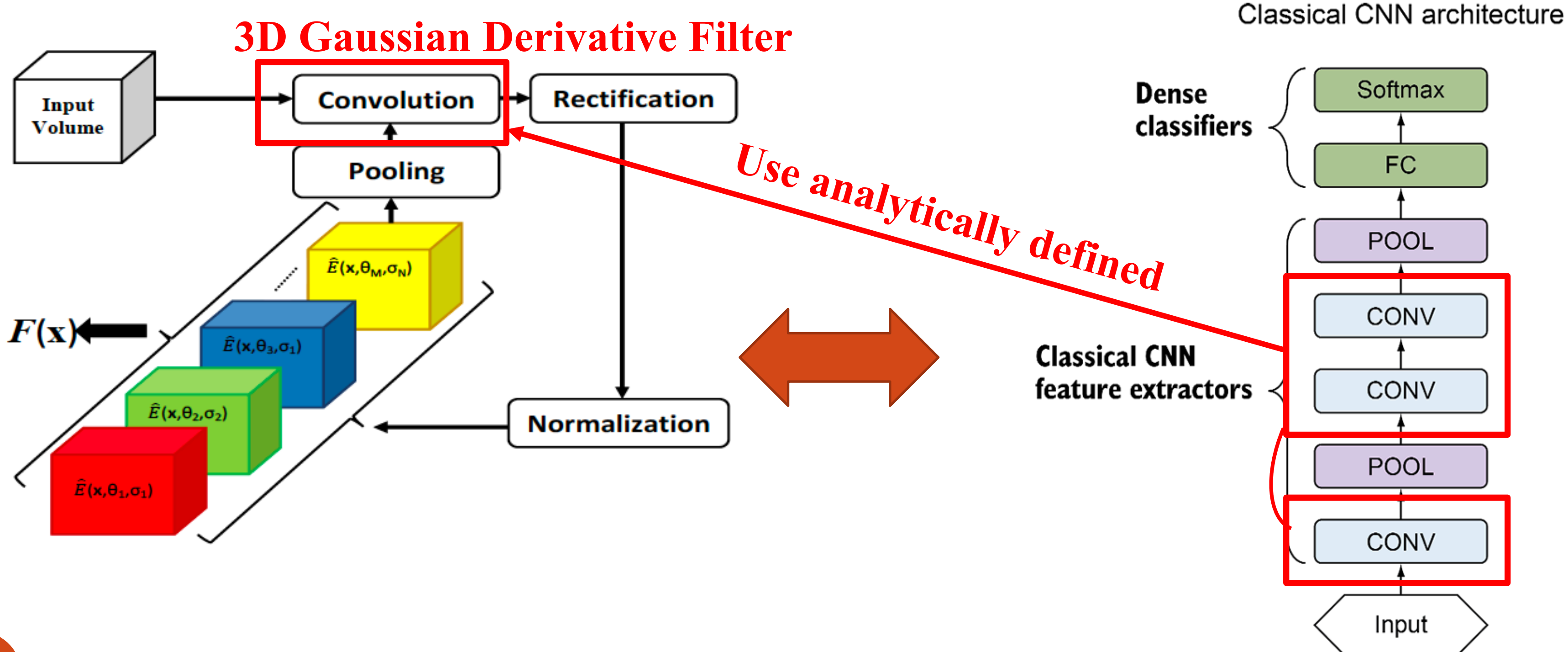
Introduction

- Construct a GNN using GraphBio [14] as graph filter:
  - Interpretable as **low-pass** graph filter with known biorthogonal conditions
  - Linear graph filter is analytically defined
    - No training for graph filter
  - Only optimize the graph via data training
    - Optimize edge weights to filter each pixel, so that the graph spectrum can be data-adaptive

[14] S. K. Narang and A. Ortega, "Compact support biorthogonal wavelet filterbanks for arbitrary undirected graphs," IEEE Transactions on Signal Processing, vol. 61, no. 19, pp. 4673– 4685, 2013.

SOE-Net : Fixed Gaussian filters

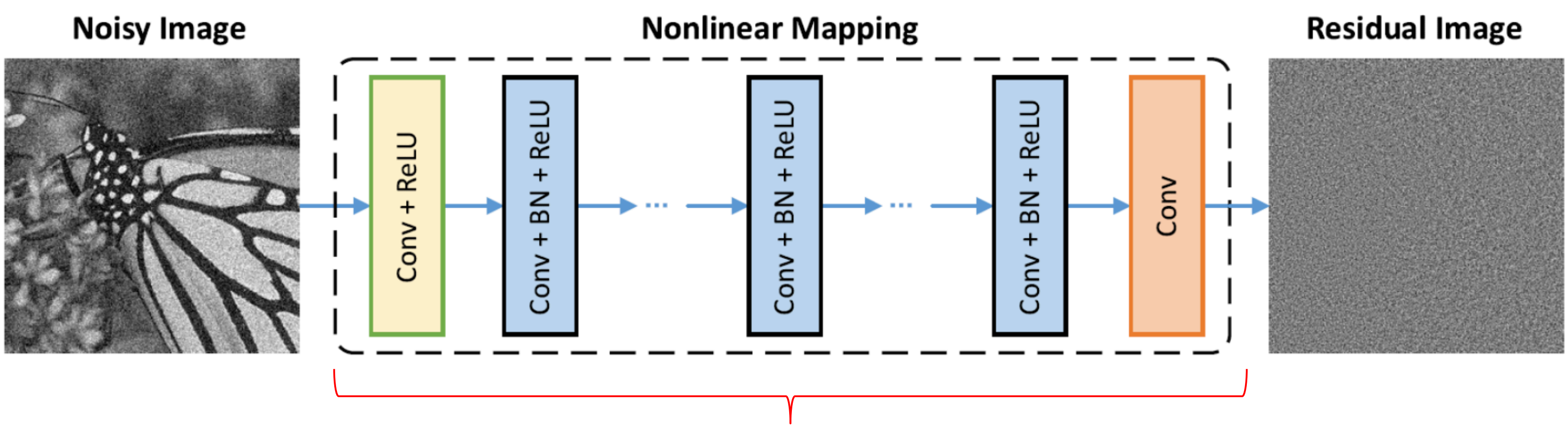
- Analytically defined filtering:
  - Use pre-defined **3D Gaussian Derivative Filter** to replace CNN that needs to be learned from data
- Refer to the advantages of CNN multi-layer architecture
  - Layers interact via a recurrent connection (output feeds back to the input)



[13] Hadji, Isma, and Richard P. Wildes. "A spatiotemporal oriented energy network for dynamic texture recognition." *Proceedings of the IEEE International Conference on Computer Vision*. 2017.

DnCNN

- Feed-forward denoising convolutional neural networks
- Deep CNN architecture
  - Include 17 CNN layers and more than 0.5 M parameters
  - Poor performance in cases of statistical mismatches (training vs. test)

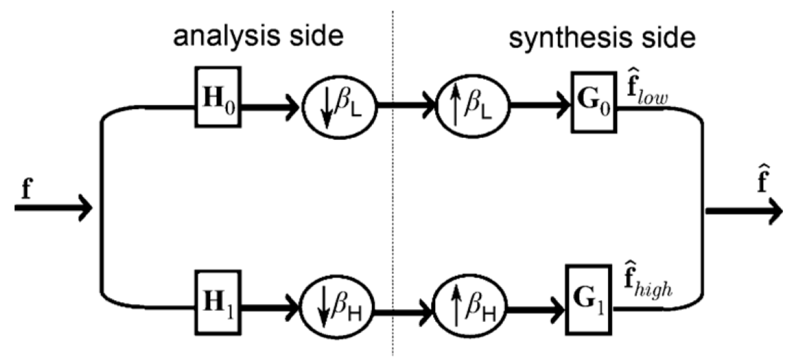


Employ 17 CNN layers

[2] K. Zhang, W. Zuo, Y. Chen, D. Meng, and L. Zhang, “Beyond a Gaussian denoiser: Residual learning of deep CNN for image denoising,” IEEE Transactions on Image Processing, vol. 26, no. 7, pp. 3142–3155, 2017.

GraphBio : Biorthogonal Wavelet Filterbanks

- Design a biorthogonal pair of graph-wavelets that can have compact spatial spread
- Exact reconstruction with polynomial filter (compact support)
  - Aliasing Cancellation
  - Perfect Reconstruction



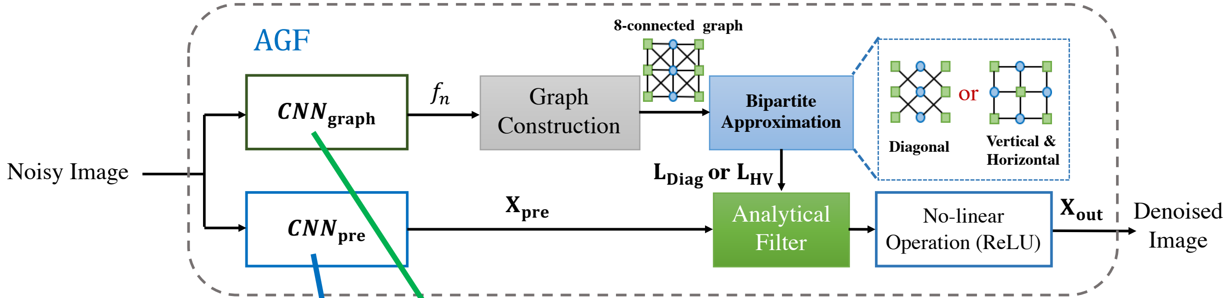
$$\mathbf{T} = \overset{\text{Low pass}}{\boxed{\frac{1}{2}\mathbf{G}_0(\mathbf{I}+\mathbf{J}_\beta)\mathbf{H}_0 +}}\overset{\text{High pass}}{\boxed{\frac{1}{2}\mathbf{G}_1(\mathbf{I}-\mathbf{J}_\beta)\mathbf{H}_1}}$$

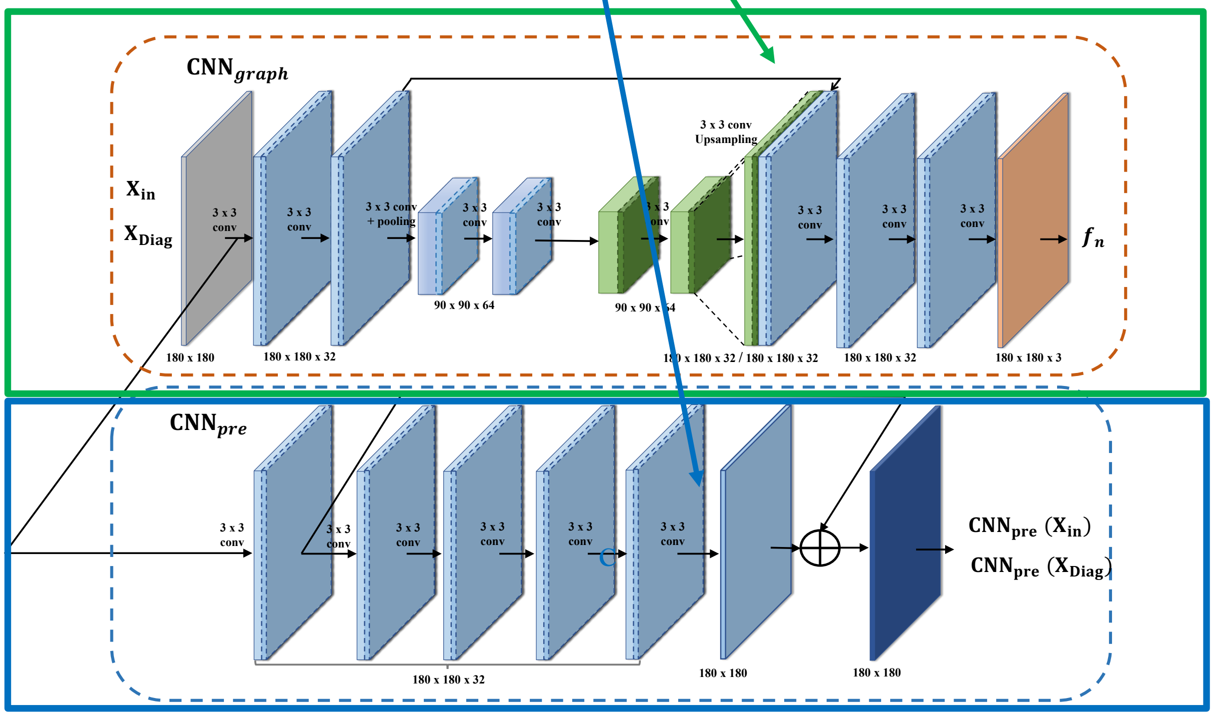
$$= \underbrace{\frac{1}{2}(\mathbf{G}_0\mathbf{H}_0 + \mathbf{G}_1\mathbf{H}_1)}_{\mathbf{T}_{eq}} + \underbrace{\cancelto{0}{\frac{1}{2}(\mathbf{G}_0\mathbf{J}_\beta\mathbf{H}_0 - \mathbf{G}_1\mathbf{J}_\beta\mathbf{H}_1)}}_{\mathbf{T}_{alias}}, \quad (6)$$

Perfect Reconstruction Aliasing Cancellation

[14] S. K. Narang and A. Ortega, "Compact support biorthogonal wavelet filterbanks for arbitrary undirected graphs," IEEE Transactions on Signal Processing, vol. 61, no. 19, pp. 4673– 4685, 2013.

Architecture Design





➢ Use U-Net [27] to constructs an underlying graph

➢ Use the light weight CNNs for pre-filtering the image

[27] O. Ronneberger, P. Fischer, and T. Brox, "U-net: Convolu-tional networks for biomedical image segmentation," in Inter-national Conference on Medical Image Computing and Com-puter Assisted Intervention. Springer, 2015, pp. 234–24

Graph Construction

- For each patch, we construct a graph G to connect pixels in the patch for graph filtering [18]:

CNN_{graph} → Calculate the edge weight w_{ij}^{k} for each patch :

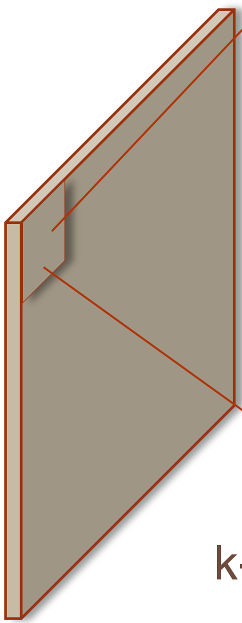
k-th patch Node i Node j

Exponential kernel:

$$w_{ij}^{k} = \exp\left(-\frac{\text{dist}(\boxed{k},\boxed{i},\boxed{j})}{2\epsilon^2}\right), 1 \leq k \leq \boxed{K},$$

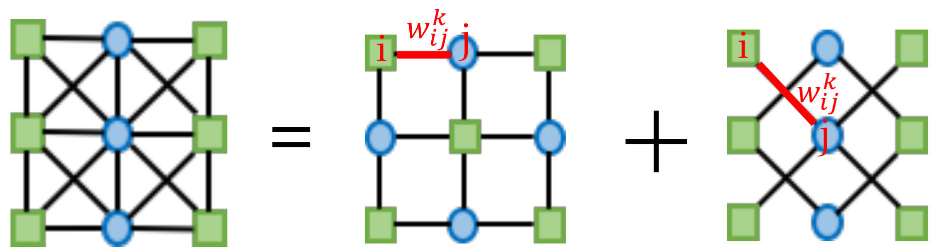
Feature distance :

$$\text{dist}(k,i,j) = \underbrace{\left(\mathbf{f}_i^k - \mathbf{f}_j^k\right)^\top \left(\mathbf{f}_i^k - \mathbf{f}_j^k\right)}_{\text{The output feature of } CNN_{graph}}, 1 \leq k \leq \boxed{K},$$



k-th patch

Noisy Image



8-connected graph **Vertical & Horizontal** **Diagonal** → **Bipartite Approximation**

[18] J. Zeng, J. Pang, W. Sun, and G. Cheung, "Deep graph Laplacian regularization for robust denoising of real images," in Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition Workshops, 2019, pp. 0–0.

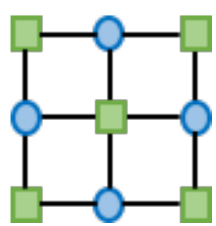
Bipartite Approximation

- GraphBio operate only on bipartite graphs
  - Spectral folding phenomenon when one partite is discarded [14]
  - Two bipartite graphs
    - **Vertical & Horizontal**
    - **Diagonal**

- Graph Laplacian L:

$$L = D - A$$

$$\{L_{HV}^{k}\}_{k=1}^{K} \qquad \{L_{Diag}^{k}\}_{k=1}^{K}$$



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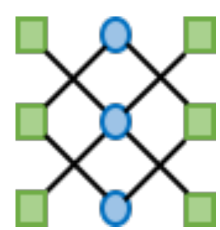

Vertical & Horizontal **Diagonal**

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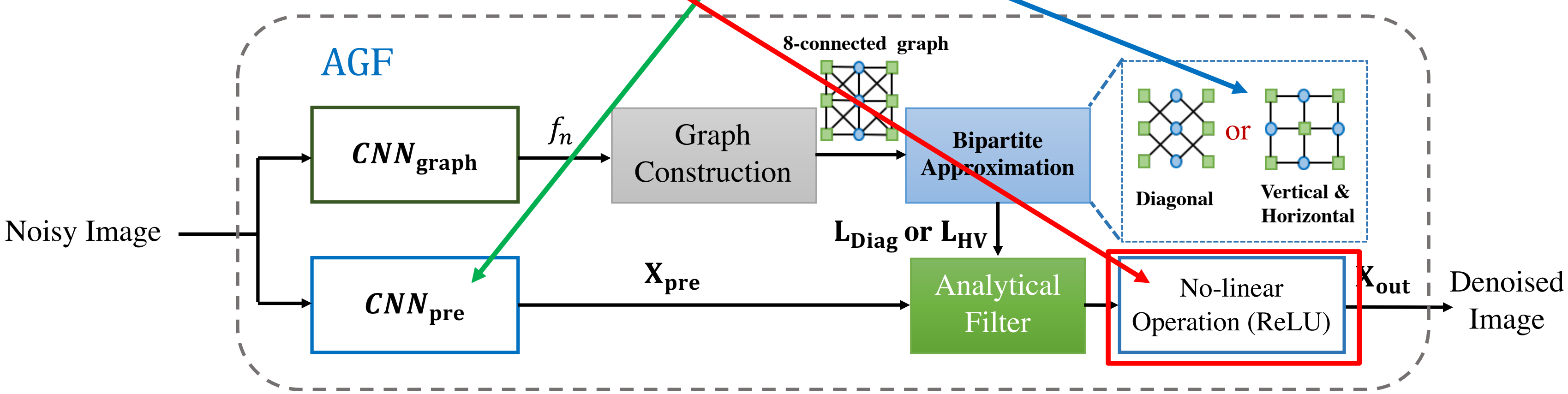
Analytical Graph Filter

- Using analytical graph filter (AGF), we compute its output, image denoised patches $\boldsymbol{X}^k_{\text{out}}$, as

$$\mathbf{X}^k_{\text{out}} = \sigma[F(\mathbf{L}^k_{\text{HV}}, \text{CNN}_{\text{pre}}(\mathbf{X}^k_{\text{Diag}}))], 1 \leq k \leq K, \quad (3)$$

$$\mathbf{X}^k_{\text{Diag}} = \sigma[F(\mathbf{L}^k_{\text{Diag}}, \text{CNN}_{\text{pre}}(\mathbf{X}^k_{\text{in}}))], 1 \leq k \leq K, \quad (4)$$

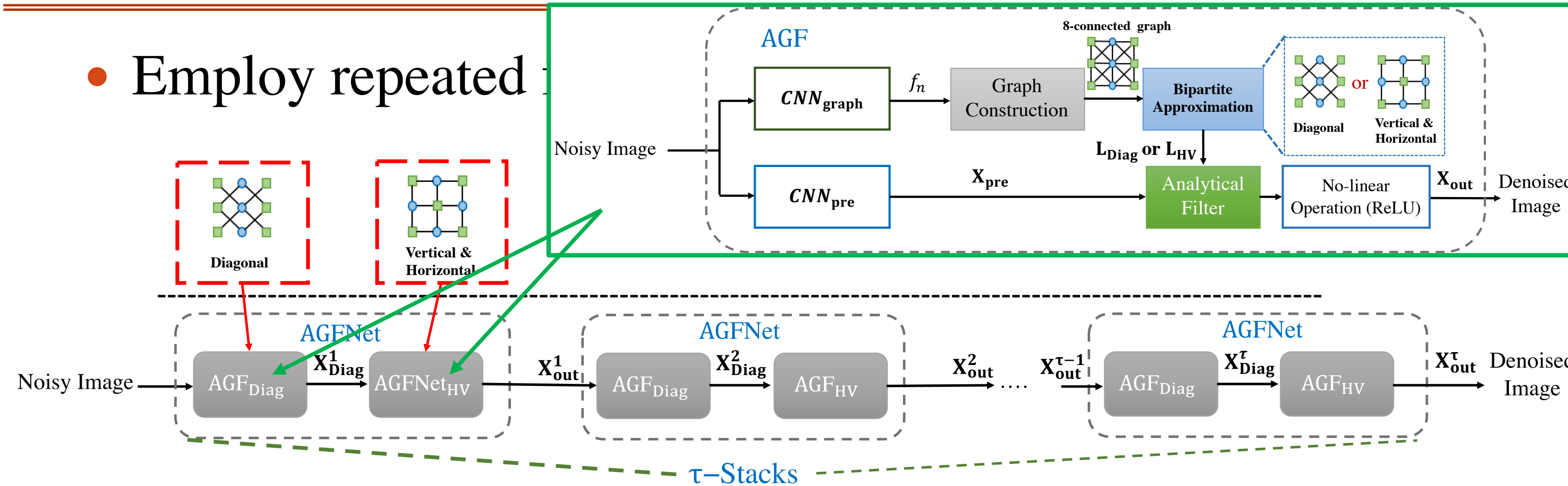
- ➢ **Use light weight CNNs for pre-filtering the image and tuning the weight parameters**
- ➢ **Using a different bipartite graph**



- ➢ No-linear operation increases model complexity

Repeated Analytical Graph Filter

- Employ repeated



- The objective function of DeepAGFNet framework can be defined as the mean squared error (MSE) :

$$\mathbf{L}_{MSE}(\mathbf{X_{gt}}, \mathbf{X}^{\tau}_{\mathbf{out}}) = \frac{1}{HW} \sum_{i=1}^{H} \sum_{j=1}^{W} \big(\underbrace{\boxed{\mathbf{X_{gt}}(i,j)}}_{\text{ground-truth image}} - \underbrace{\boxed{\mathbf{X}^{\tau}_{\mathbf{out}}(i,j)}}_{\text{denoised image,}}\big)^2,$$

Experimental Result

- Training Setting:
  - 400 gray-scale images of size 180 x 180 provided by [2]
  - Patch size to 24 x 24
  - 74k patches for training
  - Batch size : 32
  - 200 epochs

- Testing Set:
  - Set12 with sizes of 256 x 256 or 512 x 512

[2] K. Zhang, W. Zuo, Y. Chen, D. Meng, and L. Zhang, "Beyond a Gaussian denoiser: Residual learning of deep CNN for image denoising," IEEE Transactions on Image Processing, vol. 26, no. 7, pp. 3142–3155, 2017.

PSNR Comparison

Name	Method ($\sigma = 50$)				
	BM3D [10]	WNNM [11]	OGLR [12]	DnCNN-S [2]	DeepAGF
Cman	26.15	26.42	25.93	26.99	26.74
House	29.66	30.44	29.4	30.15	30.02
Peppers	26.69	26.93	26.55	27.24	27.16
Starfish	24.93	25.36	24.8	25.73	25.54
Monarch	25.78	26.17	25.62	26.86	26.90
Airplane	25.03	25.36	24.97	25.92	25.67
Parrot	25.81	26.09	25.78	26.49	26.33
Lena	29.05	29.23	28.84	29.34	29.37
Barbara	27.21	27.78	27.13	26.20	25.89
Boat	26.64	26.88	26.58	27.13	27.06
Man	26.81	26.85	26.62	27.18	27.15
Couple	26.47	26.64	26.38	26.81	26.78
Avg.	26.69	27.01	26.55	27.17	27.05

Table 1. Set12 PSNR (dB)

Although our proposed method is not the best, the CNN architecture employs only six layers for pre-filtering, which is small compared to the top performing DnCNN (17 layers)

Model Comparison

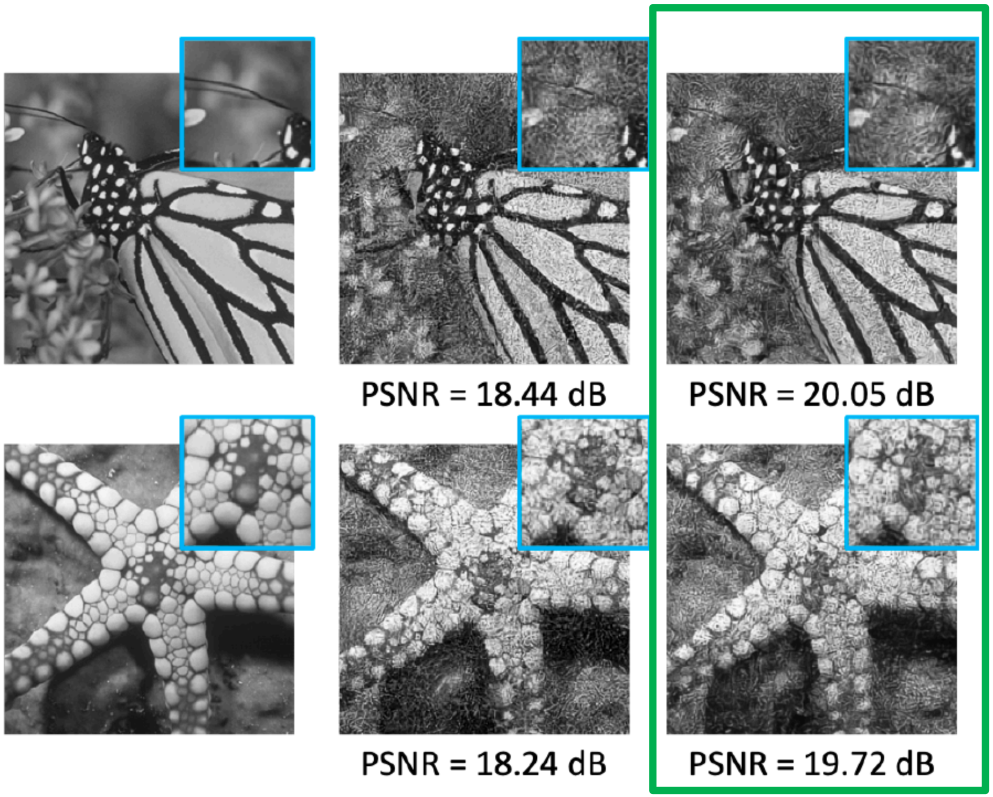
	DnCNN-S	DeepAGF
Parameters	0.55 M	0.32M

Table 2. Parameter count comparison for different methods

Our GNN model saves more than 40% parameters

Mismatch Case

- The case of statistical mismatch between training ($\sigma = 50$) and testing data ($\sigma = 70$)



Fewer artifacts and smoother results without loosing important Detail (our proposed outperforms DnCNN-S by more than 1dB in PSNR)

Fig.4. Denoising results for Monarch and Starfish (noise level σ=70). Left to right: Original, DnCNN-S, DeepAGF.

Conclusion

- Proposed a new Graph Neural Net (GNN) architecture for image denoising which employs an analytical graph wavelet filter
- Compared to conventional CNNs, our architecture has fewer degrees of freedom , i.e., only for graph learning
- Our GNN outperforms competing CNNs by more than 1dB when the statistics between training and testing data differ