Iterative Geometry Calibration from Distance Estimates for Wireless Acoustic Sensor Networks

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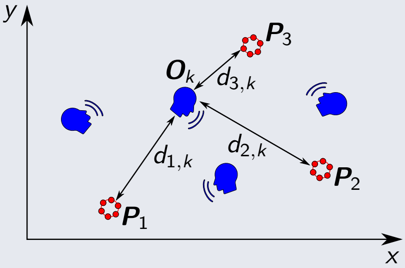
IEEE International Conference on Acoustics, Speech and Signal Processing

6-11 June 2021; Toronto, Ontario, Canada

Motivation

Wireless Acoustic Sensor Networks



- Sensor nodes
  - Positions: $\boldsymbol{P}_n$
- Acoustic sources
  - Positions: $\boldsymbol{O}_k$
- Source-node distances: $d_{n,k}$

Geometry Calibration

Estimation of the positions of the sensor nodes

Optimization Problem

Task

- Estimate all unknown positions $\Omega = \Omega_{\boldsymbol{P}} \cup \Omega_{\boldsymbol{O}}$ from distance estimates [1] $\hat{d}_{n,k}$
  - Node positions: $\Omega_{\boldsymbol{P}} = \{\boldsymbol{P}_1, \dots, \boldsymbol{P}_N\}$
  - Source positions: $\Omega_{\boldsymbol{O}} = \{\boldsymbol{O}_1, \dots, \boldsymbol{O}_K\}$

Cost Function

$$J(\Omega) = \sum_{k=1}^{K} \sum_{n=1}^{N} \left(\hat{d}_{n,k}^{2} - \|\boldsymbol{P}_n - \boldsymbol{O}_k\|_2^2 \right)^2$$

[1] T. Gburrek, J. Schmalenstroeer, A. Brendel, W. Kellermann and R. Haeb-Umbach, "Deep Neural Network based Distance Estimation for Geometry Calibration in Acoustic Sensor Networks," 2020 28th European Signal Processing Conference (EUSIPCO), Amsterdam, Netherlands, 2021, pp. 196-200, doi: 10.23919/Eusipco47968.2020.9287583.

Iterative Optimization

Cost Function

$$J(\Omega) = \sum_{k=1}^{K} \sum_{n=1}^{N} \left(\hat{d}_{n,k}^{2} - \|\boldsymbol{P}_n - \boldsymbol{O}_k\|_2^2 \right)^2$$

Alternating Optimization

1. Keep $\Omega_{\boldsymbol{P}}$ fixed and optimize $\Omega_{\boldsymbol{O}}$: $\hat{\boldsymbol{O}}_k = \underset{\boldsymbol{O}_k}{\operatorname{argmin}} \sum_{n=1}^{N} \left(\hat{d}_{n,k}^{2} - \|\boldsymbol{P}_n - \boldsymbol{O}_k\|_2^2 \right)^2$
2. Keep $\Omega_{\boldsymbol{O}}$ fixed and optimize $\Omega_{\boldsymbol{P}}$: $\hat{\boldsymbol{P}}_n = \underset{\boldsymbol{P}_n}{\operatorname{argmin}} \sum_{k=1}^{K} \left(\hat{d}_{n,k}^{2} - \|\boldsymbol{P}_n - \boldsymbol{O}_k\|_2^2 \right)^2$

Optimization of $\boldsymbol{O}_k$

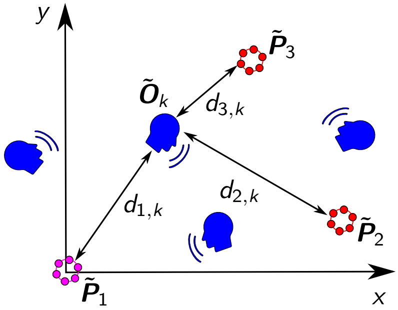
Reference Node

- Linearize the least squares problem [2]
- First node as reference node:
  - $\tilde{\boldsymbol{P}}_n = \boldsymbol{P}_n - \boldsymbol{P}_1$
  - $\tilde{\boldsymbol{O}}_k = \boldsymbol{O}_k - \boldsymbol{P}_1$

System of Equations

$$\underbrace{\left(\tilde{P}_{n,x} - \tilde{O}_{k,x}\right)^2 + \left(\tilde{P}_{n,y} - \tilde{O}_{k,y}\right)^2}_{\left\|\tilde{P}_n - \tilde{O}_k\right\|_2^2} = \hat{d}_{n,k}^2 \qquad \forall n \neq 1$$

$$\tilde{O}_{k,x}^{\ 2} + \tilde{O}_{k,y}^{\ 2} = \hat{d}_{1,k}^2 \qquad n = 1$$



[2] M. A. Spirito, "On the accuracy of cellular mobile station location estimation," in IEEE Transactions on Vehicular Technology, vol. 50, no. 3, pp. 674-685, May 2001, doi: 10.1109/25.933304.

Optimization of $\boldsymbol{O}_k$

Least Squares Problem

$$\underbrace{\begin{bmatrix} 2\tilde{P}_{2,x} & 2\tilde{P}_{2,y} \\ \vdots & \vdots \\ 2\tilde{P}_{N,x} & 2\tilde{P}_{N,y} \end{bmatrix}}_{\boldsymbol{R}} \underbrace{\begin{bmatrix} \tilde{O}_{k,x} \\ \tilde{O}_{k,y} \end{bmatrix}}_{\tilde{\boldsymbol{O}}_k} = \underbrace{\begin{bmatrix} \hat{d}_{1,k}^2 + \tilde{P}_{2,x}^2 + \tilde{P}_{2,y}^2 - \hat{d}_{2,k}^2 \\ \vdots \\ \hat{d}_{1,k}^2 + \tilde{P}_{N,x}^2 + \tilde{P}_{N,y}^2 - \hat{d}_{N,k}^2 \end{bmatrix}}_{\boldsymbol{b}}$$

Weighted Least Squares Solution

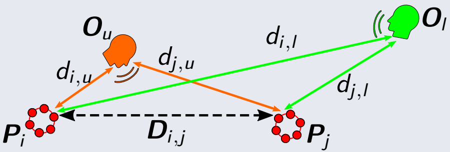
- Heteroscedastic distance estimation error:

$$J(\tilde{\boldsymbol{O}}_k) = \left(\boldsymbol{R}\tilde{\boldsymbol{O}}_k - \boldsymbol{b}\right)^T \boldsymbol{W} \left(\boldsymbol{R}\tilde{\boldsymbol{O}}_k - \boldsymbol{b}\right) \text{ with } W_{n,n} = 1/\tilde{d}_{n,k}^2$$

- Exchange roles of $\boldsymbol{P}$ and $\boldsymbol{O}$ to estimate the nodes' positions

Initialization

Multidimensional Scaling (MDS)



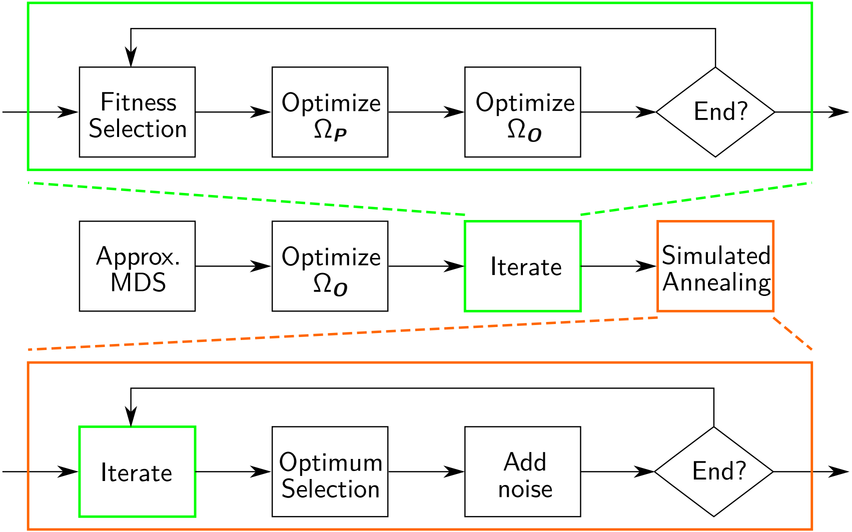
- Needed: Matrix $\boldsymbol{D}$ of inter-node distances $D_{i,j}$ with $i, j \in [1, N]$
- Given: Source-node distances $\boldsymbol{A}_P = \{\hat{d}_{1,1}, \dots, \hat{d}_{N,K}\}$

Approximate MDS

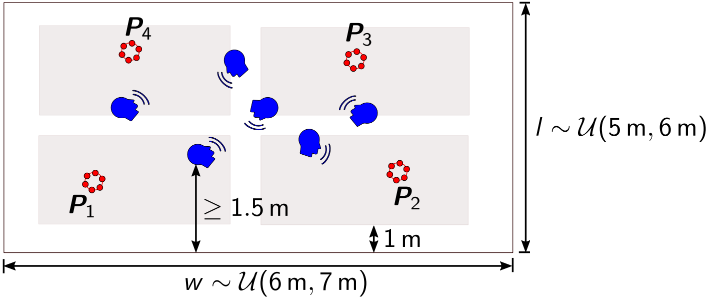
$$\max_l(|\hat{d}_{i,l} - \hat{d}_{j,l}|) \leq D_{i,j} \leq \min_u(\hat{d}_{i,u} + \hat{d}_{j,u})$$

$$\widehat{D}_{i,j} = [\max_l(|\hat{d}_{i,l} - \hat{d}_{j,l}|) + \min_u(\hat{d}_{i,u} + \hat{d}_{j,u})]/2$$

Geometry cAlibration fRom Distance Estimates (GARDE) Algorithm



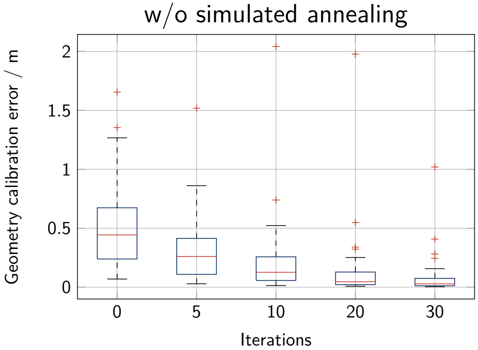
Experiments

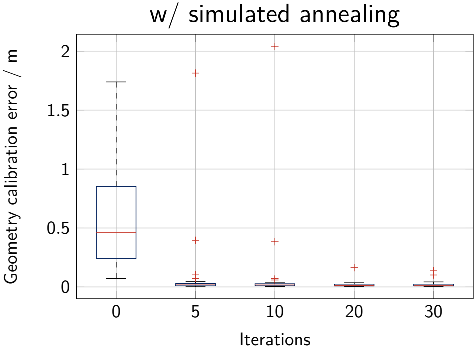


Simulated Data Set

- 40 setups
  - Sensor nodes placed in gray areas
  - Sources (speech from TIMIT database) observed at 500 positions
  - Reverberation time $T_{60} \in \{200\,\mathrm{ms}, 400\,\mathrm{ms}\}$
- Image-source method to simulate room impulse responses (RIRs)

Experiments





- Dependency of GARDE on the number of iterations and annealing (30 rounds)
  - The more iterations the smaller the geometry calibration error
  - Simulated annealing to overcome local minima → smaller errors

Experiments

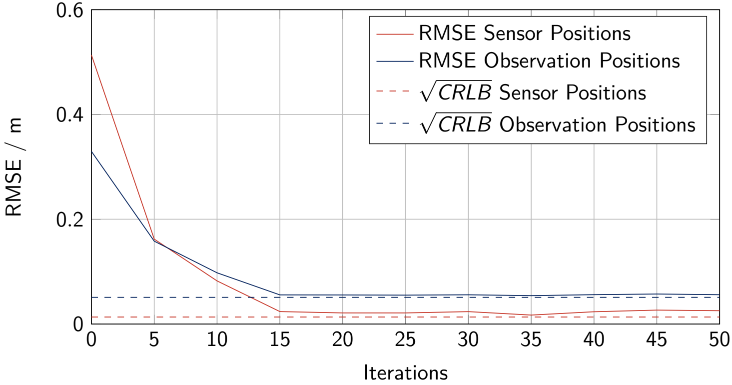
Method	$T_{60} = 200\,\mathrm{ms}$	$T_{60} = 400\,\mathrm{ms}$
DoA-based [3] + Scaling [1]	0.043 m	0.103 m
GARDE	0.017 m	0.032 m

- Root mean squared error (RMSE) of the sensor positions
  - Direction of arrival (DoA) estimates are more affected by reverberation than distance estimates

[3] F. Jacob, J. Schmalenstroeer and R. Haeb-Umbach, "Microphone Array Position Self-Calibration from Reverberant Speech Input," IWAENC 2012; International Workshop on Acoustic Signal Enhancement, Aachen, Germany, 2012, pp. 1-4.

[1] T. Gburrek, J. Schmalenstroeer, A. Brendel, W. Kellermann and R. Haeb-Umbach, "Deep Neural Network based Distance Estimation for Geometry Calibration in Acoustic Sensor Networks," 2020 28th European Signal Processing Conference (EUSIPCO), Amsterdam, Netherlands, 2021, pp. 196-200, doi: 10.23919/Eusipco47968.2020.9287583.

Experiments



- Cramer-Rao lower bounds (CRLBs) of estimator and RMSE of position estimates
  - Derivation of the CRLBs can be found in the paper
  - GARDE gets close to the CRLBs

Conclusions

- GARDE: Iterative weighted least squares based algorithm for geometry calibration
  - Input: Estimates of the source-node distances
  - Results: Positions of the sensor nodes and acoustic sources
- Derivation of the positions' CRLBs
- Simulations have shown promising precision:
  - GARDE is able to outperform our previous DoA-based approach
  - GARDE gets close to the CRLBs

Toolbox

https://github.com/fgnt/paderwasn

Questions?

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