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Iterative Quadratic Relaxation Method for Optimization of Multiple Radar Waveforms

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Introduction

- MIMO radars transmit multiple waveforms simultaneously
- Waveforms with low peak sidelobe (PSL) and peak cross-correlation (PCC) are desirable
- Waveform optimization is a demanding task
- Constraint on peak to average ratio of power (PAR) necessary to simplify transmitter front-end

Contributions

- Formulate the multiple waveform design problem as a minimax optimization on an oblique manifold
- Novel Iterative Quadratic Relaxation method with PAR constraint is proposed

Problem Formulation

- Sample cross-ambiguity function

$$\chi_{ij}(\tau, F_D, T_s) = \left| T_s \sum_k s_i(kT_s)\sqrt{\gamma} s_j^*(\gamma k T_s + \tau) e^{j2\pi F_D k T_s} \right|^2$$

  - τ is the time delay
  - T_s is the sampling interval
  - F_D is the Doppler frequency
  - γ is the compression factor
- Rewrite the sample cross-ambiguity function as

$$\chi_{ij}(\tau, F_D, T_s) = \left| \mathbf{s}_i^H \mathbf{D}(\tau, F_D, T_s) \mathbf{s}_j \right|^2.$$

  - $\mathbf{D}(\tau, F_D, T_s)$ is a time delay and Doppler matrix
  - $\mathbf{s}_i$ is a vector containing the symbols of the ith waveform
- Waveform normalization $\|\mathbf{s}_i\|^2 = 1$
- Define main lobe half-width as one symbol in delay and f_0 in normalized Doppler
- Minimizing the max PSL and PCC

$$\begin{aligned} \text{minimize} \quad & \max_{i,j,k,f} \left| \mathbf{s}_i^H \mathbf{D}_{k,f} \mathbf{s}_j \right|^2 \\ \text{s.t.} \quad & \|\mathbf{s}_i\| = 1 \qquad \forall i \\ & |f| \geq \delta_{ij}\delta_{k0} f_0 \end{aligned}$$

- Additional PAR constraints

$$|(\mathbf{s}_i)_k|^2 \leq \frac{\mathrm{PAR}_{\max}}{N_p} \qquad \forall k,$$

  where N_p is the number of symbols

Proposed Algorithm

- Iterative Quadratic Relaxation (IQR)
- Update the waveforms iteratively by solving a convex problem
- Waveform at mth iteration $\mathbf{s}_i^{(m)}$
- Solve for each i in an alternating manner

1. solve $\mathbf{x}_i$ from

$$\begin{aligned} \min \quad & \max_{j,k,f} \left| \mathbf{x}_i^H \mathbf{D}_{k,f} \mathbf{s}_j^{(m)} \right|^2, \quad |f| \geq \delta_{k0}\delta_{ij} f_0 \\ \text{s.t.} \quad & \mathbf{x}_i^H \mathbf{s}_i^{(m)} = 1 \\ & \mathrm{Re}\left[(\mathbf{x}_i)_k^* (\mathbf{s}_i^{(m)})_k \right] \leq \frac{\mathrm{PAR}_{\max}}{N_p} \qquad \forall k. \end{aligned}$$

2. let $y_k(r) = \frac{(\mathbf{x}_i)_k}{|(\mathbf{x}_i)_k|^r}$, $k = 1 \ldots N_p$, and solve r from

$$\begin{aligned} \min \quad & r \\ \text{s.t.} \quad & \frac{\max_k |y_k(r)|^2}{\frac{1}{N_p}\|\mathbf{y}(r)\|^2} \leq \mathrm{PAR}_{\max} \\ & 0 \leq r \leq 1, \end{aligned}$$

3. update the waveform

$$\mathbf{s}_j^{(m+1)} = \frac{\mathbf{y}(r)}{\|\mathbf{y}(r)\|}$$

- The basic concept of the algorithm is shown in Fig.1

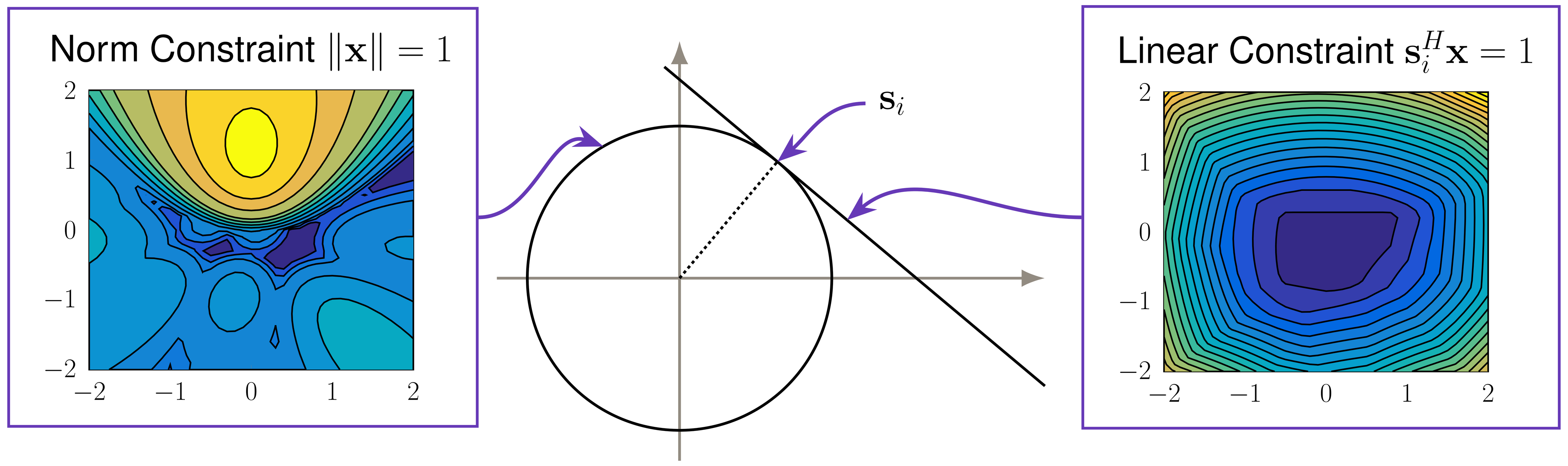

Figure 1: Illustration of the basic concept of the proposed algorithm with 2D cuts of the objective function on the constraints. The initial formulation contains a norm constraint that is nonconvex. Relaxing this into linear constraint a convex problem is obtained that can be solved iteratively.

Numerical Examples

- Optimize a set of four polyphase waveforms with 40 symbols in each
- Random inital points
- No constraint on PAR, PAR at most 1.5, or constant modulus
- Compare IQR with other methods
  - Simulated Annealing (SA)
  - Greedy algorithm
  - Quasi-Newton on Manifold
  - Maximum Block Improvement (MBI)
- Averaged PSL and PCC for 20 initial points

	PAR Constraint		
Method	∞	≤ 1.5	$= 1$
SA	-9.84	-9.62	-9.64
SA+Greedy	-9.85	-9.63	-9.64
Manifold	-9.13	-8.91	-8.61
Manifold+Greedy	-9.21	-9.08	-8.71
IQR	-10.07	-9.72	-9.83
IQR+Greedy	-10.07	-9.72	-9.84
Greedy	-7.78	-7.69	-7.60
MBI	-6.61	-6.71	-6.99

- IQR performed the best in most cases
- No much additional benefit from the greedy algorithm
- An example of the achieved PSL and PCC vs. the function evaluations is shown in Fig.2

Conclusions

- MIMO radar waveform optimization problem can be formulated as a quartic minimax problem on an oblique manifold
- IQR algorithm was proposed for minimizing waveform sidelobes and cross-correlation
- Constraints on peak to average ratio of transmit power can be included in the IQR
- IQR algorithm provided the best waveform in most of the cases

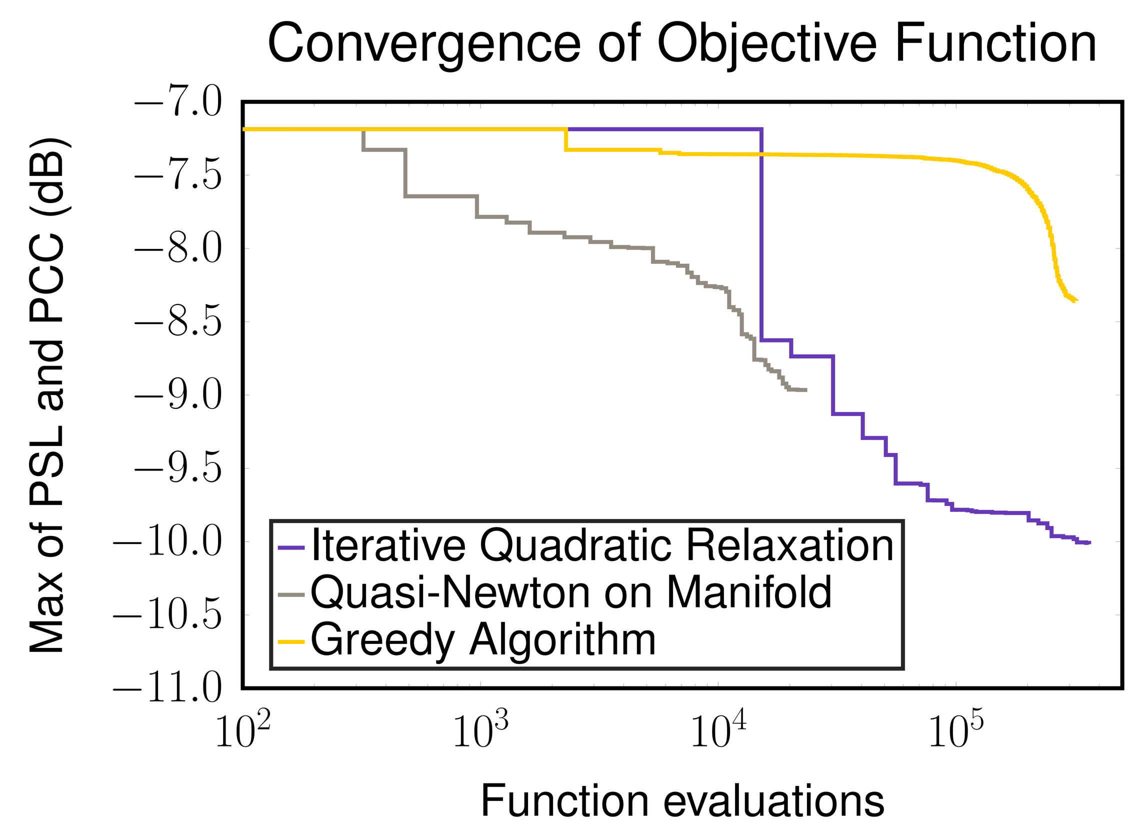

Figure 2: Number of function evaluations required to achieve certain PSL and PCC level for different optimization algorithms in one initialization. IQR requires large number of function evaluations but provides the lowest combined PSL and PCC value in this example.