COMPRESSED TRAINING ADAPTIVE EQUALIZATION

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What is Compressed Training Adaptive Equalization?

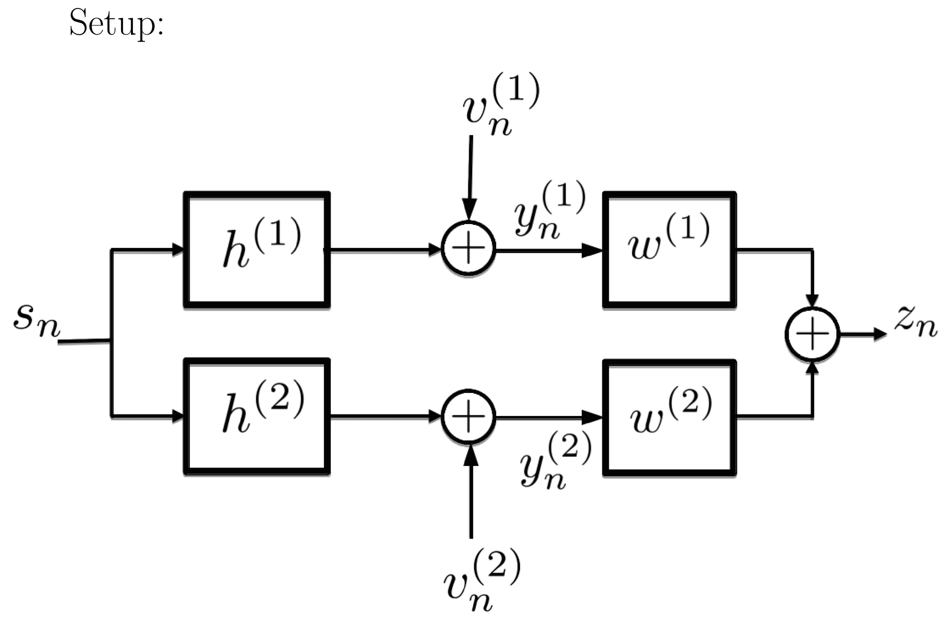
- **An adaptive equalization framework** aiming to reduce the number of training symbols in a communication packet. The equalizer coefficients are trained by exploiting
  - Training symbols,
  - Magnitude boundedness property of digital communication constellations.

Highlights of the Framework

- Direct Link with Compressed Sensing,
- Reduce Training Length,
- Prescribe Minimum Training Length ∝ log(Channel Spread),
- Algorithms Based on Convex Settings,
- DO NOT Make Sparse Channel Assumption.

Equalization Setup

- We assume the standard Fractionally-Spaced Equalization Setup:



- The receiver employs two (WLOG) diversity branches,
- $\{s_n\}$: Transmission sequence sent by the transmitter,
- For the receiver branch k:
  - $\left\{h_n^{(k)} : n \in \{0, \dots, L_C - 1\}\right\}$: Channel impulse response,
  - $\left\{w_n^{(k)} : n \in \{0, \dots, L_E - 1\}\right\}$: Equalizer coefficients,
  - $\{y_n^{(k)}\}$: Received signal,
- $\{z_n\}$ is the equalizer output sequence,
- $\{g_n\}$: Combined channel impulse response where

$$g_n = \sum_{k=1}^{2} w_n^{(k)} * h_n^{(k)}.$$

- **Perfect Equalization Condition:** $g_n = \delta_{n-d}$.

The Proposed Framework

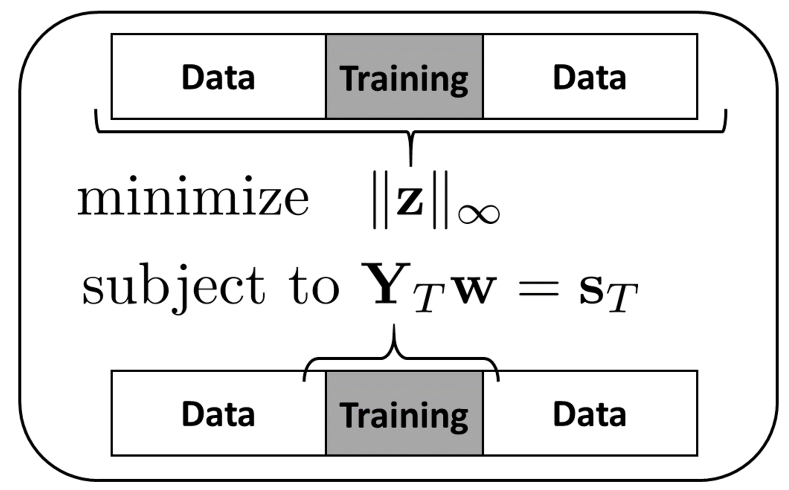
Noiseless/High SNR Case:

- The proposed optimization setting :

Setting I

$$\begin{aligned}&\text{minimize} && \|\mathbf{z}\|_\infty \\ &\text{subject to} && \mathbf{Y}_T\mathbf{w} = \mathbf{s}_T\end{aligned}$$

- $\mathbf{z}$: Vector containing all equalizer outputs for the whole packet,
- $\mathbf{Y}_T$: Equalizer input vectors corresponding to the training region,
- $\mathbf{s}_T$: Training symbols,
- $\mathbf{w}$: Equalizer coefficient vector.



Noisy Case:

- The optimization setting factoring existence of noise:

Setting ℓ_∞- CLASSO

$$\begin{aligned}&\text{minimize} && \|\mathbf{Y}_T\mathbf{w} - \mathbf{s}_T\|_2 \\ &\text{subject to} && \|\mathbf{z}\|_\infty \le \gamma\end{aligned}$$

- γ represents the knowledge about the symbol boundedness.
- Alternative convex optimization setting for the noisy case:

Setting $\ell_\infty - \ell_2$ - LASSO

$$\text{minimize} \quad \|\mathbf{Y}_T\mathbf{w} - \mathbf{s}_T\|_2 + \lambda\|\mathbf{z}\|_\infty$$

- λ is the regularization parameter.

Connection to Compressed Sensing

- For the noiseless scenario, z_n can be written as :

$$z_n = g_0 s_n + g_1 s_{n-1} + \dots g_{L_G-1} s_{n-L_G+1},$$

- For *sufficiently long* data packet and BPSK constellation,

$$\|\mathbf{z}\|_\infty = \|\mathbf{g}\|_1. \qquad (1)$$

- The corresponding dual optimization setting:

Setting Ig

$$\begin{aligned}&\text{minimize} && \|\mathbf{g}\|_1 \\ &\text{subject to} && \mathbf{S}\mathbf{g} = \mathbf{s}_T\end{aligned}$$

- We observe *Setting I* is equivalent to *Sparse Reconstruction Problem* if we consider
  - $\mathbf{s}_T$ as the observation vector,
  - $\mathbf{S}$ as the measurement matrix and,
  - $\mathbf{g}$ as the one-sparse vector to be reconstructed.

Analysis of the Proposed Approach

- The mutual coherence of the matrix $\mathbf{S} \in \Re^{L_T \times L_G}$ is defined as $\mu(\mathbf{S}) = \max_{1 \le \mathbf{i},\mathbf{j} \le \mathbf{M}, \mathbf{i} \ne \mathbf{j}} \frac{|\mathbf{S}_{:,\mathbf{i}}^T \mathbf{S}_{:,\mathbf{j}}|}{\|\mathbf{S}_{:,\mathbf{i}}\|_2 \|\mathbf{S}_{:,\mathbf{j}}\|_2}$.
- **Theorem**[2] : Let $\mathbf{S} \in \Re^{L_T \times L_G}$ be full rank with $L_T < L_G$. If the system of linear equations $\mathbf{S}\mathbf{g} = \mathbf{y}$ has a solution $\mathbf{g}_s$ which obeys

$$\|\mathbf{g}_s\|_0 < 0.5\left(1 + \mu(\mathbf{S})^{-1}\right)$$

then it is the unique solution for the optimization problem in *Setting Ig*.

COROLLARY

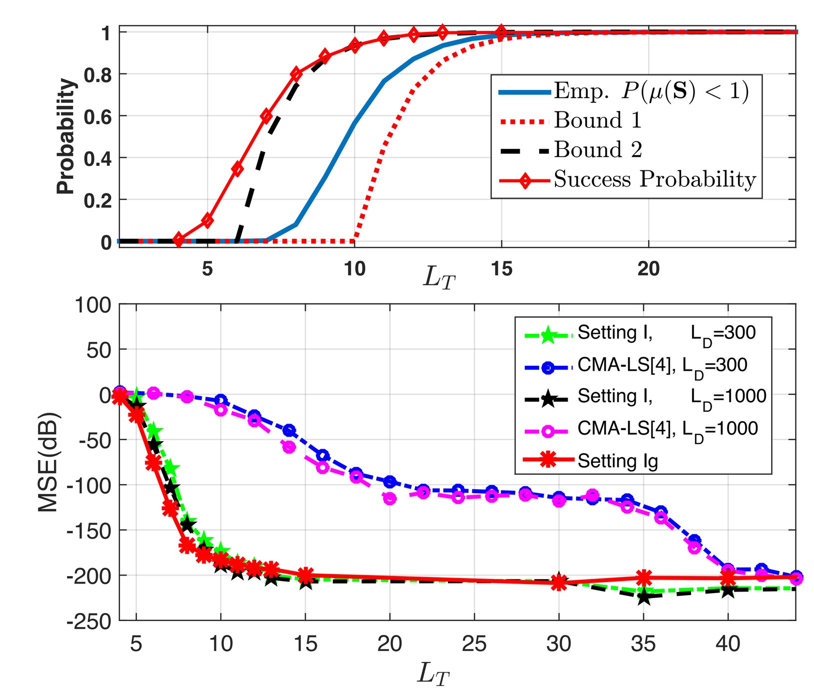
Let $\mathbf{S} \in \Re^{L_T \times L_G}$ be a Toeplitz matrix with i.i.d. Bernoulli elements. If $\mathbf{L_T} > \log_2\left(\mathbf{L_G}(\mathbf{L_G} - \mathbf{1})\right)$, then the mutual coherence condition $\mu(\mathbf{S}) < 1$ is satisfied with probability at least $1 - L_G(L_G - 1) \cdot 2^{-L_T}$.

CONCLUSION

- We introduced convex optimization based Adaptive Equalization Framework that reduces training data to $\mathcal{O}(\mathbf{log(Channel\text{-}Spread)})$ as opposed to $\mathcal{O}(\mathbf{Channel\text{-}Spread})$.
- A duality based link between the proposed approach and compressed sensing is established.

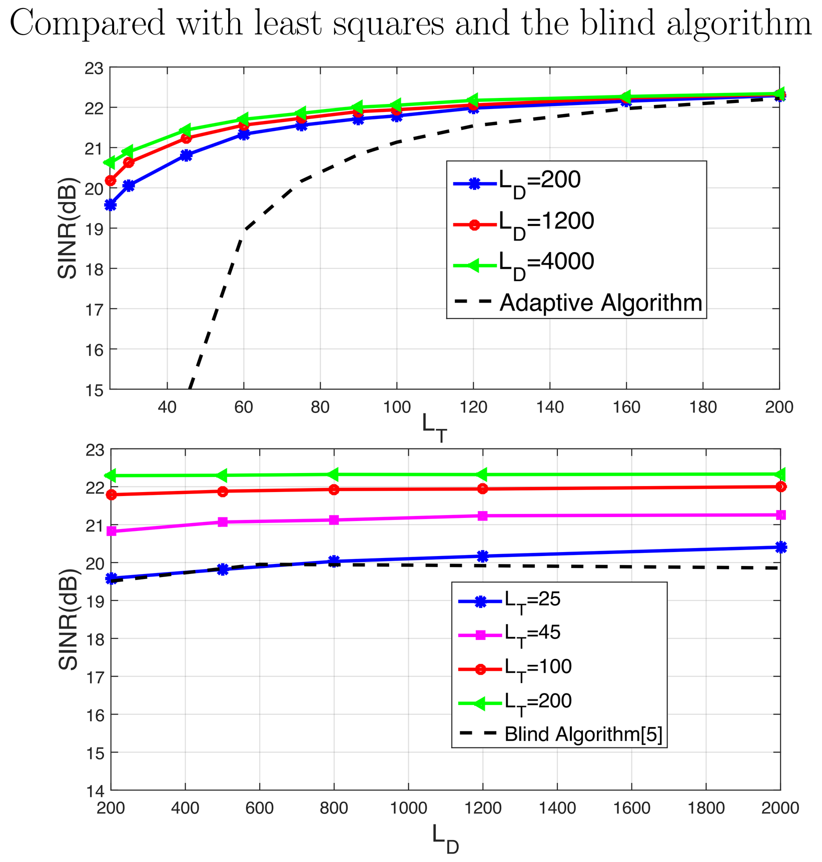
Noiseless Case Communication Example

- Channel Length=15 and Equalizer Length=20,
- Success probability is defined as $\|\mathbf{g}_* - \mathbf{e}_{d+1}\|_2 \le 10^{-5}$,
- Comparison with the algorithm in [4](CMA+LS),
- Empirical probability vs. the bounds and Mean Square Error Performances:



Noisy Case Communication Example

- SNR is chosen as $25dB$,
- Compared with least squares and the blind algorithm in [5]:



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