


Domain-Specific Language Abstractions for Compression

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The Case for a Compression DSL

DSL = domain-specific language

- Motivation:** Experts should be able to focus on the compression and not have to worry about implementation details
 - Overall lack of programming language support
 - Current hand-crafted compression systems are exceedingly complex, non-malleable, and non-portable
 - Implementation often requires efforts of many people
- Problem:** Primitives in C/C++ are a poorly matched to the structure of compression algorithms
 - Lack multidimensional data structures, cannot natively capture the complex data dependencies, mismatched control flow, require manual optimization
 - Higher-level languages (Julia, Python/Numpy, etc) provide more multidimensional support
 - Still lack ability to capture the dependencies, control flow, optimizations
- DSL Benefit:** DSLs provide a more intuitive programming interface
 - Data structures that model the multidimensional structure, data access, dependencies
 - Control flow designed for iterating through the data structures
 - Provides concise, high-level syntax
- Problem:** Optimizations
 - Optimizations are necessary for getting practical run-time performance
 - Currently, optimizations are hand-coded, often in assembly--significantly increases the complexity factor
- DSL Benefit:** DSLs enable development of an optimizing compiler
 - Can automatically generate high-performance code
 - Has intrinsic knowledge of the domain-specific abstractions, opening the door for compression-specific optimizations
 - Ex: waveform parallelism, non-SIMD vectorization
- Primary goals**
 - (This work) Develop domain-specific abstractions for block-based compression
 - The abstractions are the main building blocks of DSLs
 - Focusing on image and video: JPEG, WebP, AVC/H.264, HEVC/H.265, VVC/H.266
 - Develop an optimizing compiler based around the abstractions
 - Extend the abstractions for other categories of compression

```
INIT_YMM avx2
globl pixel_sad_64x64, 4,7,6
xorps    m0, m0
xorps    m5, m5
mov     r4d, 8
lea      r6, [r1 * 3]
lea      r6, [r3 * 3]
.loop:
    movu    m1, [r0]
    movu    m2, [r2]
    movu    m3, [r0 + 32]
    movu    m4, [r2 + 32]
    psadbw m1, m2
    psadbw m3, m4
    padddd m0, m1
    padddd m3, m4
    padddd m0, m1
    padddd m3, m4
    movu    m1, [r0 + r1]
    movu    m2, [r2 + r3]
    movu    m3, [r0 + 32 + r1]
    movu    m4, [r2 + 32 + r3]
    psadbw m1, m2
    psadbw m3, m4
    padddd m0, m1
    padddd m3, m4
    movu    m1, [r0 + 2 * r1]
    movu    m2, [r2 + 2 * r3]
    movu    m3, [r0 + 32 + r1 + 32]
    movu    m4, [r2 + 32 + r3 + 32]
    psadbw m1, m2
    psadbw m3, m4
    padddd m0, m1
    padddd m3, m4
    movu    m1, [r0 + r5]
    movu    m2, [r2 + r6]
    movu    m3, [r0 + 32 + r5]
    movu    m4, [r2 + 32 + r6]
    psadbw m1, m2
    psadbw m3, m4
    padddd m0, m1
    padddd m3, m4
    movu    m1, [r0 + 2 * r1]
    movu    m2, [r2 + 2 * r3]
    movu    m3, [r0 + 2 * r1 + 32]
    movu    m4, [r2 + 2 * r3 + 32]
    psadbw m1, m2
    psadbw m3, m4
    padddd m0, m1
    padddd m3, m4
    lea      r2, [r2 + 4 * r3]
    lea      r0, [r0 + 4 * r1]
    movu    m1, [r0]
    movu    m2, [r2]
    movu    m3, [r0 + 32]
    movu    m4, [r2 + 32]
    psadbw m1, m2
    psadbw m3, m4
    padddd m0, m1
    padddd m3, m4
    movu    m1, [r0 + r5]
    movu    m2, [r2 + r6]
    movu    m3, [r0 + 32 + r5]
    movu    m4, [r2 + 32 + r6]
    psadbw m1, m2
    psadbw m3, m4
    padddd m0, m1
    padddd m3, m4
    movd    xm0, xm0
    ret
loop:
    psadbw m1, m2
    vextracti128 xm1, xm0, 1
    psadbw xm0, xm1
    psufsd xm1, xm0, 2
    psadbw xm0, xm1
    psufsd xm0, xm1
    movd    xm0, xm0
    ret
```

Hand-coded assembly for the simple sum-of-absolute-differences operation (from x265 software)

Framework Code Size

JM (H.264 reference)

> 120,000 lines of C/C++

x264

> 68,000 lines of assembly
> 37,000 lines of C/C++

libvpx (vp8)

> 47,000 lines of assembly
> 5,000 lines of C/C++

HM (H.265 reference)

> 60,000 lines of C/C++

x265

> 179,000 lines of assembly
> 96,000 lines of C/C++

aom (AV1 reference)

> 215,000 lines of C/C++

VTM (H.266 reference)

> 134,000 lines of C/C++

kvazaar (H.265 encoder)

> 2,800 lines of assembly
> 32,000 lines of C/C++

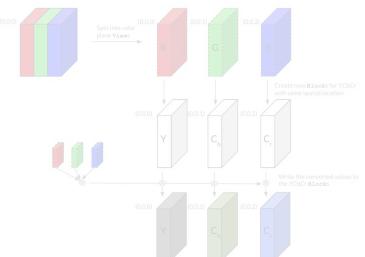
WebP

> 50,000 lines of C/C++

Compression Abstractions

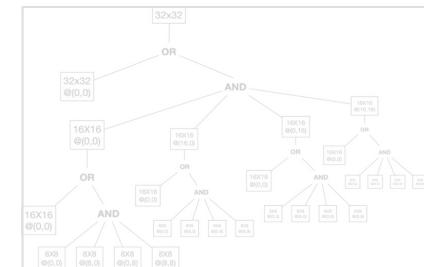
Data Representation

- Goals
 - Provide multidimensional data structures
 - Capture spatial information
 - Provide intuitive data access
 - Support elementwise and reduction operations
- Our abstractions
 - **Block, View, Stream, BitStream**



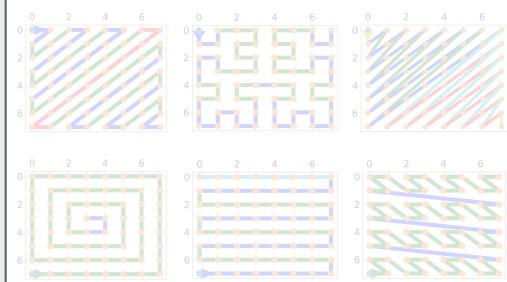
Data Partitioning

- Goals
 - Capture partition options in single tree structure
 - Provide high-level syntax for creating the tree
 - Support tree iteration
 - Preserve spatial information
- Our abstractions
 - **PTree, AndTree**



Data Traversals

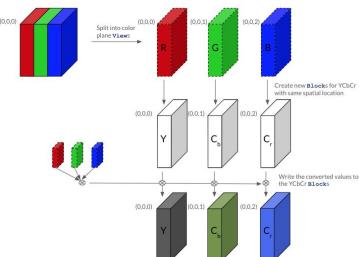
- Goals
 - Capture temporal dependencies that imply data orderings
 - Provide high-level syntax for representing dependencies
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 - **UnitTraversal**



Compression Abstractions

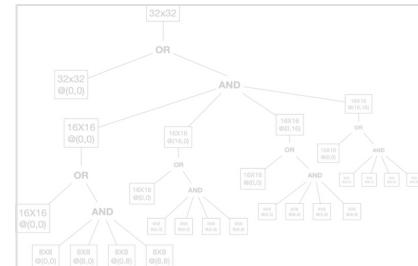
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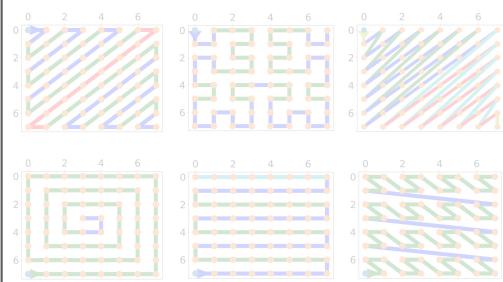
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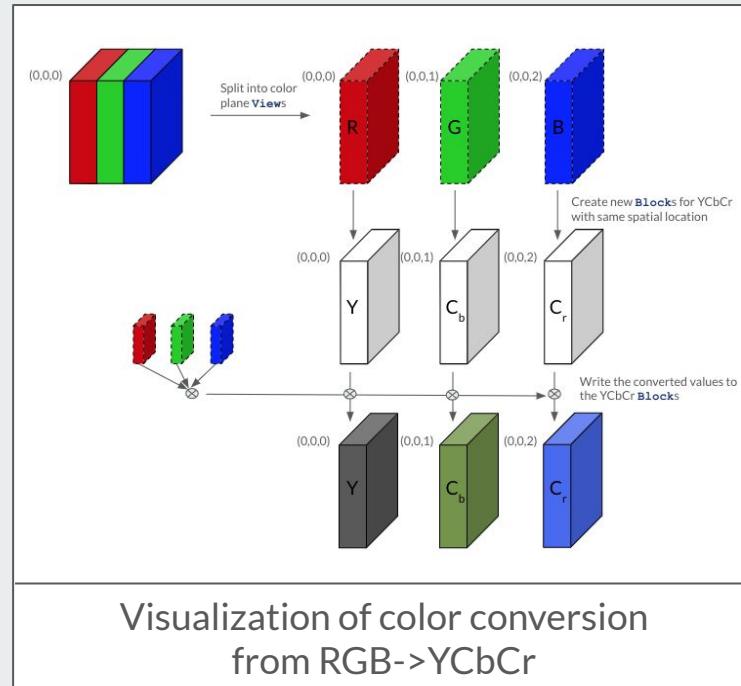
Data Traversals

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Data Representation

- **Abstraction goals**
 - Provide multidimensional data structures
 - Capture the spatial information in regions of data
 - Hide the underlying data storage format
 - Provide more intuitive access to data
 - Prevent unnecessary data copies
 - Support elementwise and reduction operations on data
- **Block abstraction**
 - A multidimensional region of data with an underlying buffer
- **View abstraction**
 - Represents a shared copy of some part of a **Block**'s buffer
 - Reads/writes automatically propagated to the underlying **Block**
 - Allows a for a different data representation without copying
 - Has a flexible slicing syntax for extracting arbitrary regions of data with varying strides
- **Stream/BitStream abstraction**
 - Models the input/output as a possibly infinite series of data elements
 - Bit-level version provides support for common bit operations such as slicing, masking, packing, shuffling, etc
- **Existing implementations**
 - Rely on low-level types like arrays which do not provide any spatial information
 - Programmer has to track all dependencies manually



RGB->YCbCr Color Conversion

```
# Color conversion function
def rgb_to_ycc(R_plane, G_plane, B_plane, rescale, rgb_ycc):
    Y_plane = Block(R_plane)
    Cb_plane = Block(G_plane)
    Cr_plane = Block(B_plane)
    Domain i,j
    Y_plane[i,j] = (rgb_ycc[rescale[R_plane[i,j]],0] +
        rgb_ycc[rescale[G_plane[i,j]],1] +
        rgb_ycc[rescale[B_plane[i,j]],2]) >> 16 - 128
    Cb_plane[i,j] = (rgb_ycc[rescale[R_plane[i,j]],3] +
        rgb_ycc[rescale[G_plane[i,j]],4] +
        rgb_ycc[rescale[B_plane[i,j]],5]) >> 16 - 128
    Cr_plane[i,j] = (rgb_ycc[rescale[R_plane[i,j]],6] +
        rgb_ycc[rescale[G_plane[i,j]],7] +
        rgb_ycc[rescale[B_plane[i,j]],8]) >> 16 - 128
    return Y_plane,Cb_plane,Cr_plane
```

```
image = Stream(w,h,3)
# Separate image into separate color planes
R = image[:, :, 0] # View
G = image[:, :, 1] # View
B = image[:, :, 2] # View
# Color conversion
Y, Cb, Cr = rgb_to_ycc(R, G, B, 3, rescale_values, rgb_ycc_table)
# Signal the type of padding to use
Y_padded = (8 * round(Y.dims[0] / 8), 8 * round(Y.dims[1] / 8))
Y = Block(Y, Y_padded, padding_type=Block.EXTEND)
Cb_padded = (8 * round(Cb.dims[0] / 8), 8 * round(Cb.dims[1] / 8))
Cb = Block(Cb, Cb_padded, padding_type=Block.EXTEND)
Cr_padded = (8 * round(Cr.dims[0] / 8), 8 * round(Cr.dims[1] / 8))
Cr = Block(Cr, Cr_padded, padding_type=Block.EXTEND)
```

With our abstractions: concise, supports multidimensional representation, intuitive access

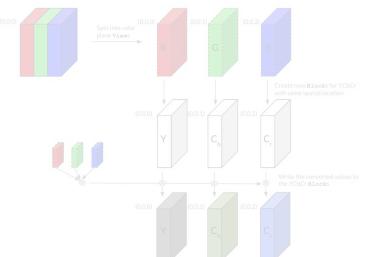
```
// Call color conversion, downsampling (which does right edge padding),
// and bottom edge padding
METHODDEF(void)
pre_process_data (j_compress_ptr cinfo,
JSAMPARRAY input_buf, JDIMENSION *in_row_ctr,
JDIMENSION rows_avail,
JSAMPLEIMAGE output_buf, JDIMENSION *out_row_group_ctr,
JDIMENSION num_rows, int numcols, cl);
JDIMENSION inrows;
my_prep_ptr my_prep = (my_prep_ptr) cinfo->prep;
int numrows;
JDIMENSION inrows;
jpeg_component_info * comptr;
// Iterate through the image rows
while (*in_row_ctr < in_rows_avail &&
*in_row_group_ctr < out_row_groups_avail) {
    // Color conversion
    inrows = in_rows_avail - *in_row_ctr;
    numrows = cinfo->max_v_samp_factor * prep->next_buf_row;
    numrows = (int) MIN((JDIMENSION) numrows, inrows);
    // Calls rgb_ycc_convert(...
    (*cinfo->convert->color_convert)(cinfo, input_buf + *in_row_ctr,
        prep->color_buf, (JDIMENSION) prep->next_buf_row,
        numrows);
    *in_row_ctr += numrows;
    prep->next_buf_row += numrows;
    prep->rows_to_go -= numrows;
    if (prep->next_buf_row == cinfo->max_v_samp_factor) {
        // Calls fullsize_downsample(...
        (*cinfo->downsample->downsample)(cinfo,
            prep->color_buf, (JDIMENSION) 0,
            output_buf, *out_row_group_ctr);
        prep->next_buf_row = 0;
        *out_row_group_ctr++;
    }
    // Pad the bottom
    if (prep->rows_to_go == 0) {
        *out_row_group_ctr < out_row_groups_avail) {
            for (cl = 0, comptr = cinfo->comp_info; cl < cinfo->num_components;
                cl++, comptr++);
            numrows = cinfo->max_v_samp_factor * comptr->DCT_v_scaled_size;
            cinfo->min_DCT_v_scaled_size;
            expand_bottom_edge(output_buf[cl],
                comptr->width_in_blocks * comptr->DCT_h_scaled_size,
                (int) (*out_row_group_ctr * numrows),
                (int) (out_rows_avail * numrows));
        }
        *out_row_group_ctr = out_rows_avail;
        break;
    }
}
// Pad right edge
METHODDEF(void)
expand_right_edge (JSAMPARRAY image_data, int num_rows,
JDIMENSION input_cols, JDIMENSION output_cols) {
register JSAMPROW ptr;
register JSAMPLE pixel;
register int count;
int row;
int numcols = (int) (output_cols - input_cols);
if (numcols > 0) {
    for (row = 0; row < num_rows; row++) {
        ptr = image_data[row] + input_cols;
        pixel = ptr[-1];
        for (count = numcols; count > 0; count--)
            *ptr++ = pixel;
    }
}
// Pad left edge
METHODDEF(void)
expand_bottom_edge (JSAMPARRAY image_data, JDIMENSION num_cols,
int input_rows, int output_rows) {
register int row;
fprintf(stderr, "In expand %d-%d, %d\n", input_rows, output_rows, num_cols);
for (row = input_rows; row < output_rows; row++) {
    copy_sample_rows(image_data, input_rows-1, image_data, row,
        1, num_cols);
}
}
// Copy data and pad the right edge
METHODDEF(void)
fullsize_downsample (j_compress_ptr cinfo, jpeg_component_info * comptr,
JSAMPARRAY input_data, JDIMENSION num_rows, JDIMENSION output_data) {
copy_sample_rows(input_data, 0, output_data, 0,
cinfo->max_v_samp_factor, cinfo->image_width);
expand_right_edge(data, cinfo->max_v_samp_factor, cinfo->image_width,
comptr->width_in_blocks * comptr->DCT_h_scaled_size);
}
// Perform the actual color conversion
METHODDEF(void)
rgb_ycc_convert (j_compress_ptr cinfo,
JSAMPARRAY input_buf, JSAMPLEIMAGE output_buf,
JDIMENSION output_row, int num_rows) {
my_convert_ptr convert = (my_convert_ptr) cinfo->convert;
register int r, g, b;
// This is what is generated by rgb_ycc_start
register int R = convert->convert->rgb_ycc_tab;
register JSAMPROW input0;
register JSAMPROW output0, output1, output2;
register JDIMENSION col;
JDIMENSION num_cols = cinfo->image_width;
// Iterate through each row of the current part of the image
while (row < num_rows) {
    // Pull out the sample for each color component
    input0 = *input_buf++;
    output0 = output_buf[0][output_row];
    output1 = output_buf[1][output_row];
    output2 = output_buf[2][output_row];
    output_row++;
    // Iterate through the columns of the row
    for (col = 0; col < num_cols; col++) {
        // Get the individual color samples from each row
        r = GETSAMPLE(input0);
        g = GETSAMPLE(input0[RGB_GREEN]);
        b = GETSAMPLE(input0[RGB_BLUE]);
        input0 += RGB_PIXELSIZE;
        // Do the actual conversion
        // ...
        output0[0] = (JSAMPLE)
            ((ctab[r+R_Y_OFFSET] + ctab[g+G_Y_OFFSET] + ctab[b+B_Y_OFFSET])
            >> SCALBITS);
        // Cb
        output1[0] = (JSAMPLE)
            ((ctab[r+R_CB_OFFSET] + ctab[g+G_CB_OFFSET] + ctab[b+B_CB_OFFSET])
            >> SCALBITS);
        // Cr
        output2[0] = (JSAMPLE)
            ((ctab[r+R_CR_OFFSET] + ctab[g+G_CR_OFFSET] + ctab[b+B_CR_OFFSET])
            >> SCALBITS);
    }
}
```

IJG implementation: long, most code dedicated to computing appropriate 1D indices

Compression Abstractions

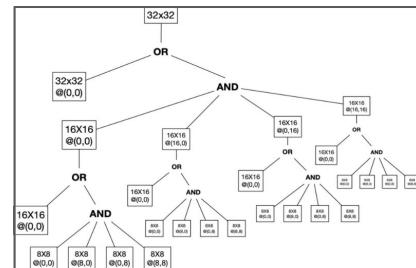
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 - Provide intuitive data access
 - Support elementwise and reduction operations
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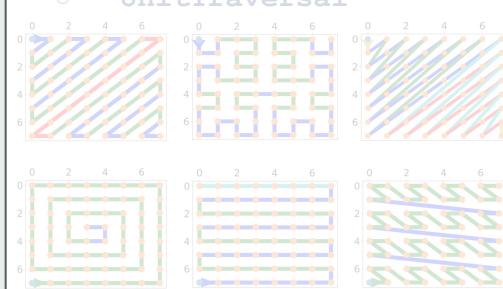
Data Partitioning

- Goals
 - Capture partition options in single tree structure
 - Provide high-level syntax for creating the tree
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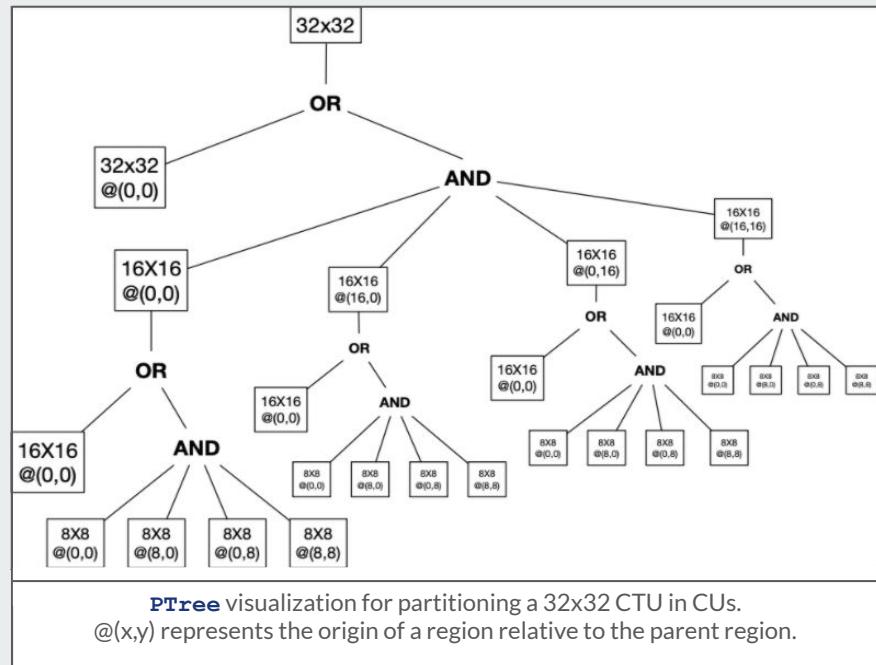
Data Traversals

- Goals
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Data Partitioning

- **Abstraction goals**
 - Provide tree data structure that compactly represents all the possible ways to partition a region into sub-regions
 - Provide compact syntax for creating the tree
 - Support iteration through all sub-regions
 - Relates to data traversal abstraction described later
 - Preserve all spatial information needed for data representation
 - **Ptree abstraction**
 - An And-Or tree representing the partition options
 - And node: defines one way to partition a region
 - Or node: defines options for partitioning a region
 - Iteration returns each unique partition as an **AndTree**
 - **AndTree abstraction**
 - Represents one unique partition of the **Ptree** root
 - Iteration returns the leaf sub-regions
 - **Existing implementations**
 - May not explicitly express the partition
 - Wrapped into index calculations, utilize lookup tables
 - Inflexible!



CU and PU Partitions in HEVC/H.265

```
# define PTree for CTU->CU
def CTU_to_CUs(ctu):
    M = ctu.dims[0]
    ptree = PTree()
    pt_root(ctu, ptree):
        pt_or:
            pt_leaf(ctu)
        if M > 8:
            pt_and:
                q0 = ctu[:M/2, :M/2]
                q1 = ctu[M/2:, :M/2]
                q2 = ctu[:M/2, M/2:]
                q3 = ctu[M/2:, M/2:]
            pt_leaf(CTU_to_CUs(q0))
            pt_leaf(CTU_to_CUs(q1))
            pt_leaf(CTU_to_CUs(q2))
            pt_leaf(CTU_to_CUs(q3))
        return ptree
```

```
CTU = ...
# partition CTU into CUs
cu_ptree = CTU_to_CUs(CTU)
# iterate through each option (AndTree)
for cu_andtree in scan(cu_ptree):
    # iterate through each CU
    for CU in scan(cu_andtree):
        # partition CU into PUs
        pu_tree = CU_to_PUs(CU)
        # iterate through each PU
        for PU in scan(pu_tree):
            # get the data to the above and right
            # using location relative to the PU
            above_right = data[PU[-1, right_range]]
        return ptree
```

With our abstractions: high-level and compact recursive notation, easy to modify tree, straightforward iteration

```
# define PTree for CU->PU
def CU_to_PUs(cu):
    M = cu.dims[0]
    ptree = PTree()
    pt_root(cu, ptree):
        pt_or:
            pt_leaf(cu)
        pt_and:
            pt_leaf(cu[:, :M/2])
            pt_leaf(cu[:, M/2:])
        pt_and:
            pt_leaf(cu[:M/2, :])
            pt_leaf(cu[M/2:, :])
        if M == min_CU_size:
            pt_and:
                pt_leaf(cu[:M/2, :M/2])
                pt_leaf(cu[:M/2, M/2:])
                pt_leaf(cu[M/2:, :M/2])
                pt_leaf(cu[M/2:, M/2:])
        if M > 8:
            pt_and:
                pt_leaf(cu[:, :M/4])
                pt_leaf(cu[:, M/4:])
            pt_and:
                pt_leaf(cu[:, :3M/4])
                pt_leaf(cu[:, 3M/4:])
            pt_and:
                pt_leaf(cu[:M/4, :])
                pt_leaf(cu[M/4:, :])
            pt_and:
                pt_leaf(cu[:3M/4, :])
                pt_leaf(cu[3M/4:, :])
        return ptree
```

```
m_cuabove = (m_cuAddr >= widthInCU) && !m_pFirstRowInSlice ?
    m_endData->getPicCU(m_cuAddr - widthInCU) : NULL;
m_cubelowRight = (m_cuabove && (m_cuAddr % widthInCU) < (widthInCU - 1)) ?
    m_endData->getPicCU(m_cuAddr - widthInCU + 1) : NULL;

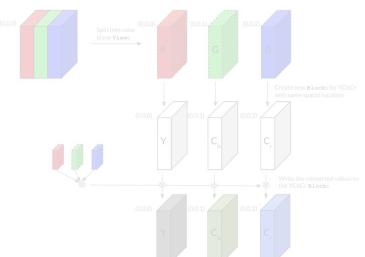
const CUData* CUData* g_getPUaboveRight(uint32_t_k arPartUnitIdx, uint32_t_k
    cuPartUnitIdx) const {
    if (!m_endData->getPicCU(m_cuAddr) && !m_cubelowRight && g_zscanToPelix[curPartUnitIdx] +
        UNIT_SIZE) >= m_slice.m_sps->picWidthInLumaSamples)
        return NULL;
    uint32_t absPartUnitIdx = g_zscanToRaster[curPartUnitIdx];
    if ((absPartUnitIdx < m_absIdxInCTU) && (absPartUnitIdx < m_numPartInCUSize - 1)) {
        if (absPartUnitIdx > g_rasterToZScan[absPartUnitIdx - RASTER_SIZE + 1]) {
            uint32_t absEndCUIdx = g_zscanToRaster[m_absIdxInCTU] + (1 <<
                m_log2CUSize[0] - LOG2_UNIT_SIZE) - 1;
            arPartUnitIdx = g_rasterToZScan[absPartUnitIdx - RASTER_SIZE + 1];
            if (isEqualRowOrCol(absPartUnitIdx, absEndCUIdx))
                return m_endData->getPicCU(m_cuAddr);
            else {
                arPartUnitIdx -= m_absIdxInCTU;
                return this;
            }
        }
        return NULL;
    }
    arPartUnitIdx = g_rasterToZScan[absPartUnitIdx] + ((s_numPartInCUSize - 1) <<
        LOG2_RASTER_SIZE + 1);
    return m_cuabove;
}

const uint32_t_k g_rasterToZScan[MAX_NUM_PARTITIONS] =
{
    0x00, 0x01, 0x04, 0x05, 0x10, 0x11, 0x14, 0x15, 0x40, 0x41, 0x44, 0x45, 0x50, 0x51, 0x54, 0x55,
    0x02, 0x03, 0x06, 0x07, 0x12, 0x13, 0x16, 0x17, 0x42, 0x43, 0x46, 0x47, 0x52, 0x53, 0x56, 0x57,
    0x08, 0x09, 0x0C, 0x0D, 0x18, 0x19, 0x1C, 0x1D, 0x48, 0x49, 0x4C, 0x4D, 0x58, 0x59, 0x5C, 0x5D,
    0x0A, 0x0B, 0x0E, 0x0F, 0x1A, 0x1B, 0x1F, 0x20, 0x44, 0x45, 0x53, 0x54, 0x5A, 0x5B, 0x5E, 0x5F,
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    0x22, 0x23, 0x26, 0x27, 0x32, 0x33, 0x36, 0x37, 0x63, 0x64, 0x67, 0x68, 0x72, 0x73, 0x76,
    0x24, 0x25, 0x28, 0x29, 0x34, 0x35, 0x38, 0x39, 0x66, 0x67, 0x6A, 0x6B, 0x74, 0x75, 0x78,
    0x26, 0x27, 0x30, 0x31, 0x35, 0x36, 0x39, 0x3A, 0x68, 0x69, 0x6B, 0x6C, 0x76, 0x77, 0x7A,
    0x28, 0x29, 0x32, 0x33, 0x36, 0x37, 0x3A, 0x3B, 0x6A, 0x6B, 0x6C, 0x6D, 0x78, 0x79, 0x7B,
    0x2A, 0x2B, 0x2E, 0x2F, 0x32, 0x33, 0x36, 0x37, 0x65, 0x66, 0x69, 0x6A, 0x7C, 0x7D, 0x7F,
    0x2C, 0x2D, 0x2F, 0x2G, 0x33, 0x34, 0x37, 0x38, 0x67, 0x68, 0x6B, 0x6C, 0x75, 0x76, 0x79,
    0x2E, 0x2F, 0x30, 0x31, 0x34, 0x35, 0x38, 0x39, 0x69, 0x6A, 0x6B, 0x6C, 0x77, 0x78, 0x7B,
    0x2G, 0x2H, 0x31, 0x32, 0x35, 0x36, 0x39, 0x3A, 0x6B, 0x6C, 0x6D, 0x7A, 0x7B, 0x7C, 0x7D,
    0x2I, 0x2J, 0x33, 0x34, 0x37, 0x38, 0x3B, 0x3C, 0x6B, 0x6C, 0x6D, 0x76, 0x77, 0x7A, 0x7B,
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    0x2M, 0x2N, 0x37, 0x38, 0x3B, 0x3C, 0x6D, 0x6E, 0x6F, 0x7A, 0x7B, 0x7C, 0x7D, 0x7E,
    0x2O, 0x2P, 0x39, 0x3A, 0x6E, 0x6F, 0x6G, 0x7C, 0x7D, 0x7E, 0x7F, 0x7B,
    0x2Q, 0x2R, 0x3B, 0x3C, 0x6F, 0x6G, 0x6H, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D,
    0x2S, 0x2T, 0x3D, 0x3E, 0x6B, 0x6C, 0x6D, 0x77, 0x78, 0x79, 0x7A, 0x7B, 0x7C, 0x7D, 0x7E,
    0x2U, 0x2V, 0x3F, 0x3G, 0x6B, 0x6C, 0x6D, 0x79, 0x7A, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D,
    0x2W, 0x2X, 0x3H, 0x3I, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
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    0x2W, 0x2X, 0x3H, 0x3I, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
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    0x2A, 0x2B, 0x3L, 0x3M, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
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    0x2E, 0x2F, 0x3P, 0x3Q, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2G, 0x2H, 0x3R, 0x3S, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
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    0x2O, 0x2P, 0x3Z, 0x3A, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2Q, 0x2R, 0x3B, 0x3C, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2S, 0x2T, 0x3D, 0x3E, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2U, 0x2V, 0x3F, 0x3G, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2W, 0x2X, 0x3H, 0x3I, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2Y, 0x2Z, 0x3J, 0x3K, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
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    0x2Y, 0x2Z, 0x3J, 0x3K, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2A, 0x2B, 0x3L, 0x3M, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2C, 0x2D, 0x3N, 0x3O, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2E, 0x2F, 0x3P, 0x3Q, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2G, 0x2H, 0x3R, 0x3S, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2I, 0x2J, 0x3T, 0x3U, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2K, 0x2L, 0x3V, 0x3W, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2M, 0x2N, 0x3X, 0x3Y, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2O, 0x2P, 0x3Z, 0x3A, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2Q, 0x2R, 0x3B, 0x3C, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2S, 0x2T, 0x3D, 0x3E, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2U, 0x2V, 0x3F, 0x3G, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2W, 0x2X, 0x3H, 0x3I, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2Y, 0x2Z, 0x3J, 0x3K, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2A, 0x2B, 0x3L, 0x3M, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2C, 0x2D, 0x3N, 0x3O, 0x6B, 0x6C, 0x6D, 0x7B, 0x7C, 0x7D, 0x7E, 0x7F, 0x7D, 0x7E,
    0x2E, 0x2F, 0x3P, 0x3Q, 0x6B, 0
```

Compression Abstractions

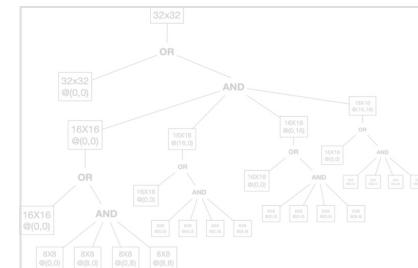
Data Representation

- Goals
 - Provide multidimensional data structures
 - Capture spatial information
 - Provide intuitive data access
 - Support elementwise and reduction operations
- Our abstractions
 - **Block, View, Stream, BitStream**



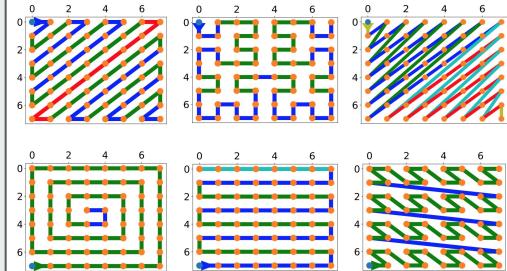
Data Partitioning

- Goals
 - Capture partition options in single tree structure
 - Provide high-level syntax for creating the tree
 - Support tree iteration
 - Preserve spatial information
- Our abstractions
 - **PTree, AndTree**



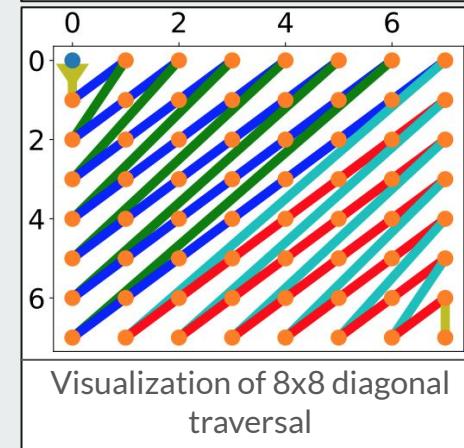
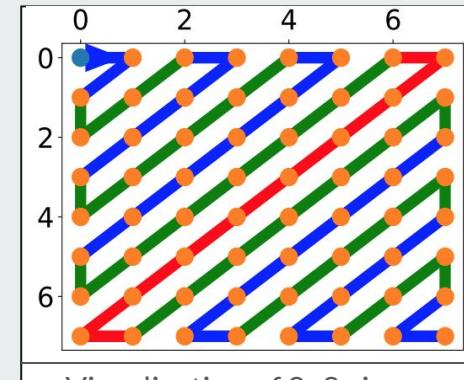
Data Traversals

- Goals
 - Capture temporal dependencies that imply data orderings
 - Provide high-level syntax for representing dependencies
 - Support iteration with the orderings
- Our abstractions
 - **UnitTraversal**



Data Traversal

- **Abstraction goals**
 - Capture the intricate temporal dependencies that define a data ordering
 - Orderings can be within or across multidimensional data regions
 - Provide syntax for succinctly representing these temporal dependencies
 - Support iteration across **Blocks**, **Views**, **Streams**, and **BitStreams** using a specified traversal
- **AndTree abstraction**
 - Same **AndTree** from data partitioning
 - The order of the leaves imposes an iteration order across regions
 - Changing the traversal order just requires changing the leaves
- **UnitTraversal abstraction**
 - Defines an iteration order within a region
 - We utilize a recursive notation based on parametric Lindenmayer systems¹ to compactly define the order
- **Existing implementations**
 - Manually enumerate each coordinate or require loop nests with many conditionals
 - Manual enumerations introduce indirect data accesses
 - Lack a common syntax

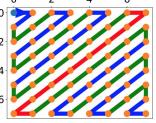


Zigzag and Diagonal Traversals

```

N = ...region size...
zz = UnitTraversal(blk)
zz.axiom([A(1)])
zz.rule(A(i), [ST,135,ST*i,315,B(i+1),45,ST*i,225,ST], i<N)
zz.rule(B(i), [ST,225,ST*i,45,A(i+1),315,ST*i,135,ST], i<N)
zz.rule(A(i), [ST,135,ST*(size-1),225,ST], i==N)
zz.rule(B(i), [ST,225,ST*(size-1),135,ST], i==N)

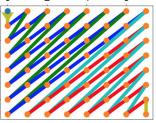
```



```

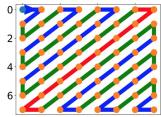
N = ...region size...
diagonal = UnitTraversal(blk)
diagonal.axiom([90,ST,A(1,N-1),135,ST])
diagonal.rule(A(i,j),[225,ST*i,B(i+1,j)],i<N)
diagonal.rule(B(i,j),[225,SK*(i-1),270,SK*i,A(i,j)],i<N)
diagonal.rule(A(i,j),[225,ST*(j-1),B(i,j-1)],(i==N) & (j>1))
diagonal.rule(B(i,j),[225,SK*(j-1),270,SK*j,A(i,j)],(i==N) & (j>1))

```

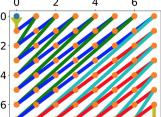


With our abstraction: high-level recursive notation, defines traversal as series of steps and rotations, easy to modify

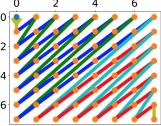
```
// IJG implementation
const int jpeg_natural_order[DCTSIZE2+16] = {
0, 1, 8, 16, 9, 2, 3, 10,
17, 24, 32, 25, 18, 11, 4, 5,
12, 19, 26, 33, 40, 48, 41, 34,
27, 20, 13, 6, 7, 14, 21, 28,
35, 42, 49, 56, 57, 50, 43, 36,
29, 22, 15, 23, 30, 37, 44, 51,
58, 59, 52, 45, 38, 31, 39, 46,
53, 60, 61, 54, 47, 55, 62, 63};
```



```
// HM implementation
if ((m_column == (m_blockWidth - 1)) || (m_line == 0)) {
    m_line    += m_column + 1;
    m_column  = 0;
    if (m_line >= m_blockHeight) {
        m_column += m_line - (m_blockHeight - 1);
        m_line    = m_blockHeight - 1;
    }
} else {
    m_column++;
    m_line--;
}
```



x265 implementation



IJG, HM, x265 implementations: manual enumerations (for each block size) or conditional blocks

Looking Ahead: DSL Compiler Support

- An optimizing compiler for a DSL can automatically generate high-performance code
 - Optimization by hand is hard, especially when there are multiple possible optimizations that can be combined
 - The DSL compiler is designed around the set of abstractions
 - Allows it to perform optimizations beyond what you would get with a general compiler
- Three categories of optimizations we are interested in: parallelization, vectorization, speculation
 1. Parallelization
 - Wavefront, frame/tile/slice, mode selection, etc
 2. Vectorization
 - Generate vector instructions for expensive computation kernels such as transforms
 - Provide support for both SIMD and non-SIMD vectorization
 - Reduce the reliance on hand-coded vector instructions
 3. Speculation
 - Relieve sequential bottlenecks by speculatively performing later computations

Put the Focus on the Algorithm!

- Experts should be able to focus on the compression and not have to worry about implementation details
- Benefits of a DSL
 - More intuitive programming interface
 - Primitive data structures, control flow, operations, etc. that capture the structure of compression algorithms
 - Automatic generation of high-performance code
- Primary goals
 1. (This work) Develop domain-specific abstractions for block-based compression
 - The abstractions are the main building blocks of DSLs
 - Focusing on image and video: JPEG, WebP, AVC/H.264, HEVC/H.265, VVC/H.266
 2. Develop an optimizing compiler based around the abstractions
 3. Extend the abstractions for other categories of compression
- Faced your own compression-related implementation challenges?
 - We'd love to hear from you!
- Contact Jessica Ray: jray@csail.mit.edu
- Comparisons with existing frameworks: <http://jray.mit.edu/block-based-compression>