DETECTION OF SHIP WAKES IN SAR IMAGERY USING CAUCHY REGULARISATION

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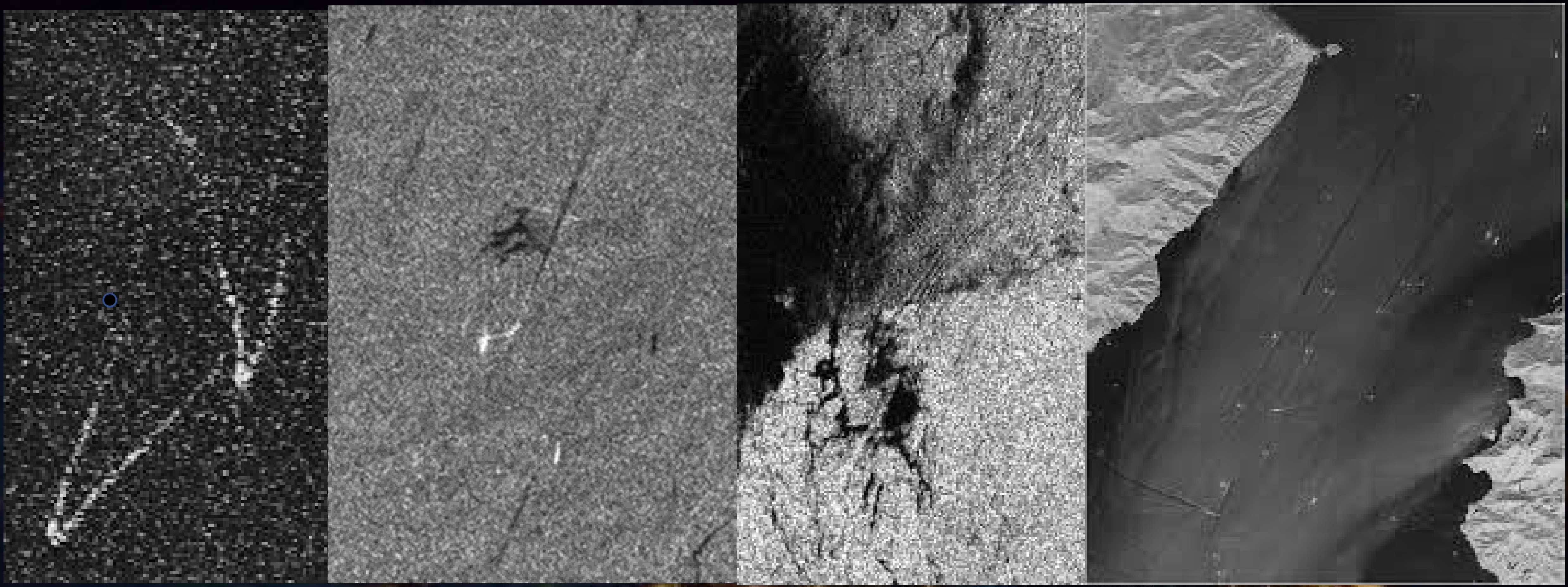
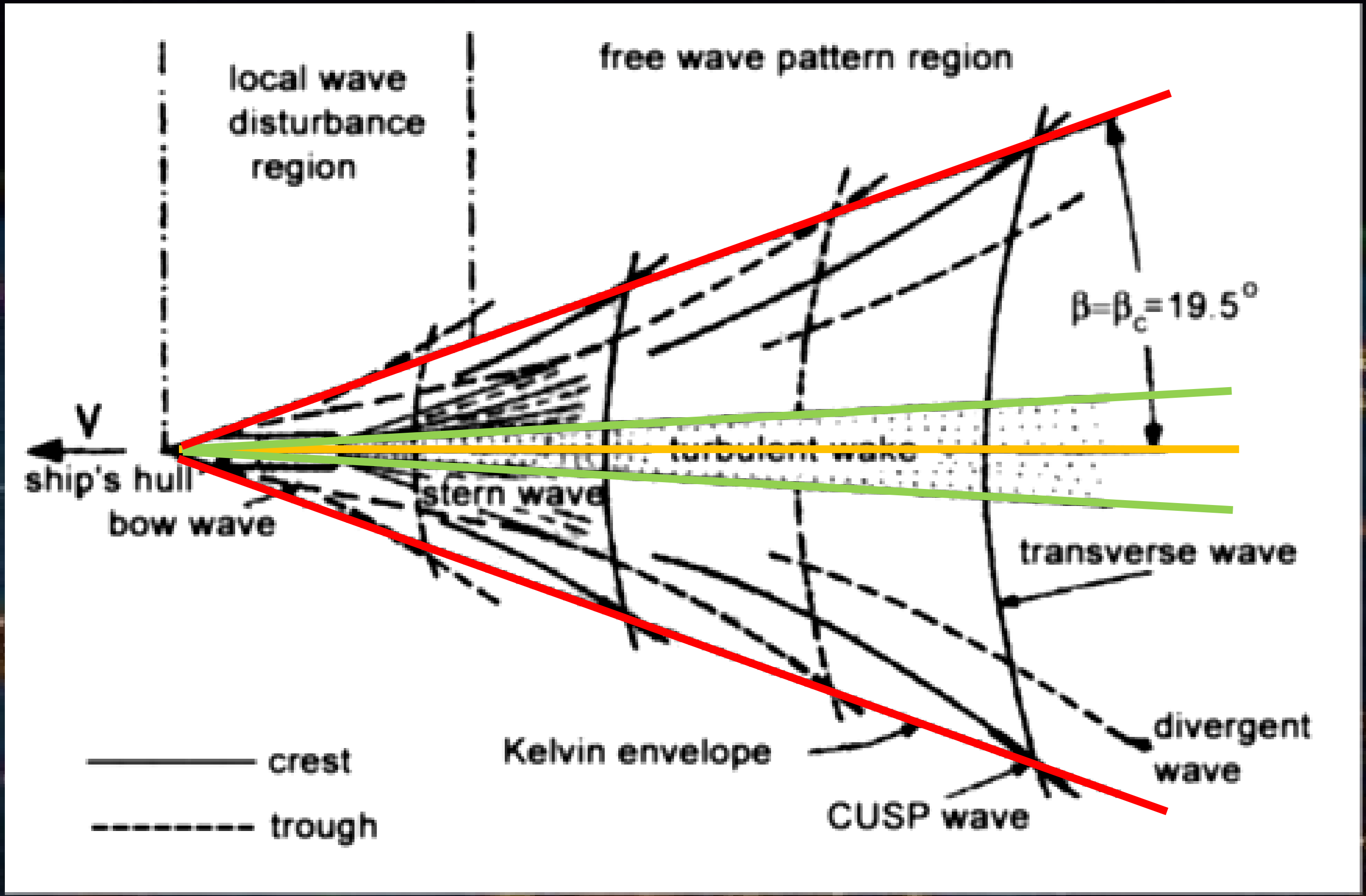
CONTENT

01 Background



Background


local wave
disturbance
region
free wave pattern region
$\beta=\beta_c=19.5^\circ$
V
ship's hull
bow wave
stern wave
turbulent wake
transverse wave
divergent
wave
Kelvin envelope
CUSP wave
crest
trough

Background

- SAR image formation model :

$$Y = CX + N.$$

where $C = R^{-1}$ represents the inverse Radon transform.

- $$p(X|Y) = \frac{p(Y|X)p(X)}{\int p(Y|X)p(X)dX}$$

- the unnormalised posterior

$$p(X|Y) \propto p(Y|X)p(X).$$

Background

- Using maximum a-posterior (MAP) estimator in optimization algorithms :

$$\hat{X}_{MAP} = arg\max_{X} p(X|Y) = arg\min_{X} F(X)$$

where $F(X)$ is denoted as the cost function.

$$F(X) \propto f(x) + g(x)$$

$$\begin{cases} f(x) = \|Y - CX\|_2^2 \\ g(x) = -log(p(X)) \end{cases}$$

- Probability density function of Cauchy distribution :

$$p(X) \propto \frac{\gamma}{\gamma^2 + X^2}$$

- The minimization with Cauchy regularization :

$$\hat{X}_{Cauchy} = arg\,\min_{x} \|Y - CX\|_2^2 - \sum_{i,j} log\left(\frac{\gamma}{\gamma^2 + {X_{i,j}}^2}\right)$$

Moreau-Yoshida unadjusted Langevin algorithm (MYULA)

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Algorithm I MYULA for Cauchy regularized cost function

Input: SAR image Y, $\gamma \in [0.0001, 0.1]$

Output: Radon image X

Set: $\delta = 1/25L$, $\omega = 1/4L$

do

$$Z^{(i+1)} \sim N(0, \mathbb{I}_d)$$

$$X^{(i+1)} = \left(1 - \frac{\delta}{\omega}\right) X^{(i)} - \delta \nabla f\left(X^{(i)}\right) + \frac{\delta}{\omega} prox_g^{\omega}\left(X^{(i)}\right) + \sqrt{2\delta} Z^{(i+1)}$$

while $\epsilon^{(i)} > 10^{-3}$ or $i < MaxIter$

Cauchy proximal operator :

$$prox_g^{\omega}(x) = arg\min_u \left[-log\left(\frac{\gamma}{\gamma^2 + u^2}\right) + \frac{\|u - x\|^2}{2\omega} \right]$$

By using Cardano's method :

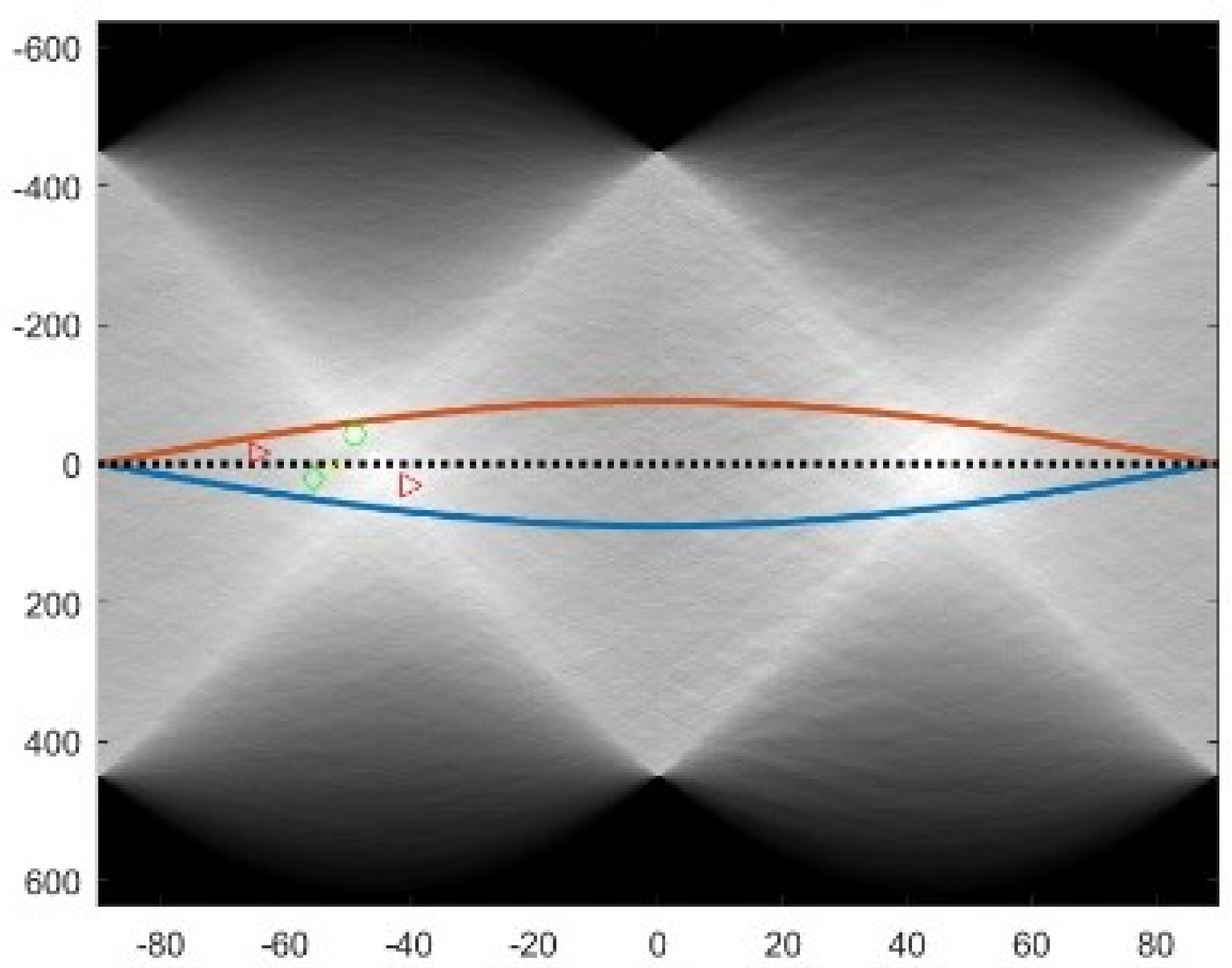
$$prox_g^{\omega}(x) = \frac{x}{3} + s + t$$

$$s = \sqrt[3]{\frac{q}{2} + \Delta}, \qquad t = \sqrt[3]{\frac{p}{2} - \Delta}, \qquad \Delta = \sqrt{\frac{p^3}{27} + \frac{q^2}{4}}$$

$$p = \gamma^2 + 2\omega - \frac{x^2}{3}$$

$$q = x\gamma^2 + \frac{2x^3}{27} - \frac{x}{3}(\gamma^2 + 2\omega)$$

02 Methodology



- The confirmation of the candidate :

$$F_I = \bar{I}_w / \bar{I} - 1.$$

where $\bar{I}_w$ is the mean value over the un-confirmed wake, and $\bar{I}$ is the mean intensity of the image.

$$\begin{cases} F_I < 0 \text{ for turbulent wakes,} \\ F_I > 0.1 \text{ for narrow-V and Kelvin wakes} \end{cases}$$

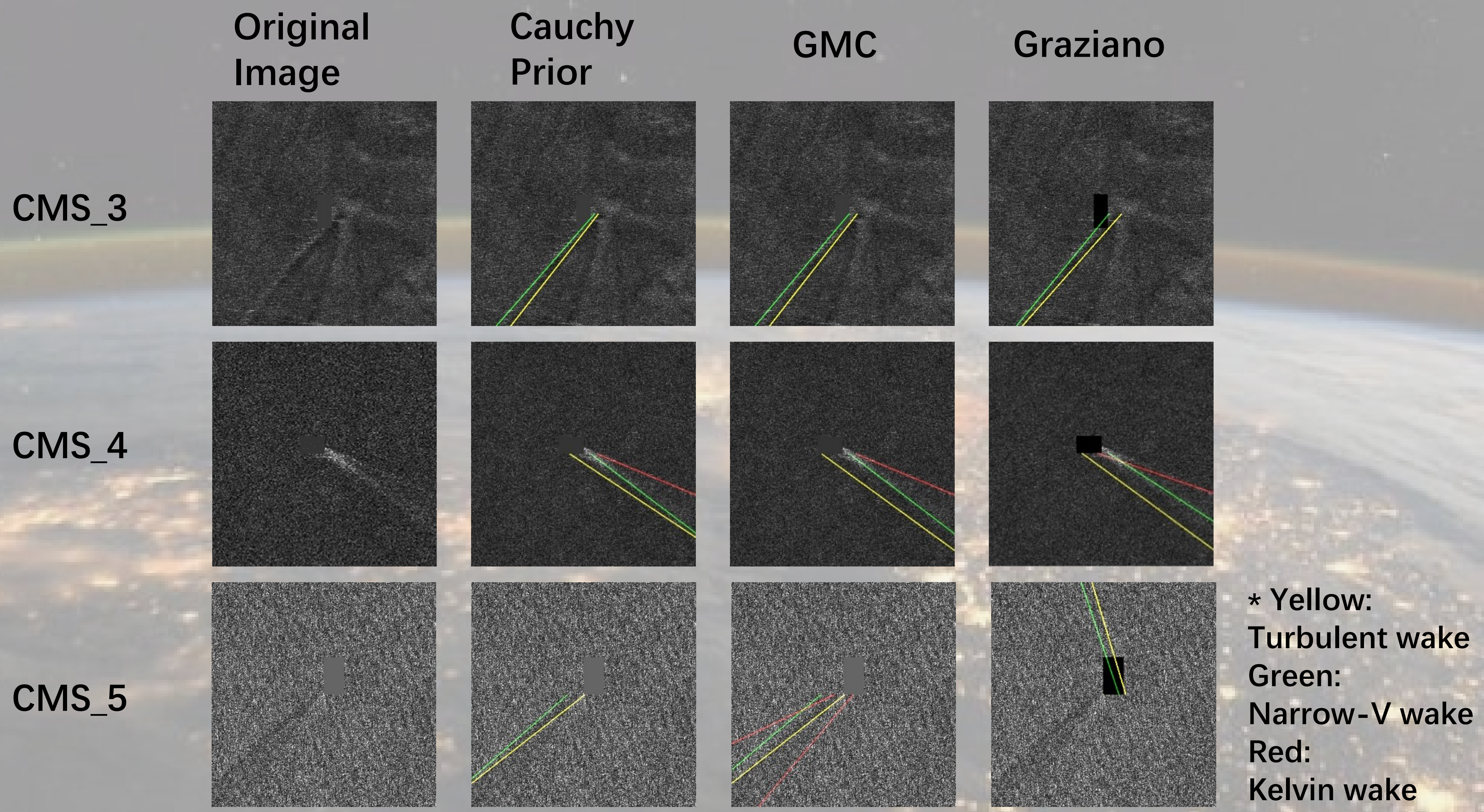
Table 1. Visible wakes in used image dataset *

Image	Turbulent	1st Narrow	2nd Narrow	1st Kelvin	2nd Kelvin
CSM_1	1	1	0	0	0
CSM_2	1	1	0	0	0
CSM_3	1	1	0	1	0
CSM_4	1	1	0	1	0
CSM_5	1	1	0	0	0
CSM_6	1	1	0	0	0

* 1 means visible and 0 represents invisible

Table 2. Detection results over 6 COSMO-SkyMed images

	TP	TN	FP	FN	%Accuracy
Cauchy	40.0%	46.7%	6.7%	6.7%	86.7%
GMC	36.7%	40%	20%	3.3%	76.7%
Graziano	33.3%	36.7%	16.7%	13.3%	70.0%


Original Image
Cauchy Prior
GMC
Graziano
CMS_3
CMS_4
CMS_5
* Yellow:
Turbulent wake
Green:
Narrow-V wake
Red:
Kelvin wake

04 Summary

- The use of Cauchy distribution in ship wake detection problem.
- Realization of MYULA in image reconstruction from SAR imagery.
- Implementation of proximal Cauchy operator in solving inverse problem.

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Thank You !

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