Analog Mappings for Correlated Sources over AWGN Channels

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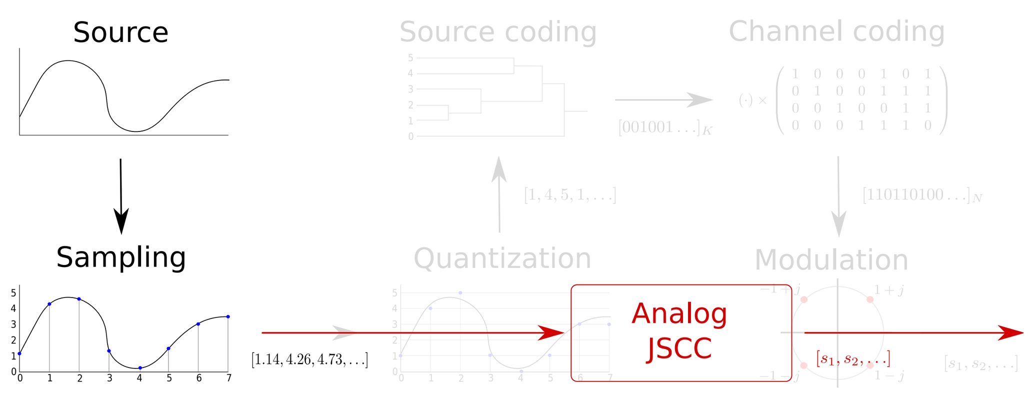
Motivation



In analog Joint Source Channel Coding (JSCC), the source and channel encoders are jointly optimized in a single step:

- The complexity and delay are drastically reduced.
- This strategy is very robust in time-varying environments.
- Analog JSCC is useful for real-time transmissions in wireless communications.

System model

Source symbols $\mathbf{s} = [s_1, s_2]^T$ follow a bivariate Gaussian distribution with zero mean and covariance matrix

$$\mathbf{C_s} = E\left[\mathbf{s}\mathbf{s}^H\right] = \begin{bmatrix} \sigma_s^2 & \rho\sigma_s^2 \\ \rho\sigma_s^2 & \sigma_s^2 \end{bmatrix}$$

Source symbols are compressed with a 2:1 analog JSCC mapping $g(\cdot)$ and sent over an AWGN channel

$$y = g(\mathbf{s}) + n = x + n. \quad (1)$$

General non-parametric mappings

If distortion criterion is MSE, then the optimal decoder is given by:

$$h(y) = \mathbb{E}[s|y] = \frac{\int \mathbf{s}\ p_s(\mathbf{s})\ p_n(y - g(\mathbf{s}))\ \mathrm{d}\mathbf{s}}{\int p_s(\mathbf{s})\ p_n(y - g(\mathbf{s}))\ \mathrm{d}\mathbf{s}} \quad (2)$$

A steepest descent algorithm is used to find the optimal mapping function, subject to a power constraint, with the gradient vector

$$\nabla J[g] = \lambda p_s(\mathbf{s}) g(\mathbf{s}) - \int h'(g(\mathbf{s}) + n)[\mathbf{s} - h(g(\mathbf{s}) + n)] p_n(n) p_s(\mathbf{s}) \mathrm{d}n.$$

Proposed parametric mapping

After examining the resulting non parametric mappings, we propose to use the following parametric curve

$$\mathbf{K}(t) = \mathbf{U}\boldsymbol{\Sigma}\begin{bmatrix} t - \frac{1}{2\alpha}\sin(\alpha t) \\ \Delta\sin(\alpha t) \end{bmatrix} \quad (3)$$

- $K(t)$ represents the point into the bidimensional space corresponding to the parameter t on the curve.
- The parameters α and Δ represent the frequency and the amplitude of the mapping.
- $\mathbf{U}$ and $\boldsymbol{\Sigma}$ are obtained from the eigendecomposition of the source covariance matrix $\mathbf{C}_s = \mathbf{U}^H\boldsymbol{\Sigma}\mathbf{U}$.

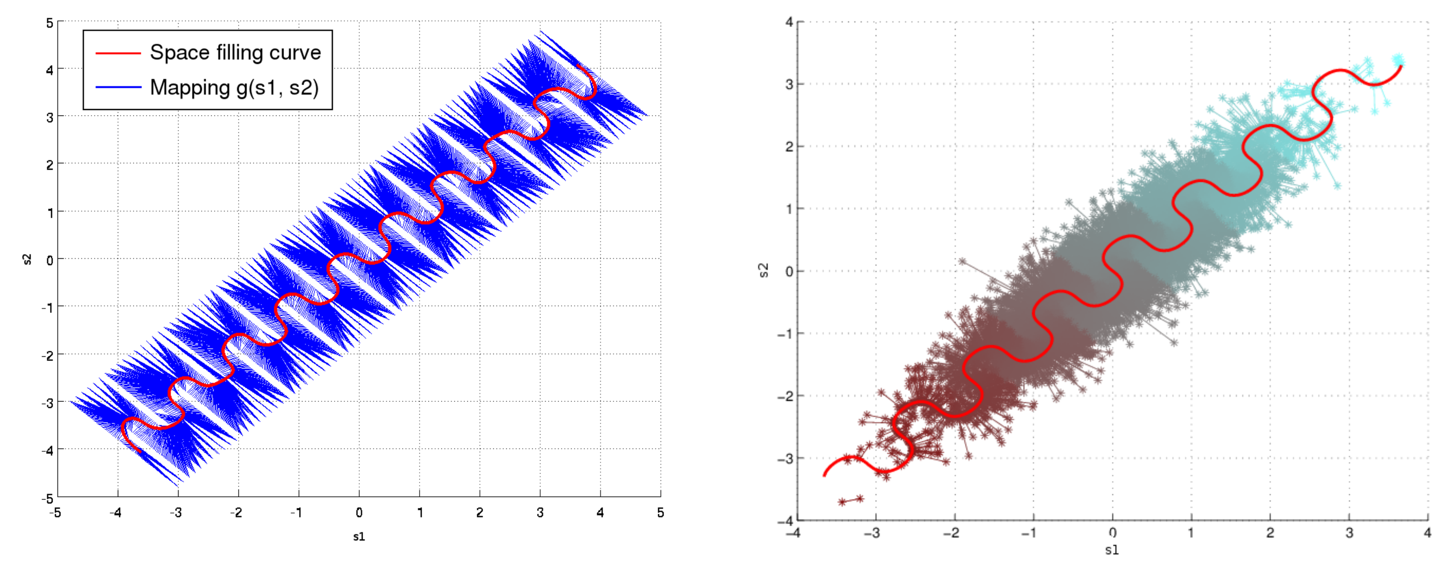


Fig. 2: Non-parametric and parametric mappings for 2:1 compression of the sources symbols.

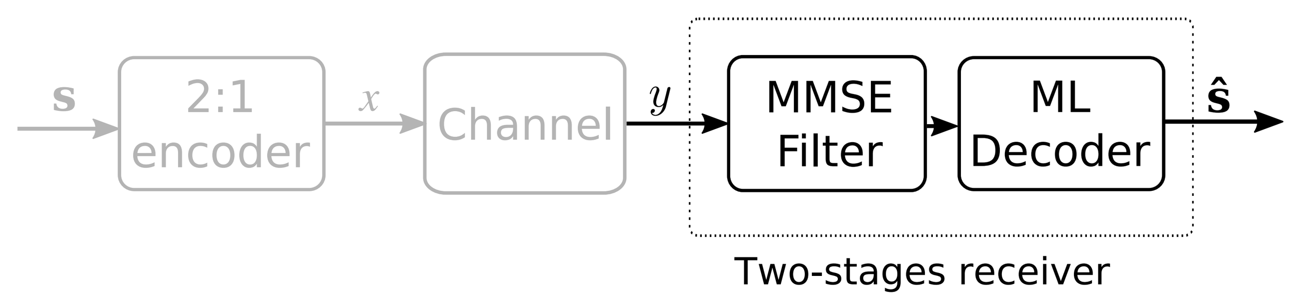
The encoder maps the symbols to the closest point on the curve

$$x = g_1(\mathbf{s}) = \arg\min_t \ \|\mathbf{s} - \mathbf{K}(t)\|^2. \quad (4)$$

This encoder can be enhanced by considering the noise variance:

$$x = g_2(\mathbf{s}) = \arg\min_t \int_{-\infty}^{\infty} \|\mathbf{s} - \mathbf{K}(u)\|^2 p_n(u - t)\mathrm{d}u \quad (5)$$

The decoding operation is implemented by using a two-stage receiver



This approach achieves similar performance to that of the general MMSE decoder with lower computational complexity.

Results

Performance evaluation of the parametric mappings:

- 2:1 compression of the source with the encoding algorithm given by Equation (5).
- Source symbols are estimated by using the two-stage receiver.
- Comparison with non-parametric mappings, alternating SQLC, spiral-like mappings and the Optimum Performance Theoretically Attainable (OPTA).

The performance of analog JSCC is measured in terms of the Signal-to-Distortion Rate (SDR) given by

$$\mathrm{SDR(dB)} = 10\log_{10}(\sigma_s^2/\mathrm{MSE}),$$

where $\mathrm{MSE} = 1/2\sum_{i=1}^{2} E[\|\hat{s}_i - s_i\|^2]$.

The OPTA bound arises from equating the channel capacity and the rate-distortion function of a correlated bivariate Gaussian

$$\mathrm{OPTA} = \begin{cases} \frac{2\sigma_n^2+2}{2\sigma_n^2+(1-\rho)} & \mathrm{SNR} < \frac{2\rho}{1-\rho} \\ \sqrt{\frac{\sigma_n^2+1}{\sigma_n^2(1-\rho^2)}} & \mathrm{SNR} \geq \frac{2\rho}{1-\rho}. \end{cases} \quad (6)$$

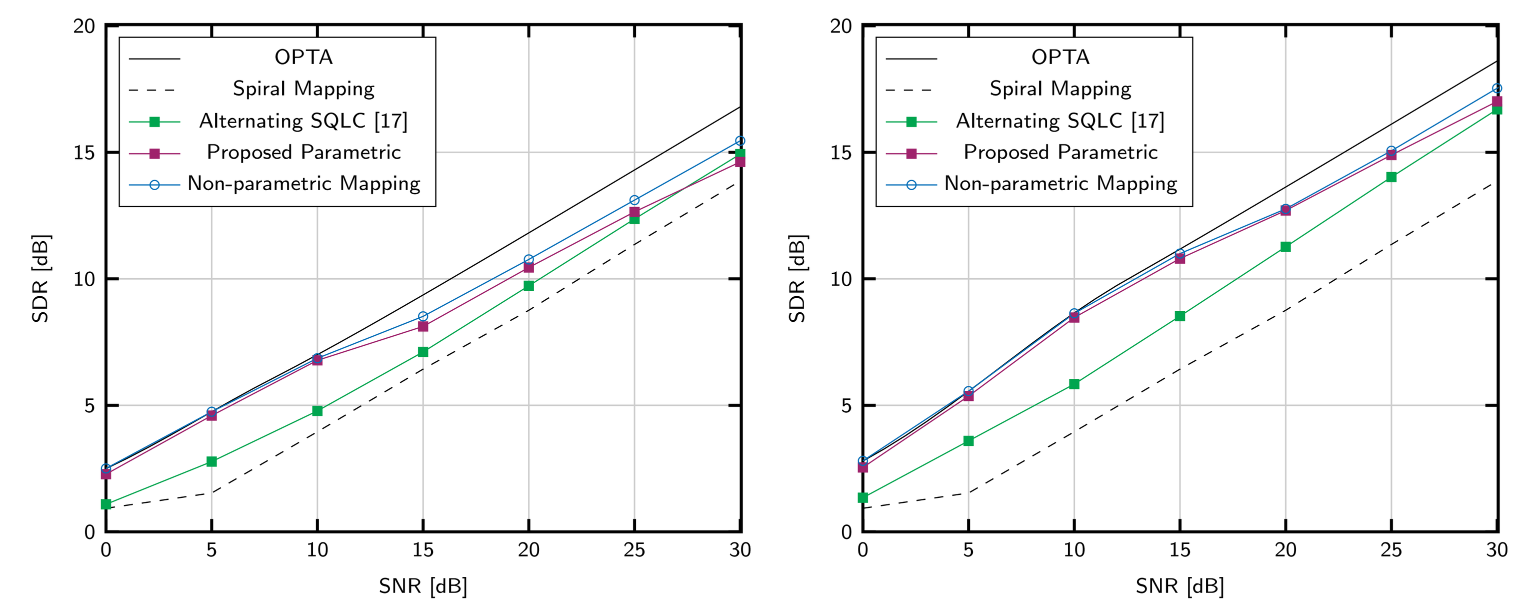


Fig. 4: SDR vs SNR for correlation factors $\rho = 0.90$ and $\rho = 0.75$

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