Optimal measurement budget allocation for particle filtering

Session: Multidimensional Signal Processing - Algorithms and Multidimensional Transforms

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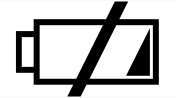
Introduction



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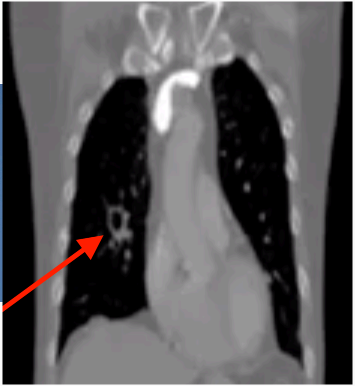

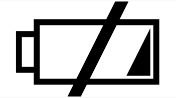

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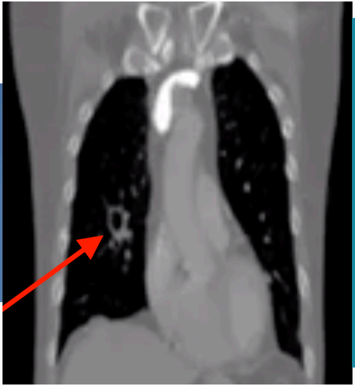
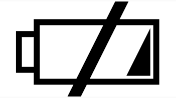




Contents

Intermittent particle filter

Dynamical system

$$
\begin{aligned}
x(t+1) &= f_t(x(t), w(t)) \quad \text{for} \quad t = 0, \ldots, T-1 \\
y(t) &= g_t(x(t), v(t)) \quad \text{for} \quad t \in \mathcal{M} \\
x(0) &\sim \mathcal{F}
\end{aligned}
$$

Random variables $x(0),\ w(t)$ and $v(t)$ follow known distributions

An intermittent particle filter estimates $x(t)$ from previous measurements $y(t)$:

$$\hat{x}_{\mathcal{M}}(t) = PF[\,y(\tau) \mid \tau \le t,\ \tau \in \mathcal{M}\,]$$

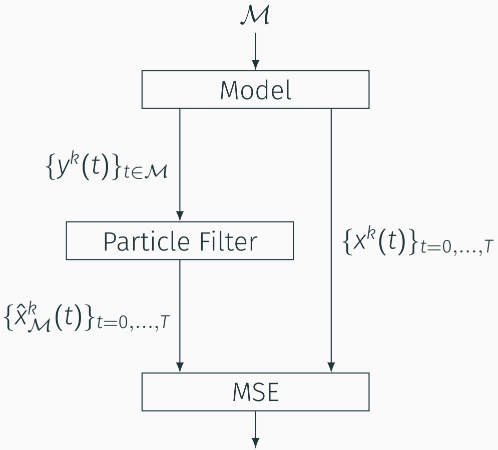
Optimal measurement times

Goal: Measurement time set $\mathcal{M}$ that minimizes the expected mean square error over the complete horizon

$$\min_{\mathcal{M}\subseteq\{0,\ldots,T\}} \mathbb{E}\left[\sum_{t=0}^{T} \|x(t) - \hat{x}_{\mathcal{M}}(t)\|^2\right] \text{ subject to } |\mathcal{M}| = N$$

Monte Carlo estimation

Repeat for $k = 1, \ldots, K$:



$$\mathrm{MSE}^k = \sum_{t=0}^{T} \|x^k(t) - \hat{x}^k_{\mathcal{M}}(t)\|^2$$

Approximate the objective function:

$$\mathbb{E}\left[\sum_{t=0}^{T} \|x(t) - \hat{x}_{\mathcal{M}}(t)\|^2\right] \approx \frac{1}{K}\sum_{k=1}^{K} \mathrm{MSE}^k$$

Optimization

$$\min_{\mathcal{M} \subseteq \{0,\dots,T\}} \mathbb{E}\left[\sum_{t=0}^{T} \|x(t) - \hat{x}_{\mathcal{M}}(t)\|^2\right] \text{ subject to } |\mathcal{M}| = N$$

Combinatorial optimization problem with $\frac{(T+1)!}{(T+1-N)!N!}$ possibilities

Computationally intractable $\longrightarrow$ Genetic algorithm

Summary

1. The problem of optimal measurement budget allocation is modeled as a combinatorial optimization problem

2. Objective function hard to compute $\longrightarrow$ Monte Carlo estimation

3. Combinatorial optimization problem hard to solve $\longrightarrow$ Genetic algorithm

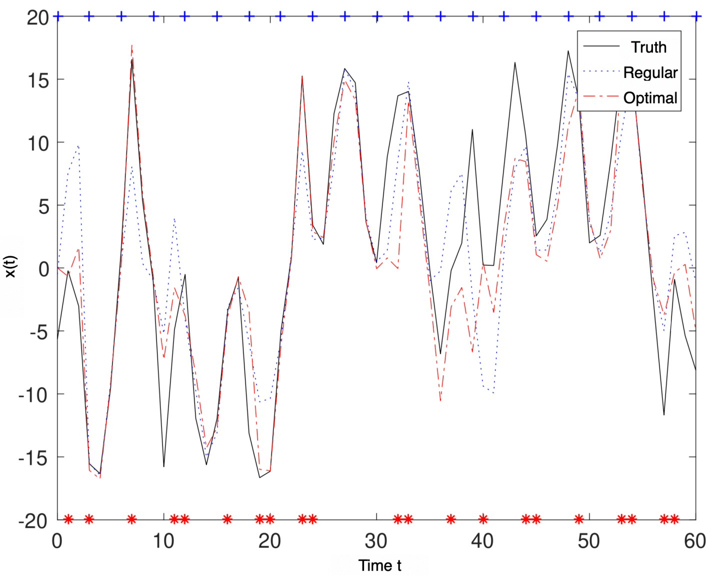
Example

Common Benchmark model [Arulampalam et al., 2002; Carlin et al., 1992; Kadirkamanathan et al., 2002; Kitagawa, 1996]

$$
\begin{aligned}
x(t+1) &= \frac{x(t)}{2} + \frac{25x(t)}{1+x(t)^2} + 8\cos(1.2t) + w(t) \quad \text{for} \;\; t = 0, \ldots, T-1 \\
y(t) &= \frac{x(t)^2}{20} + v(t) \quad \text{for} \;\; t \in \mathcal{M} \\
x(0) &\sim \mathcal{N}(0, 5^2) \\
w(t) &\sim \mathcal{N}(0, 1) \\
v(t) &\sim \mathcal{N}(0, (\sin(0.25t) + 2)^2)
\end{aligned}
$$

For $T = 60$ time intervals and $|\mathcal{M}| = N = 21$ measurements

One trajectory



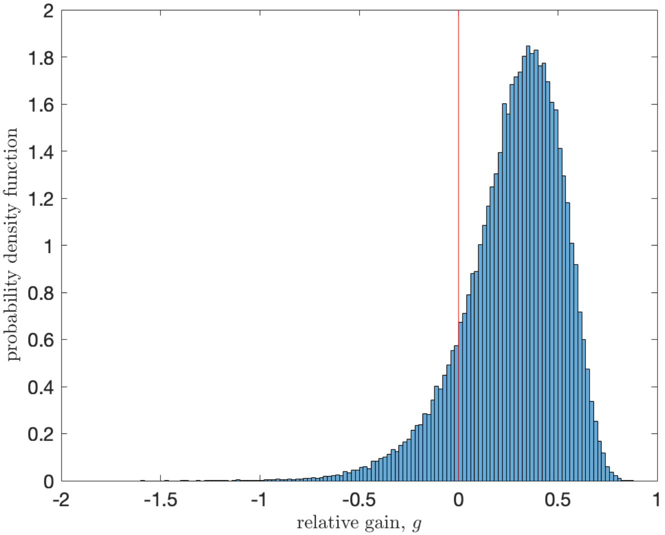
Comparison with regular particle filter

Relative gain

$$g = \frac{\text{MSE}_{\text{Regular}} - \text{MSE}_{\text{Optimal}}}{\text{MSE}_{\text{Regular}}}$$

Positive in 87.5% of cases

Mean is 27.7%



Take home message

- Filtering can be improved by choosing optimal measurement times
- A genetic algorithm works well for this purpose

Further work

- Select the measurement times *online* instead of fixing them *a priori*
- Robustness analysis

Thank you !

Questions ?

Acknowledgements

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