Quaternion Harris for Multispectral Keypoint Detection

G. Sfikas[1] D. Ioannidis[2] D. Tzovaras[2]

[1]University of Ioannina
Ioannina, Greece

[2]CERTH
Thessaloniki, Greece

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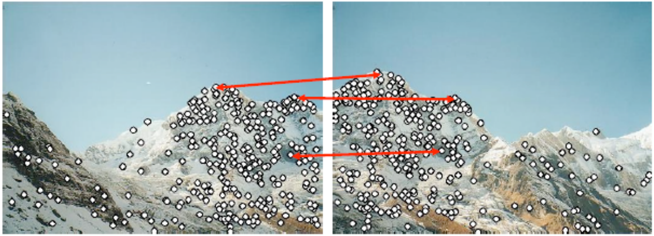
Overview

- Contribution: Keypoint detection for multispectral images
- Quaternionic autocorrelation - quaternionic eigenvectors, real eigenvalues. Result: channel cross-correlation taken into account
- Tested succesfully on color/NIR images

Keypoint detection

- Keypoints very useful for various vision tasks
- Hand-crafted detectors (e.g. SIFT, SURF, ORB) still very relevant despite deep learning



Keypoint detection

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A disadvantage of standard detectors

- Detectors typically rely on gradient information of a single input channel
- Image must be converted to grayscale first
- Loss of information!

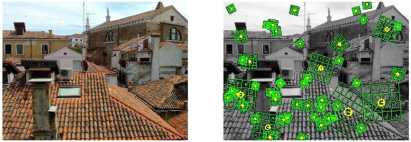

Figure: Left: Color input image. Right: Grayscale image with overlaid detected keypoints. Color information / crosschannel correlation is of no consequence to standard keypoint detectors, and practically discarded

Our contribution

We propose a keypoint detection scheme that can handle multispectral inputs by using a *quaternion* image representation. Hence, the imaged multispectral cues can be treated each as a single entity per pixel. The multimodal nature of the input, including cross-channel correlations can consequently be taken into account in a natural manner.

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Elements

Quaternions are mathematical objects that form a skew-field $\mathbb{H}$, *i.e.* quaternion addition and multiplication are defined with all the properties of a field, except that of multiplication commutativity.

Basic form

Quaternions $q \in \mathbb{H}$ share the basic form:

$$q = a + b\boldsymbol{i} + c\boldsymbol{j} + d\boldsymbol{k}$$

where $a, b, c, d \in \mathbb{R}$ and $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are independent imaginary units

Multiplication factors do not commute

$$pq \neq qp \quad \text{for} \quad p, q \in \mathbb{H}$$

Properties

$$\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1$$

$$\boldsymbol{ij} = -\boldsymbol{ji} = \boldsymbol{k}, \boldsymbol{jk} = -\boldsymbol{kj} = \boldsymbol{i}, \boldsymbol{ki} = -\boldsymbol{ik} = \boldsymbol{j}$$

$$|q| = \sqrt{q\bar{q}} = \sqrt{\bar{q}q} = \sqrt{a^2 + b^2 + c^2 + d^2}$$

$$\bar{q} = a - b\boldsymbol{i} - c\boldsymbol{j} - d\boldsymbol{k}$$

Caley-Dickson form

$$q = A + B\boldsymbol{j}$$

$$A = a + b\boldsymbol{i}, B = c + d\boldsymbol{i}.$$

Complex adjoint of quaternionic matrix

- An analogous operation can be performed for quaternion matrices, which can be written as tuples of complex matrices
- We take A_1, A_2 such that $A = A_1 + A_2\boldsymbol{j}$

$$\chi_A = \begin{bmatrix} A_1 & A_2 \\ -\bar{A}_2 & \bar{A}_1 \end{bmatrix}$$

Matrix χ_A is called the adjoint of A.

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Good keypoints should maximize..

$$E(x) = \sum_{x_n \in N(x)} g(x_n)|I(x_n) - I(x_n + \Delta x)|^2$$

After quaternionic Taylor expansion and $|x|^2 = x\bar{x}$

$$E(x) = \sum_{x_n \in N(x)} g(x_n)\Delta x^T \nabla I(x_n)\overline{\Delta x^T \nabla I(x_n)}$$

$$= \sum_{x_n \in N(x)} g(x_n)\Delta x^T \nabla I(x_n)\overline{\nabla I(x_n)^T}\Delta x$$

$$= \Delta x^T [\sum_{x_n \in N(x)} g(x_n)\nabla I(x_n)\nabla I(x_n)^H] \ \Delta x$$

$$A_q = \sum_{x_n \in N(x)} g(x_n) \nabla I(x_n) \nabla I(x_n)^H$$

We then aim to construct a detection criterion based on the eigenvalues of A_q:

$$c(A_q) = (\lambda_1 \lambda_2) - \kappa(\lambda_1 + \lambda_2)^2.$$

Problem!

Due to quaternion non-commutativity for multiplication, the eigenvalue problem is more complex!

- $Ax = \lambda x$ different than $Ax = x\lambda$
- $|A - \lambda I|$ to compute eigenvalues (left or right ?) requires a definition of a quaternion determinant, which is itself problematic

Solution

- The *right* eigenvalues of A_q are real and exactly two
- A diagonalisation $U^H A_q U = \Lambda$ exists

The adjoint of the diagonalisation

We can proceed by using the adjoint and its properties:

$$\chi_U^H \chi_{A_q} \chi_U = \chi_\Lambda,$$

and compute eigenvalues of χ_{A_q}.

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Proof of concept

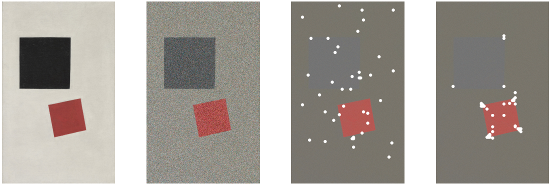
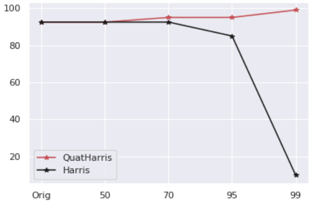


Figure: From left to right: Original image, image degraded with LUN=70% and WGN $\sigma^2 = 10$, result of Harris detector on image degraded with LUN=99% and WGN $\sigma^2 = 1$, result of proposed QuatHarris detector on the same image.

Proof of concept, numerical experiments



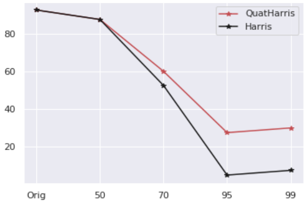


Figure: Comparison of detector accuracy (higher values are better). Tests were run on degraded versions of the image in Fig. 2. The horizontal axis corresponds to increasing levels of *LUN* degradation. *WGN* levels are $\sigma^2 = 1$ and 10 on the left and right plots respectively.

Visualization of quaternionic eigenvectors

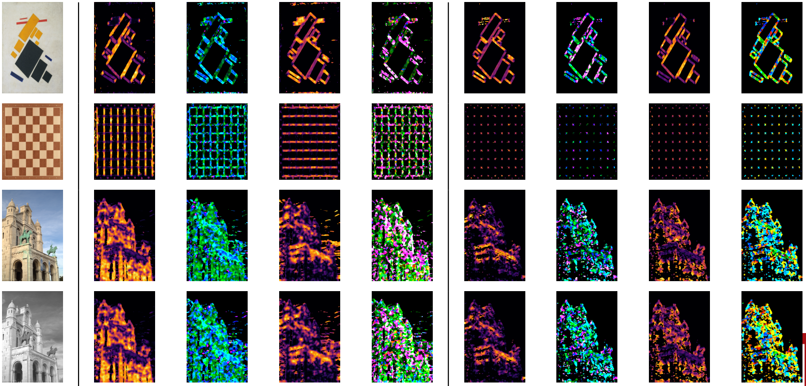


Image matching

Figure: Samples of the CERTH dataset.

Table: Comparison between QuatHarris and Multispectral Harris. The number of CERTH dataset tuples where one detector outperforms the other is reported.

Method / #kpoints	50	100	150	200	250
QuatHarris	**2**	**9**	**7**	6	7
Multispectral	1	4	3	**7**	**9**

Object detection

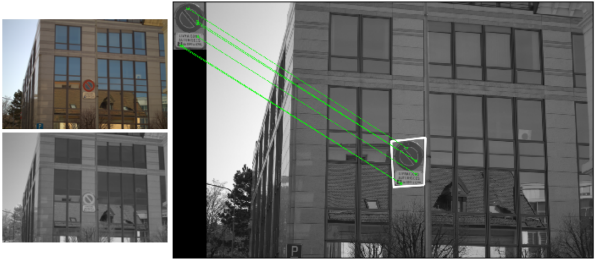


Figure: Matching a template for object detection. Information from all four channels (RGB+NIR, shown at top left and bottom respectively) is taken into account.

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Summary

- We have presented keypoint detector that can take advantage of the content and cross-correlations of multichannel inputs, unlike standard hand-crafted detectors.
- Our detector treats images as quaternionic matrices, and defines a quaternionic autocorrelation matrix.
- Quaternionic autocorrelation eigenstructure is computed and used to decide on which points to characterize as keypoints.
- Our experiments validate the usefulness of the proposed detector.

Future work

- Plan to work on coupling with a quaternionic analysis-based descriptor, examining uses with a learning-based scheme (e.g. HardNet).
- Possible extensions to more complex algebras such as octonions.
- The potential of using quaternionic eigenvectors as a basis for a better detector or an image cue.

Thank you

Thank you! Questions?