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BROADBAND HYPERSPECTRAL PHASE RETRIEVAL FROM NOISY DATA

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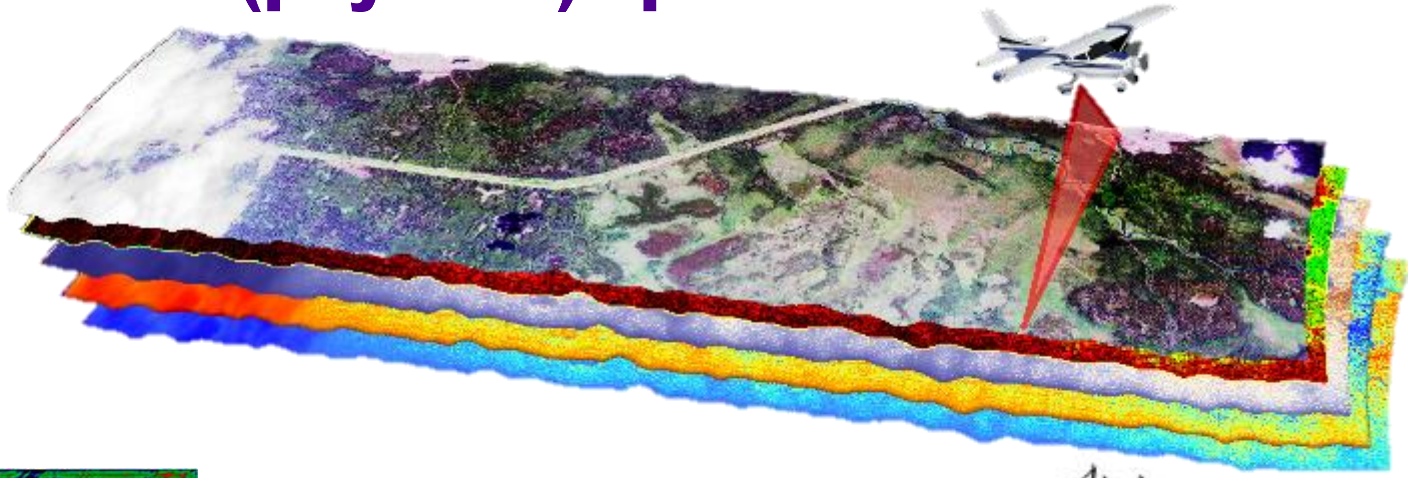
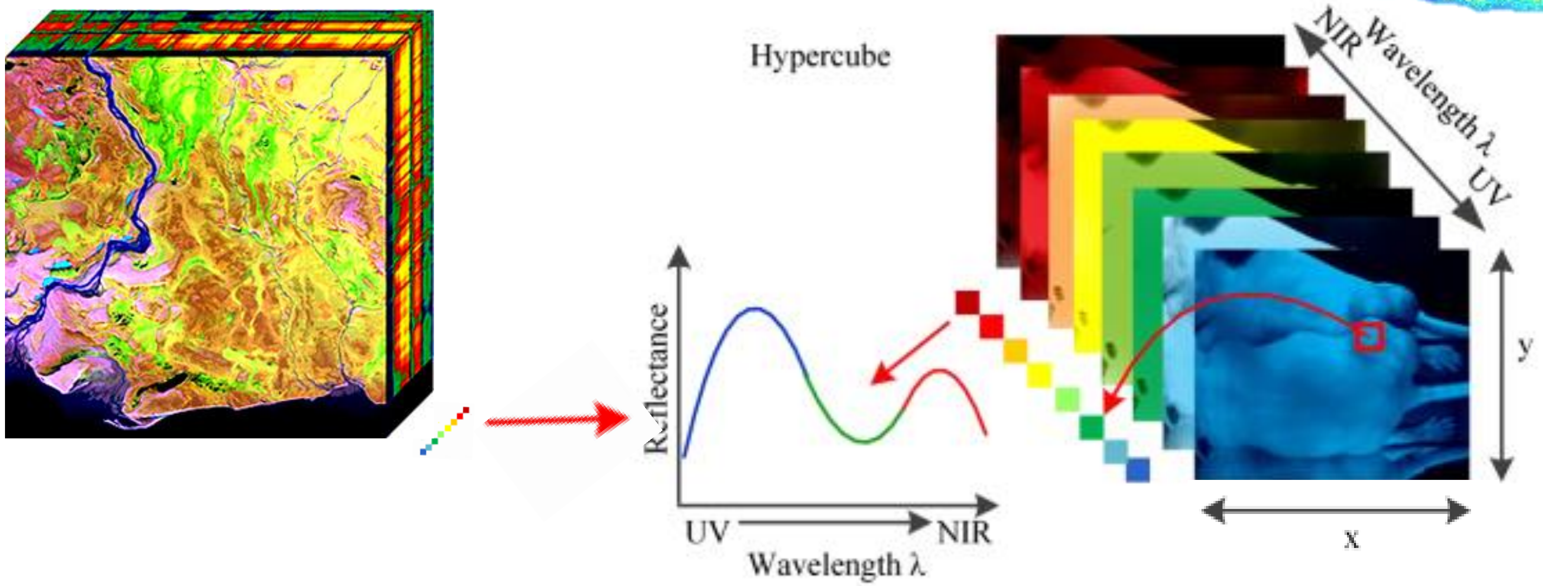
Outline

- Introduction
  - Hyperspectral Imaging
  - Fourier spectroscopy
  - Phase retrieval
- Mathematical background
  - Image and observation formation
  - Variational formulation of phase retrieval
  - Solutions
- Developed algorithm
- Test results
  - Simulation
  - Experiment

Hyperspectral Imaging (HSI)
Direct (physical) spectral measurements



AVIRIS - Airborne Visible InfraRed Imaging spectrometer



Typical HSI system employs filters for wavelength separation

*Chang, C. I. *Hyperspectral imaging: techniques for spectral detection and classification* (Vol. 1). Springer Science & Business Media, 2003
**Mehta N et.al. Single-Cell Analysis Using Hyperspectral Imaging Modalities. ASME. J Biomech Eng. 2018;140(2):020802

Hyperspectral Phase Imaging –indirect spectral measurements, Computational spectral analysis.

Phase, Why and What for ? **Fourier Spectroscopy**

(1) **Digital holography HSI with the reference beam**

$$J(t) = \int_0^\infty |A(\omega) + R\exp(2\pi\omega t)|^2 d\omega =$$

$$\int_0^\infty \left(|A(\omega)|^2 + R^2 + 2|A(\omega)| \times R \cos\left(\varphi_A(\omega) - 2\,\pi\omega t\right)\right) d\omega$$

where $A(\omega)$ is a complex-valued transfer function of the object (for transparent object) to be analyzed with the amplitude $|A(\omega)|$ and the phase $\varphi_A(\omega)$, $A(\omega) = |A(\omega)|\exp(j\ \varphi_A(\omega))$.

(2) **Phase retrieval HSI without the reference beam**

$$J(t) = \int_0^\infty |A(\omega) + A(\omega)\exp(2\pi\omega t)|^2 d\omega =$$

$$2\int_0^\infty |A(\omega)|^2 \left(1 + \cos\left(2\pi\omega t\right)\right) d\omega.$$

Problem formulation (1)

1. **Object to be reconstructed**:

$\boldsymbol{U_0(x,y,k)} \in \mathbb{C}^{\boldsymbol{n\times m}}$ is a 2D slice of 3D cube $\boldsymbol{Q_K(x,y)} = \{\boldsymbol{U_0(x,y,k), k} \in \boldsymbol{K}\}$.
The total size of the cube is $n \times m \times l_K$,
where $\boldsymbol{k}$ denotes the spectral components of the length l_K.

2. **Intensity measurements**: $$\boldsymbol{Y_t} = \sum_{\boldsymbol{k}\in\boldsymbol{K}} \left|\boldsymbol{U_{t,k}}\right|^2, \qquad \boldsymbol{U_{t,k}} = \boldsymbol{A_{t,k}\, U_{o,k}}, \quad \boldsymbol{t} \in \boldsymbol{T}.$$

Here and what follows, we use the vectorized representation for slices $\boldsymbol{U_{0,k}} \in \mathbb{C}^{\boldsymbol{N}}$, $\boldsymbol{N} = \boldsymbol{n} \times \boldsymbol{m}$, and $\boldsymbol{A_{t,k}} \in \mathbb{C}^{\boldsymbol{M\times N}}$ are linear operators of image formation object images from the object plane to the sensor plane.

3. **HS phase retrieval problem**:

Reconstruction of the complex-valued cube $\boldsymbol{Q_K(x,y)}$ from noisy intensity observations:

$$\boldsymbol{Z_t} = \boldsymbol{Y_t} + \boldsymbol{\varepsilon_t}\,,\ \boldsymbol{t} \in \boldsymbol{T}.$$

Problem formulation (2)

We restrict the class of the operators $\boldsymbol{A_{t,k}}$ to the form appeared in Fourier Spectroscopy with the measured intensities $\boldsymbol{Y_t}$ of the from:

$$\boldsymbol{Y_t} = \sum_{\boldsymbol{k \in K}} \left| \boldsymbol{A_{t,k}} \, \boldsymbol{U_{o,k}} \right|^2, \qquad \boldsymbol{A_{t,k}} = \left(1 + e^{-\frac{2\pi}{N} jkt}\right) \boldsymbol{A_k}, \ \boldsymbol{t} \in \boldsymbol{T}.$$

For noisy intensity observations $\boldsymbol{Z_t} = \boldsymbol{Y_t} + \boldsymbol{\varepsilon_t}, \boldsymbol{t} \in \boldsymbol{T}$, where $\boldsymbol{\varepsilon_t} \sim \boldsymbol{N}(0,\sigma)$

This intensity can be presented in the form

$$\boldsymbol{Y_t} = 2 \sum_{\boldsymbol{k \in K}} \left(1 + \boldsymbol{cos}\left(\frac{2\pi}{N} \boldsymbol{kt}\right)\right) |\boldsymbol{B_k}|^2, \boldsymbol{t} \in \boldsymbol{T},$$

where $\boldsymbol{B_k} = \boldsymbol{A_k} \, \boldsymbol{U_{o,k}}$ and $\boldsymbol{N}$ is a number of experiments on varying $\boldsymbol{t}$.

Algorithm development (1)

Criterion

$$J \triangleq \frac{1}{\sigma^2}\sum_{t=0}^{N-1}\left\|Z_t - 2\sum_{k=0}^{\frac{N}{2}-1}|B_k|^2\left(1+cos\left(\frac{2\pi}{N}kt\right)\right)\right\|_2^2 + 1/\gamma\sum_{k=1}^{\frac{N}{2}-1}\|B_k - A_k U_{o,k}\|_2^2 + f_{reg}\left(\{U_{o,k}\}_1^{\frac{N}{2}-1}\right).$$

The first summand is a fidelity term for Gaussian noise.
The last one is a penalty formalizing a non-local patch-wise complex-domain sparsity for the object cube images.

The second summand penalizes residuals between the splitting variable B_k and $A_k U_{o,k}$.

The developed algorithm of iterates $min_{\{B_k\}} J$ on B_k provided given $\{U_{o,k}\}_1^{\frac{N}{2}-1}$ and $min_{\{U_{o,k}\}} J$ provided given $\{B_k\}$.

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Algorithm development (2)

(1) Minimization on B_k

For minimization $\min_{\{B_k\}} J$, we solve the equations $\frac{\partial J}{\partial B_l^*} = 0, \quad l = 1, \dots, \frac{N}{2} - 1$

$$\left[-\frac{4}{\sigma^2}\sum_{t=0}^{N-1} Z_t \cos\left(\frac{2\pi}{N}kt\right) + \frac{4N}{\sigma^2}|B_l|^2 + \frac{1}{\gamma}\right] B_l = \frac{1}{\gamma} A_l U_{o,l}.$$

With the solution for B_l of the form for the phase and amplitude, respectively:

$$\varphi_{B_l} = \varphi_{A_l U_{o,l}};$$

Non-negative solutions of the cubic polynomial equations calculated by Cardano formulas:

$$\left[-\frac{4}{\sigma^2}\sum_{t=0}^{N-1} Z_t \cos\left(\frac{2\pi}{N}kt\right) + \frac{4N}{\sigma^2}|B_l|^2 + \frac{1}{\gamma}\right] |B_l| - \frac{1}{\gamma}|A_l U_{o,l}| = 0.$$

Important features of this solution:

(1) It produces the spectral analysis giving the complex-valued spectral estimates from intensity measurements;
(2) Filters the Gaussian noise in observations (can be done for Poissonian noise);
(3) This solution can be interpreted as an original **proximity operator** $B_\lambda^{(s)} = prox_{f\gamma}(A_l U_{o,l}), \quad l = 1, \dots, \frac{N}{2} - 1,$
where f stays for the fidelity term of the criterion J.

Algorithm development (3)

(2) Minimization $\min_{\{U_{o,k}\}} J$ defines regularization (filtering) of the HS cube in complex domain.

Instead of variational approach, we use the specially designed sparsity based filter for joint and separate filtering spectral slices of the HS object cube.

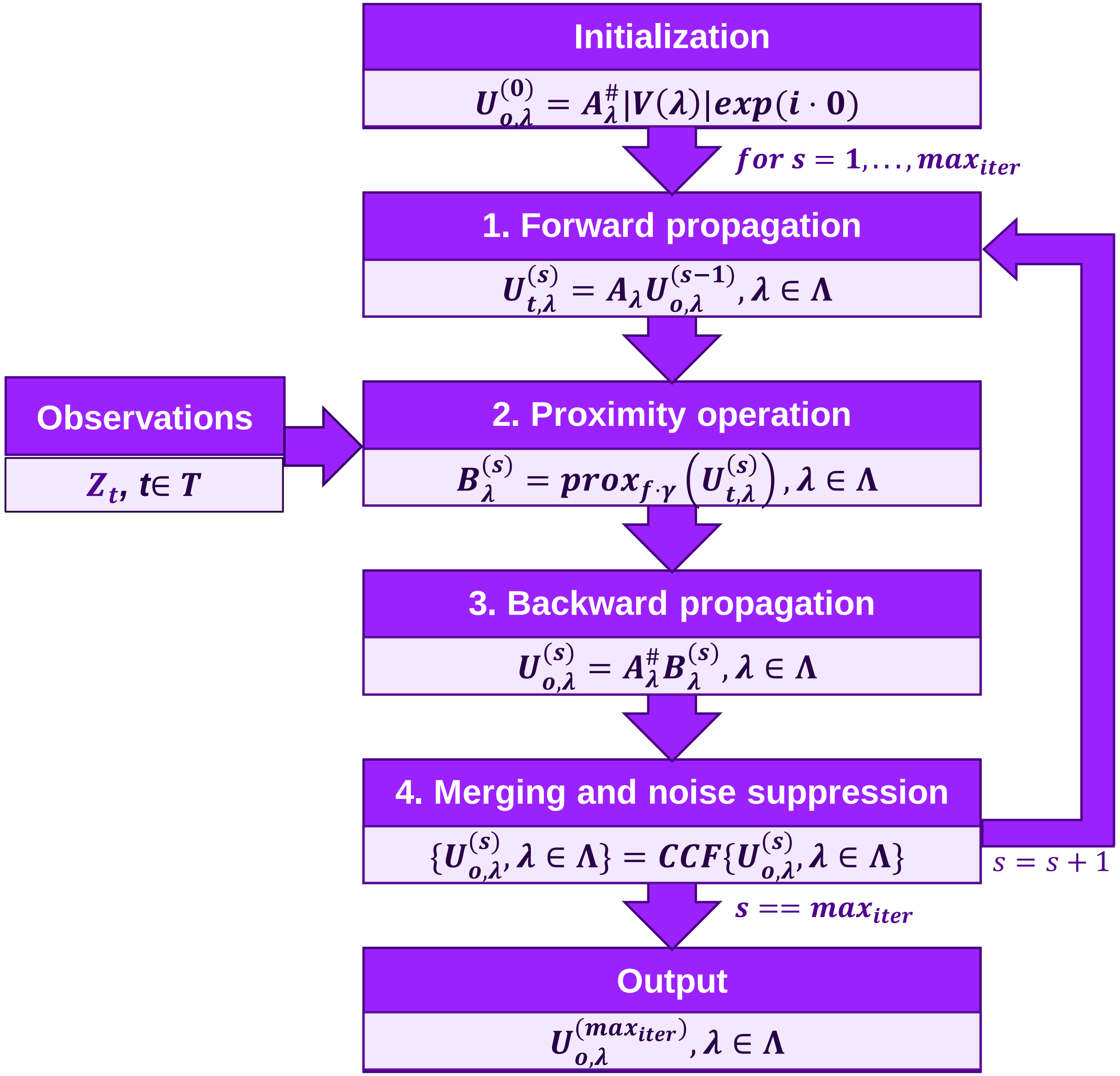
$$\{\hat{U}_{o,k}, k \in K\} = CCF\{U_{o,k}, k \in K\}$$

Complex domain Cube Filter (CCF) processes the cube data $\{U_{o,k}, k \in K\}$ jointly and provide the estimates $\{\hat{U}_{o,k}, k \in K\}$ for all k.

(a) The CCF algorithm is based on **SVD** of the HS cube which identifies an optimal subspace for the HS image representation including both the dimension of the eigenspace and eigenimages in this space.

(b) The Complex-Domain Block-Matching 3D (CDBM3D) algorithm filters this small number of eigenimages. Going from the eigenimage space back to the original image space we obtain the reconstruction of the object cube $\{\hat{U}_{o,k}, k \in K\}$.

Iterative HyperSpectral Phase Retrieval Algorithm



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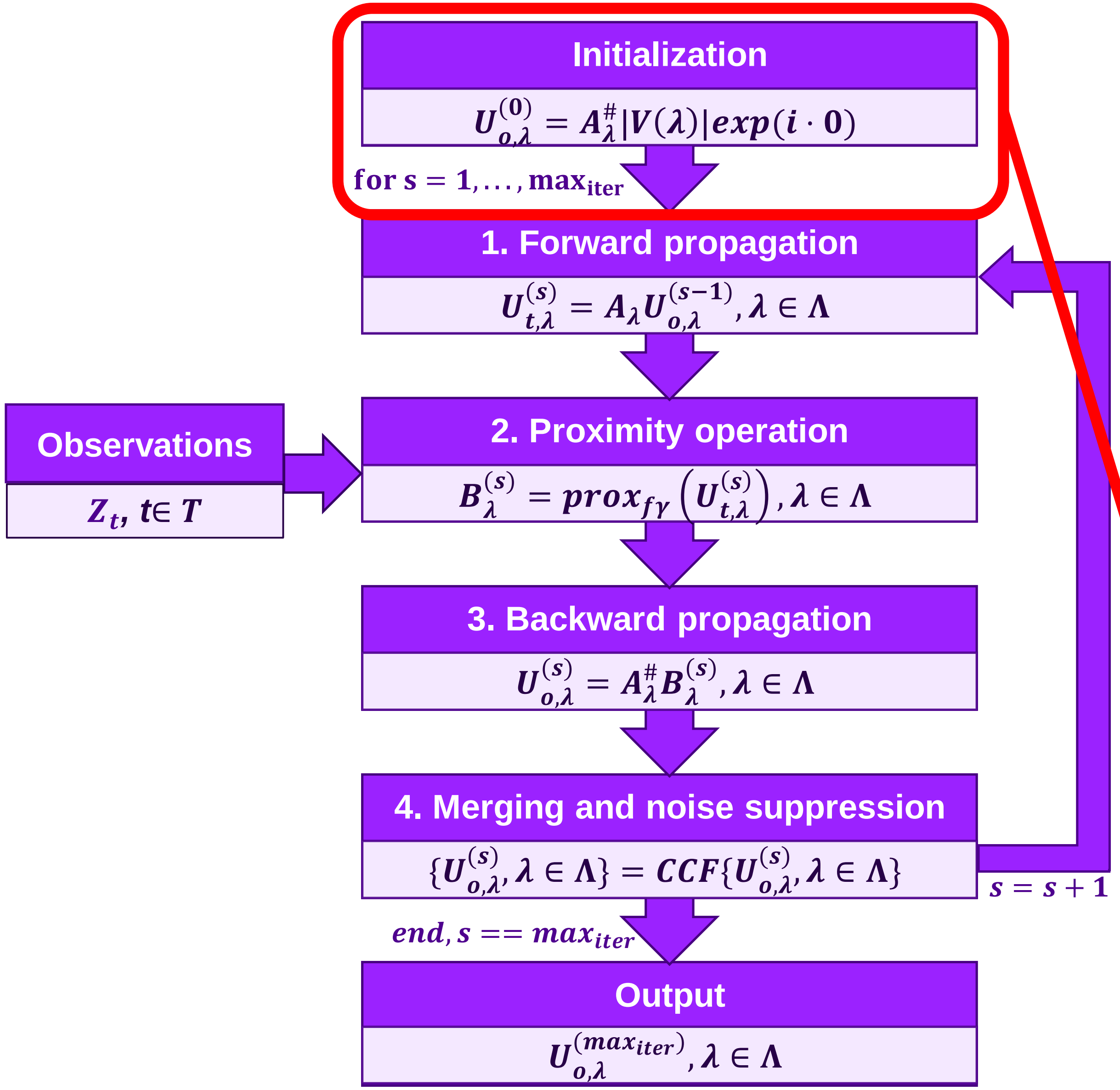
Iterative HyperSpectral Phase Retrieval Algorithm



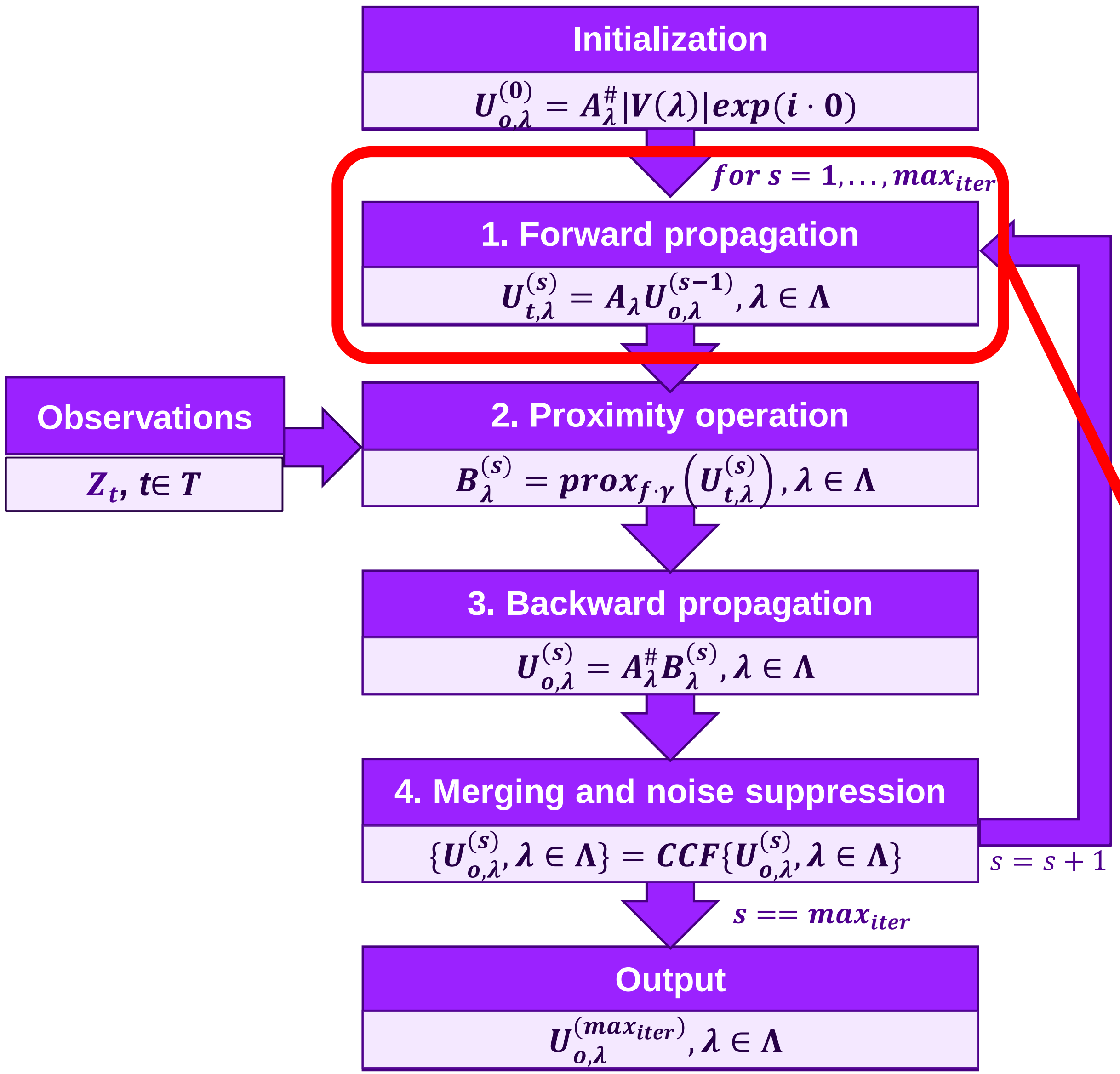
Initialization

Initial guess of object wavefronts

$$U_{o,\lambda}^{(0)}(x,y) = A_\lambda^{\#}\{|V(x,y,\lambda)|\}\, exp(i \cdot 0)$$

Here $U_{o,\lambda}^{(0)}$ - object wavefront, superscript (0) is for the initial iteration number;
subscripts o, λ are notations for object plane and wavelength, respectively;
$|V(x,y,\lambda)|$ (or $|V(\lambda)|$) is the intensity spectra of observations;
$A_\lambda^{\#}\{|V(x,y,\lambda)|\}$ – backward propagation of $|V(x,y,\lambda)|$ to the object plane;
superscript # stays for backward propagation;
$exp(i \cdot 0)$ - multiplier for the phase guess, which is zero in our case.

Iterative HyperSpectral Phase Retrieval Algorithm



1. Image Formation (Forward propagation)

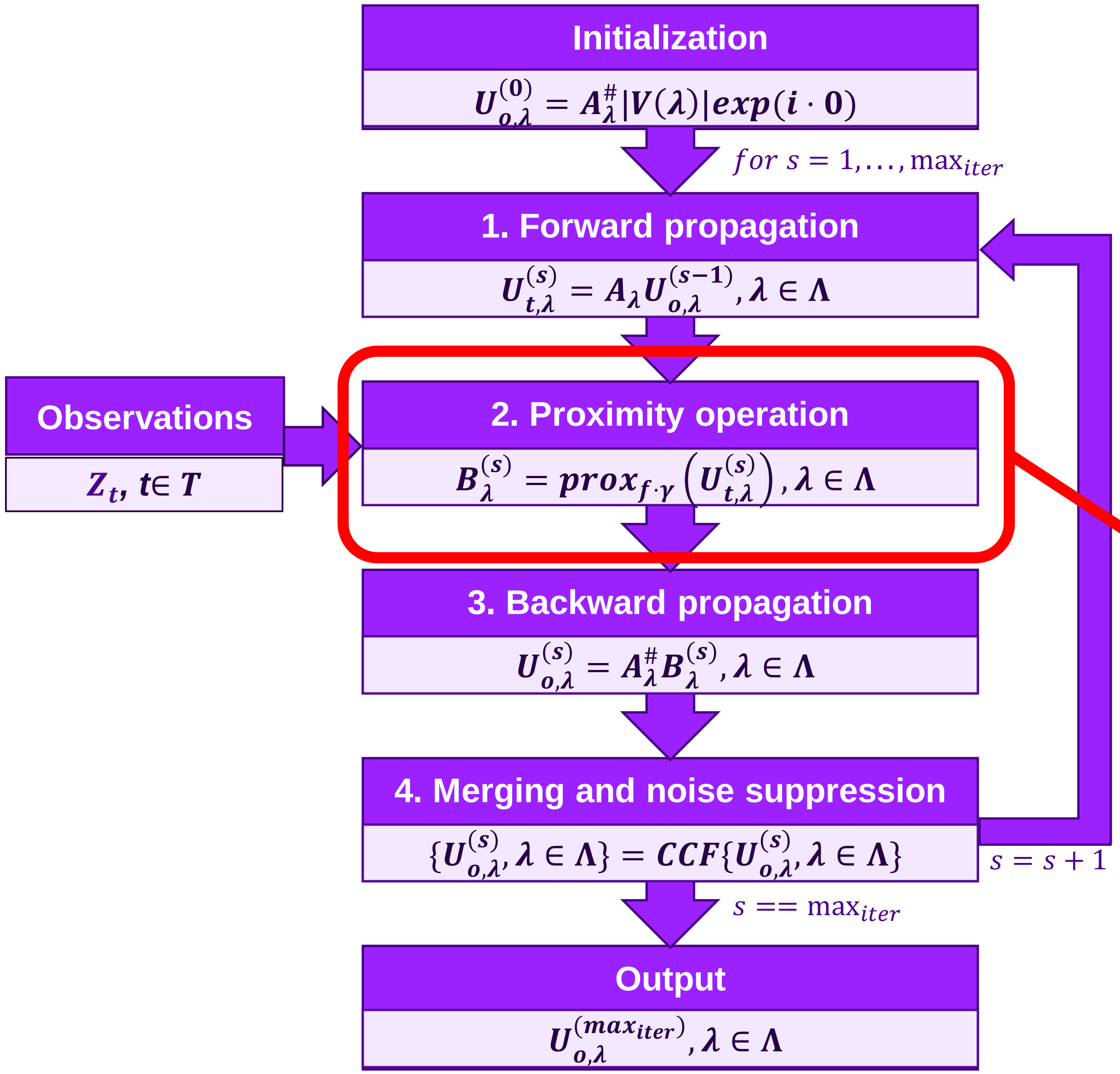
The Rayleigt-Sommerfeld integral solution)n in **Angular Spectrum (AS)** model is used:

$$U_{t,\lambda}^{(s)} = A_\lambda\left\{U_{o,\lambda}^{(s-1)}\right\} = \mathcal{F}^{-1}\left[H(f_x, f_y, d) \cdot \mathcal{F}\left\{U_{o,\lambda}^{(s-1)}\right\}\right]$$

$$H(f_x, f_y, d) = \begin{cases} exp\left[i\frac{2\pi}{\lambda}d\sqrt{1-\lambda^2(f_x^2+f_y^2)}\right], & f_x^2+f_y^2 \le \frac{1}{\lambda^2}, \\ 0 & , otherwise \end{cases}$$

$U_{o,\lambda}^{(s-1)}$ – object wavefront at obect's plane,
$U_{t,\lambda}^{(s)}$ – object wavefront at sensor's plane,
$\mathcal{F}$ and $\mathcal{F}^{-1}$ stay for the Fourier and inverse Fourier transforms,
d is a propagation distance,
f_x and f_y are spatial frequencies,
λ – wavelength.

Iterative HyperSpectral Phase Retrieval Algorithm



2. Proximity operation

$$B_{\lambda}^{(s)} = prox_{f\gamma}\left(U_{t,\lambda}^{(s)}\right)$$

where $\gamma > 0$ is a relaxational parameter and f stays for the minus log-likelihood part of the criterion:

$$\frac{1}{\sigma^2} \sum_{t=0}^{N-1} \left\| Z_t - 2 \sum_{k=0}^{\frac{N}{2}-1} |B_k|^2 \left(1 + \cos\left(\frac{2\pi}{N} kt\right)\right) \right\|_2^2$$

The proximity solution $B_{\lambda}^{(s)}$ resolves two problems:

1) Complex domain spectral components B^l are extracted from the intensity observations. Thus, we obtain the spectral analysis of the observed intensities averaged over the wavelengths.

2) The noisy observations are filtered with the power controlled by the parameter γ compromising the noisy observations Z^t and the power of the predicted signal $U_{t,\lambda}^{(s)}$ at the sensor plane.

Iterative HyperSpectral Phase Retrieval Algorithm

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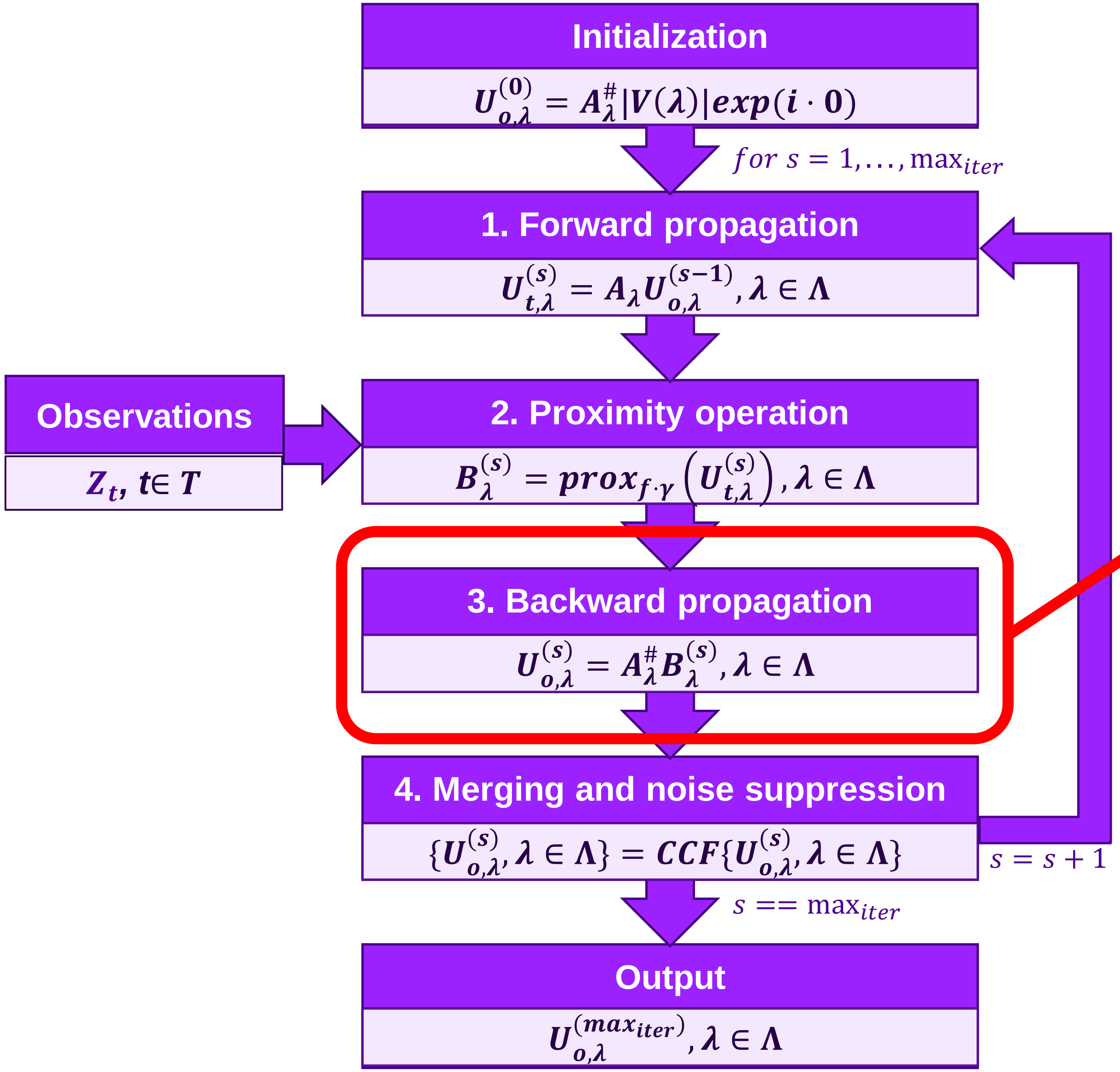


3. Backward propagation

Same Angular spectrum approach, but for backward propagation (#) the sign of the propagated distance is inverted.

Iterative HyperSpectral Phase Retrieval Algorithm

Initialization

$U_{o,\lambda}^{(0)} = A_{\lambda}^{\#}|V(\lambda)|exp(i \cdot 0)$

$for\ s = 1, \ldots, \max_{iter}$

1. Forward propagation

$U_{t,\lambda}^{(s)} = A_{\lambda} U_{o,\lambda}^{(s-1)}, \lambda \in \Lambda$

Observations

$Z_t, t \in T$

2. Proximity operation

$B_{\lambda}^{(s)} = prox_{f \cdot \gamma}\left(U_{t,\lambda}^{(s)}\right), \lambda \in \Lambda$

3. Backward propagation

$U_{o,\lambda}^{(s)} = A_{\lambda}^{\#} B_{\lambda}^{(s)}, \lambda \in \Lambda$

4. Merging and noise suppression

$\{U_{o,\lambda}^{(s)}, \lambda \in \Lambda\} = CCF\{U_{o,\lambda}^{(s)}, \lambda \in \Lambda\}$

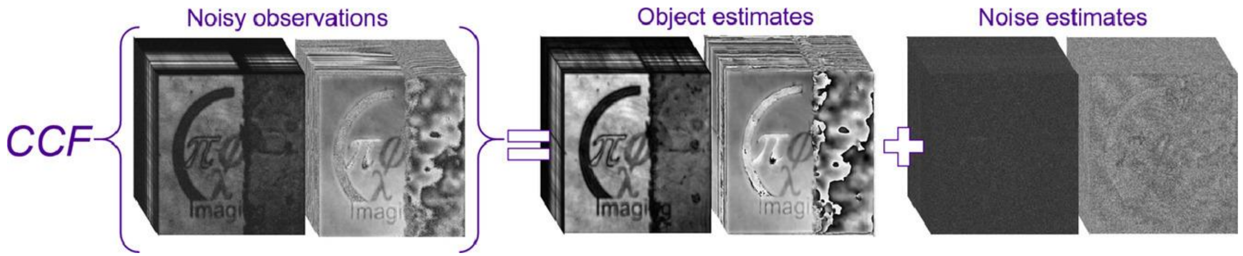
$s = s + 1$

$s == \max_{iter}$

Output

$U_{o,\lambda}^{(max_{iter})}, \lambda \in \Lambda$

4. Complex Cube Filtering (CCF) Algorithm



1. Transform data to a smaller subspace by SVD (Singular Value Decomposition);
2. Complex-Domain noise suppression by CDBM3D in this subspace;
3. Transform data back to original space.

*I. Shevkunov, et al. "Hyperspectral phase imaging based on denoising in complex-valued eigensubspace." *Optics and Lasers in Engineering* 127 (2020): 105973.
**V. Katkovnik, K. Egiazarian. "Sparse phase imaging based on complex domain nonlocal BM3D techniques." Digital Signal Processing 63 (2017): 72-85.

Simulation test (1)

$$U_o(x,y,\lambda) = A_0(x,y)e^{i\Delta\varphi_\lambda(x,y)}$$

$$\Delta\varphi_\lambda(x,y) = \frac{2\pi}{\lambda}(n_\lambda - 1)\cdot h(x,y), \quad A_0=1$$

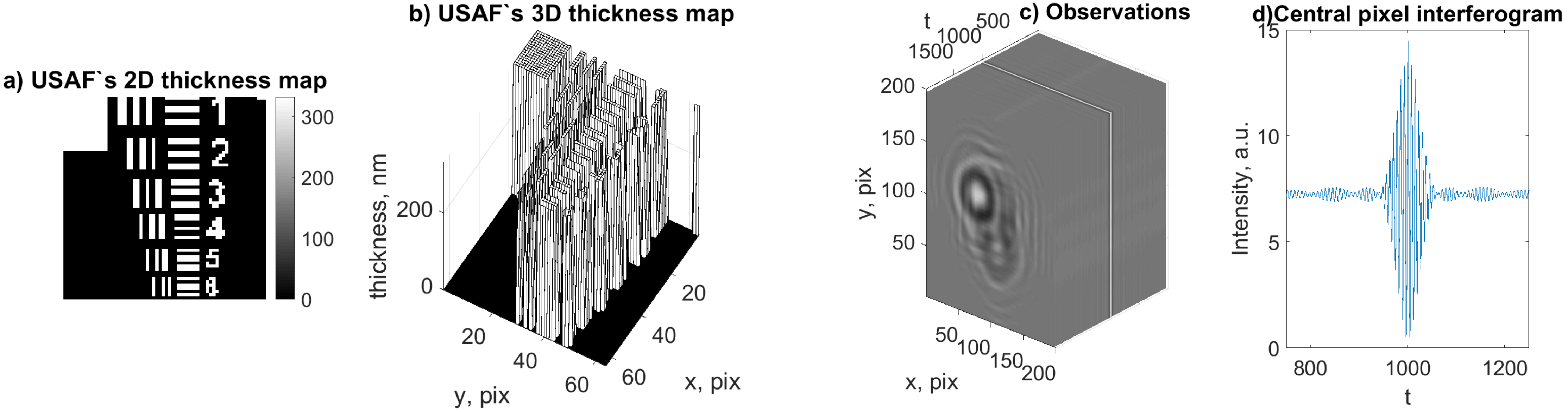
$$\varepsilon_\Lambda(\mathrm{x},\mathrm{y}) = \frac{\sigma}{\sqrt{2}}(\varepsilon_{re}(x,y) + i\varepsilon_{im}(x,y))$$

n_λ is a wavelength depending refractive index,

λ is a wavelength,

$h(x,y)$ is a thickness of the object.

250 wavelengths in the range of $\Lambda \in [450 : 900]$ nm, uniformly distributed laser spectra.
The beam goes through the object, propagates to the sensor and intensity observations are presented as 3D cube $Q_K(x,y)$ of the length $l_K = 250$.



Simulation test (2)

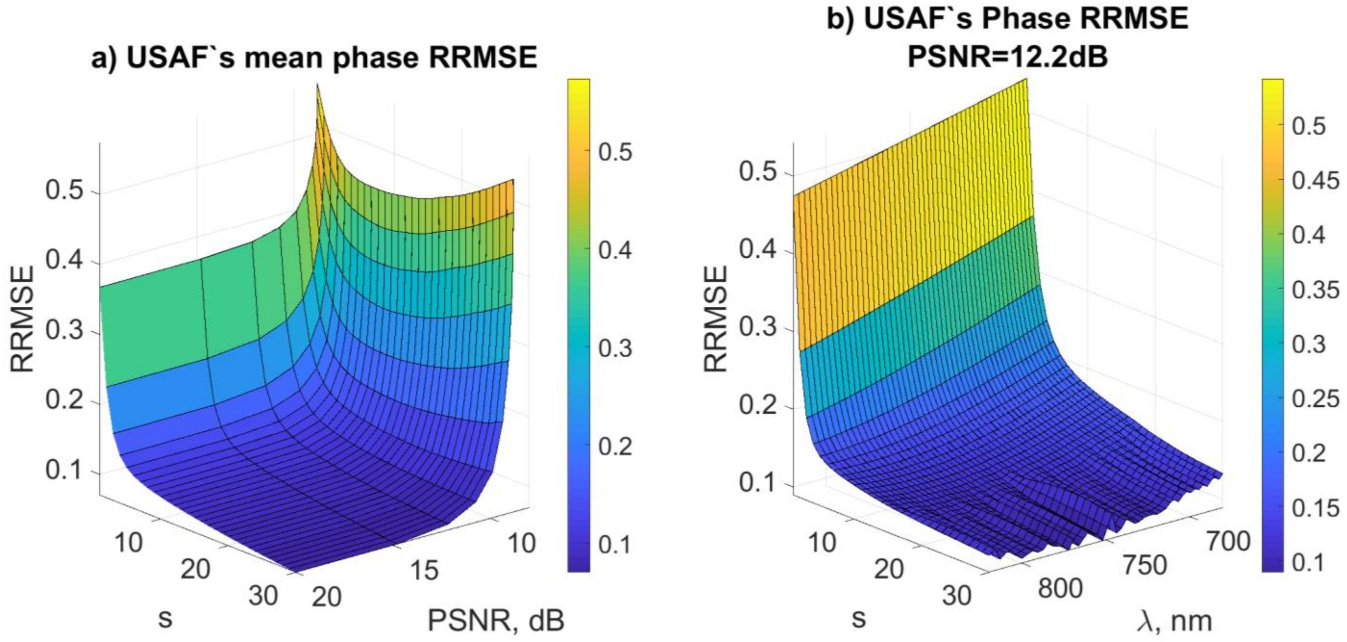


Fig. 1. RRMSE maps for the reconstructions in different noise conditions. (a) Map of RRMSE values averaged over λ as function of iteration number s and PSNR; (b) RRMSE for each λ depending on iteration number s for the case of PSNR= 12.2 dB.

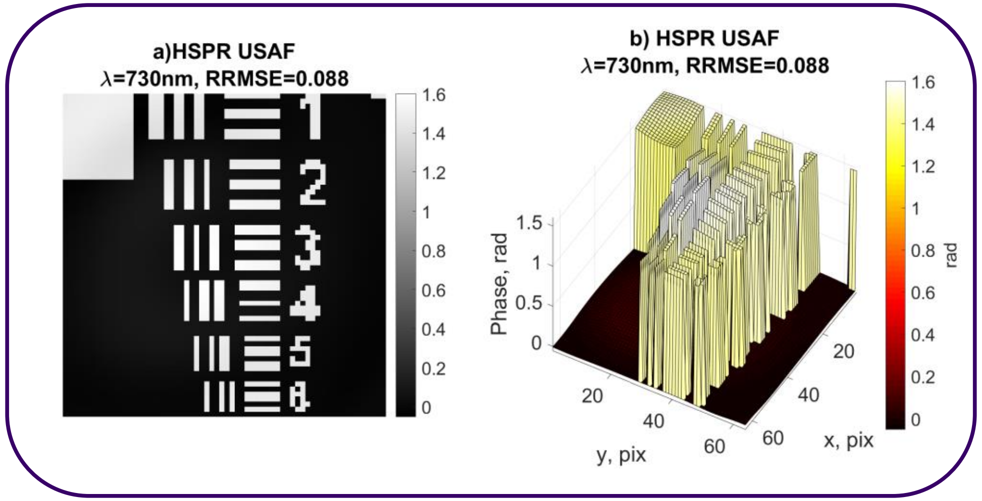


Fig. 2 Reconstruction of the object thickness obtained from the phase retrieval for the wavelength λ = 730 nm, σ = 2.5 and iteration number s = 30

$$\boldsymbol{RRMSE} = \frac{\sqrt{\sum_{x,y} |\hat{\varphi}(x,y) - \varphi(x,y)|^2}}{\sqrt{\sum_{x,y} |\varphi(x,y)|^2}}$$

Simulation test (3)

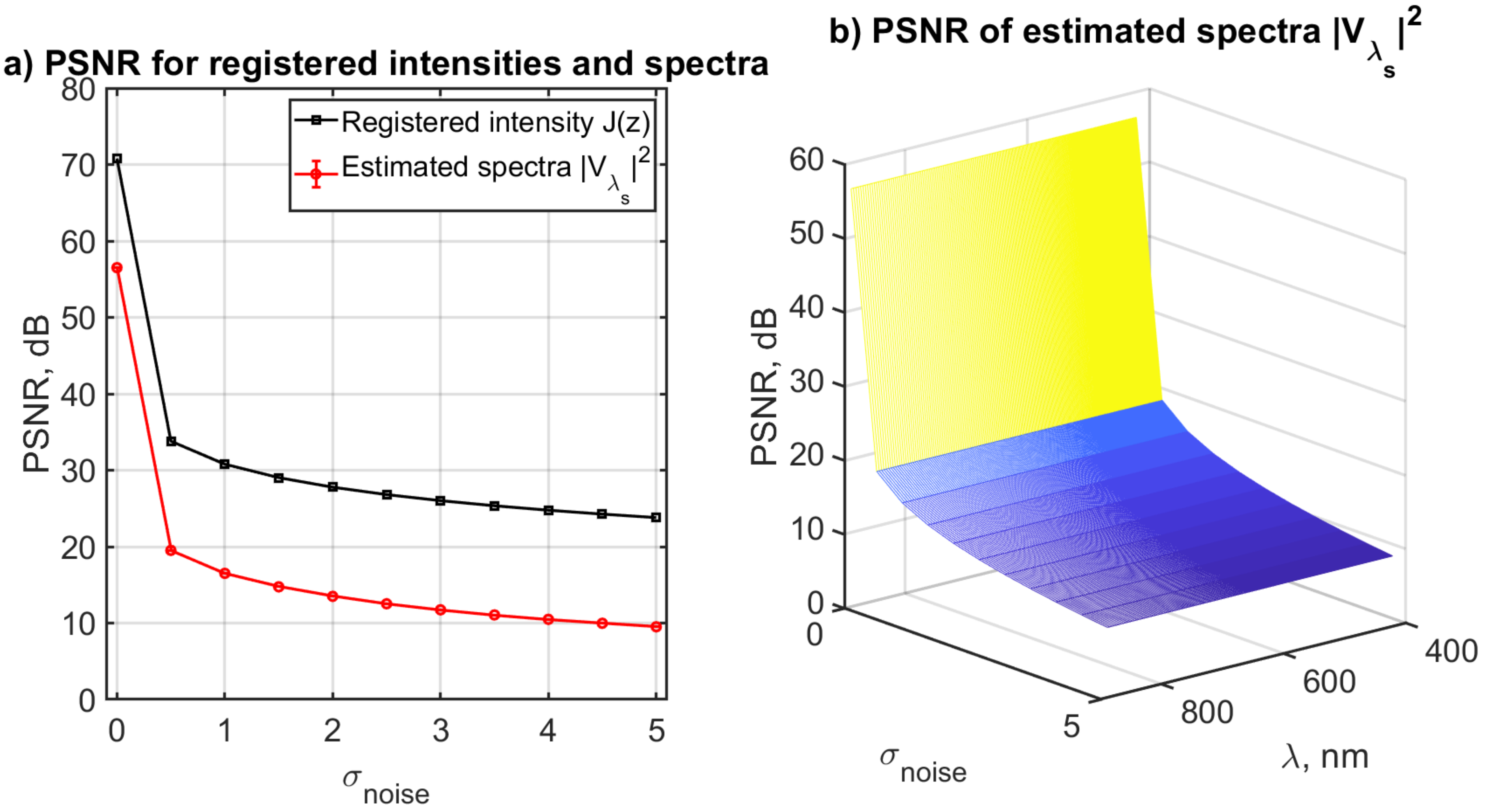


Fig. 1. (a) PSNR dependencies from noise standard deviation σ_{noise} for registered intensities $J(z)$, black squares curve, and for estimated spectra $|\boldsymbol{V}(\boldsymbol{\lambda})|^2$, red circles. (b) PSNR map of the estimated spectra for different σ_{noise} and wavelengths λ.

Experimental results, setup

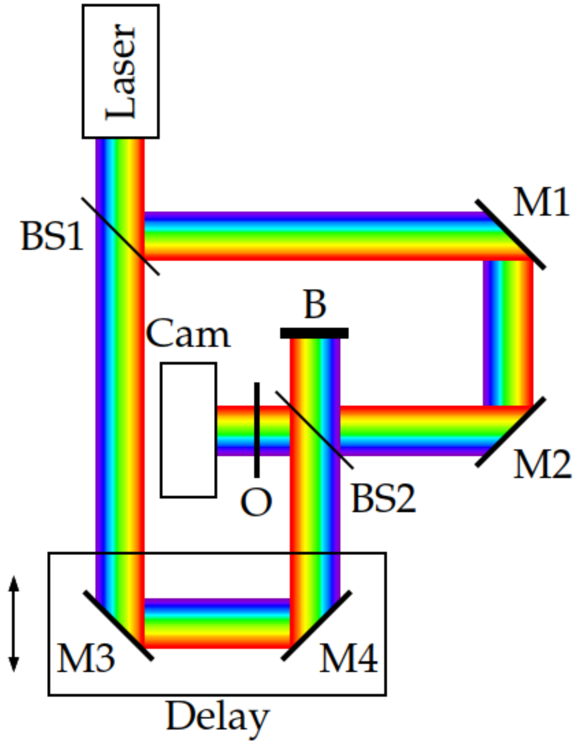


Fig.1 Setup for spectrally resolved digital holography with the supercontinuum laser source. a) Hyperspectral phase retrieval setup. BS1-2 are beamsplitters, M1-4 are mirrors, ``O'' is a transparent object, ``Cam'' is a registering sensor, ``B'' is a light blocker. ``Delay'' is a moving delay stage.

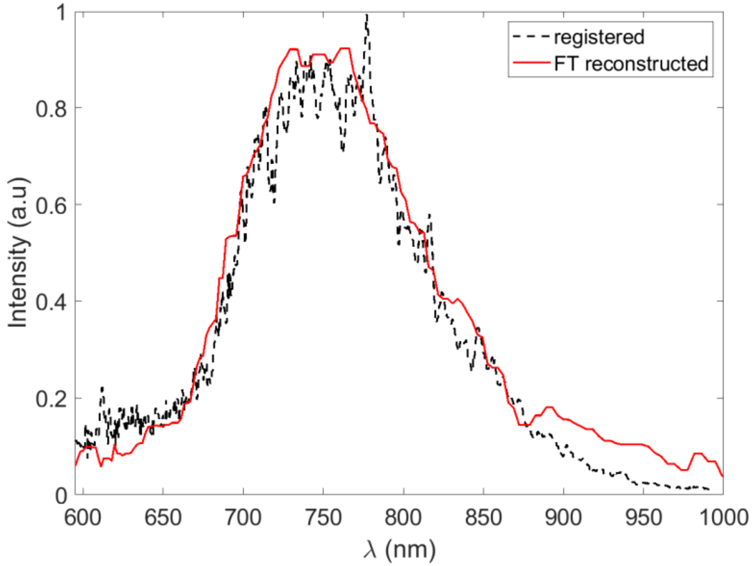
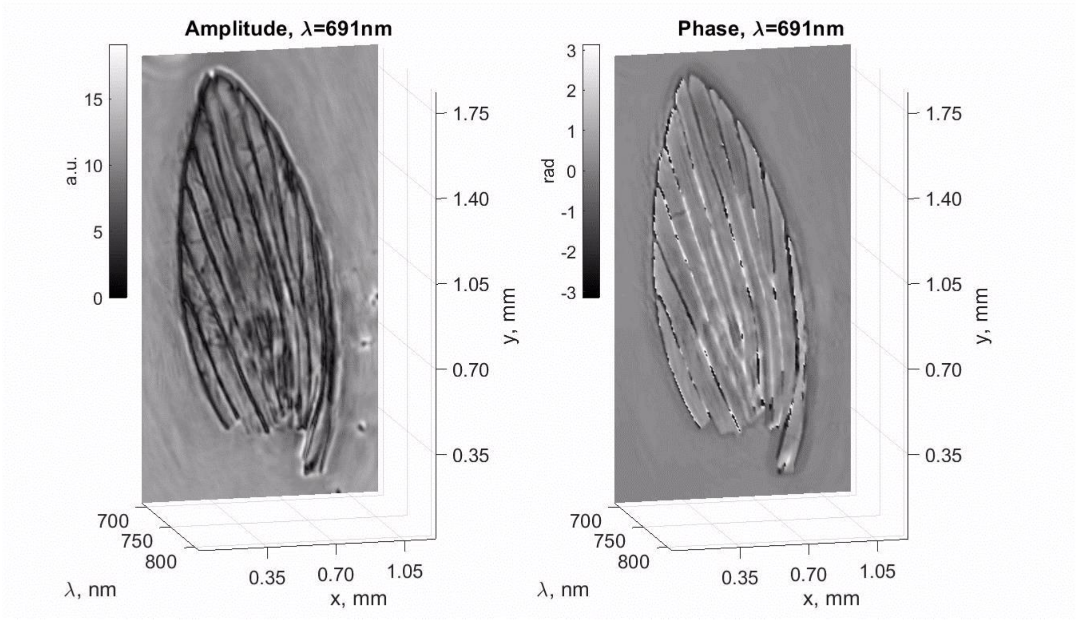


Fig.2 Used spectrum: a black dash curve is a registered spectrum by a spectrometer and multiplied by camera quantum efficiency, a red solid curve is for the Fourier transform reconstructed spectrum.

Experimental results

Object



Video 1. The amplitude and phase reconstructions.

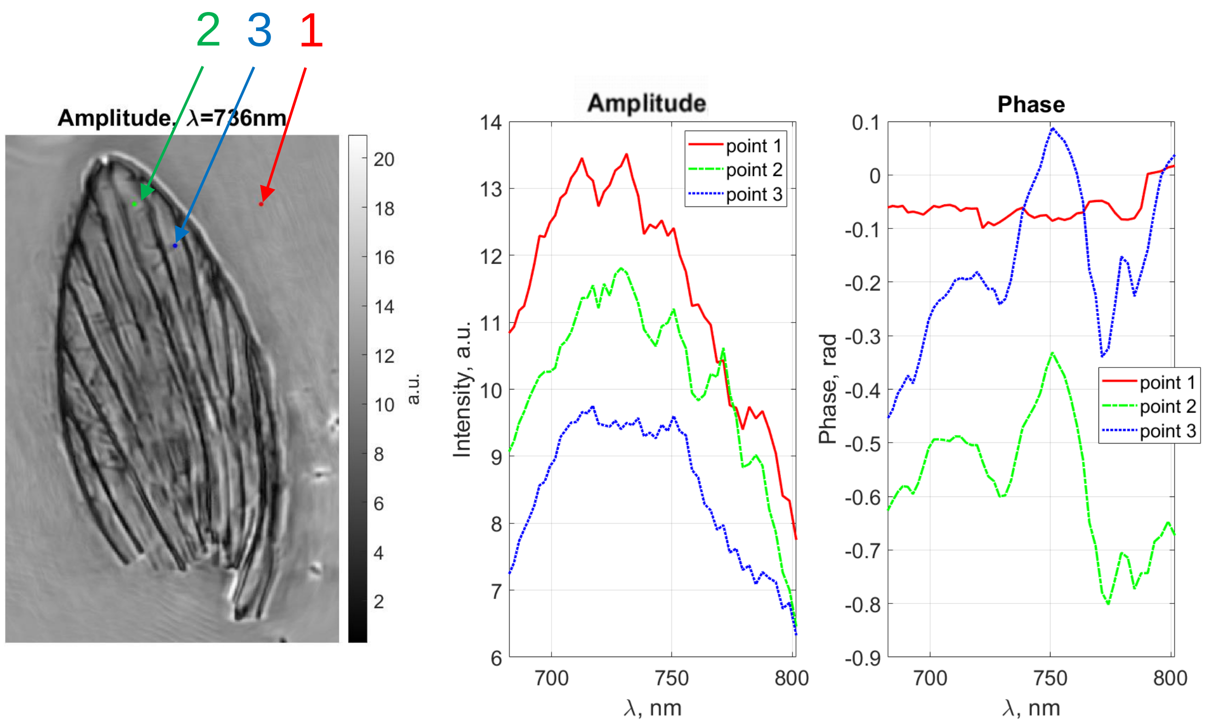


Fig 1.Spectra and phase for different points of the amplitude object image. The curves in the plots correspond to the points numbered in the amplitude image on the left.

Main Results

- A novel class of the HS phase retrieval problems is presented where both object and image formation operators are spectrally varying;

- The hyperspectral phase retrieval algorithm has been developed for Fourier Spectroscopy scenario which allows to reconstruct a complex amplitude of object in lensless optical setup;

- The developed iterative algorithm uses the original proximity spectral analysis operator and the HS sparsity modeling for complex-valued 3D cubes;

- With growing interest in hyperspectral imaging, we expect that the developed technique finds application in various bio- and medical tasks for non-invasive quantitative phase imaging.

Thank you for your attention

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