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Compressive Sensing via Unfolded ℓ_0-constrained Convolutional Sparse Coding

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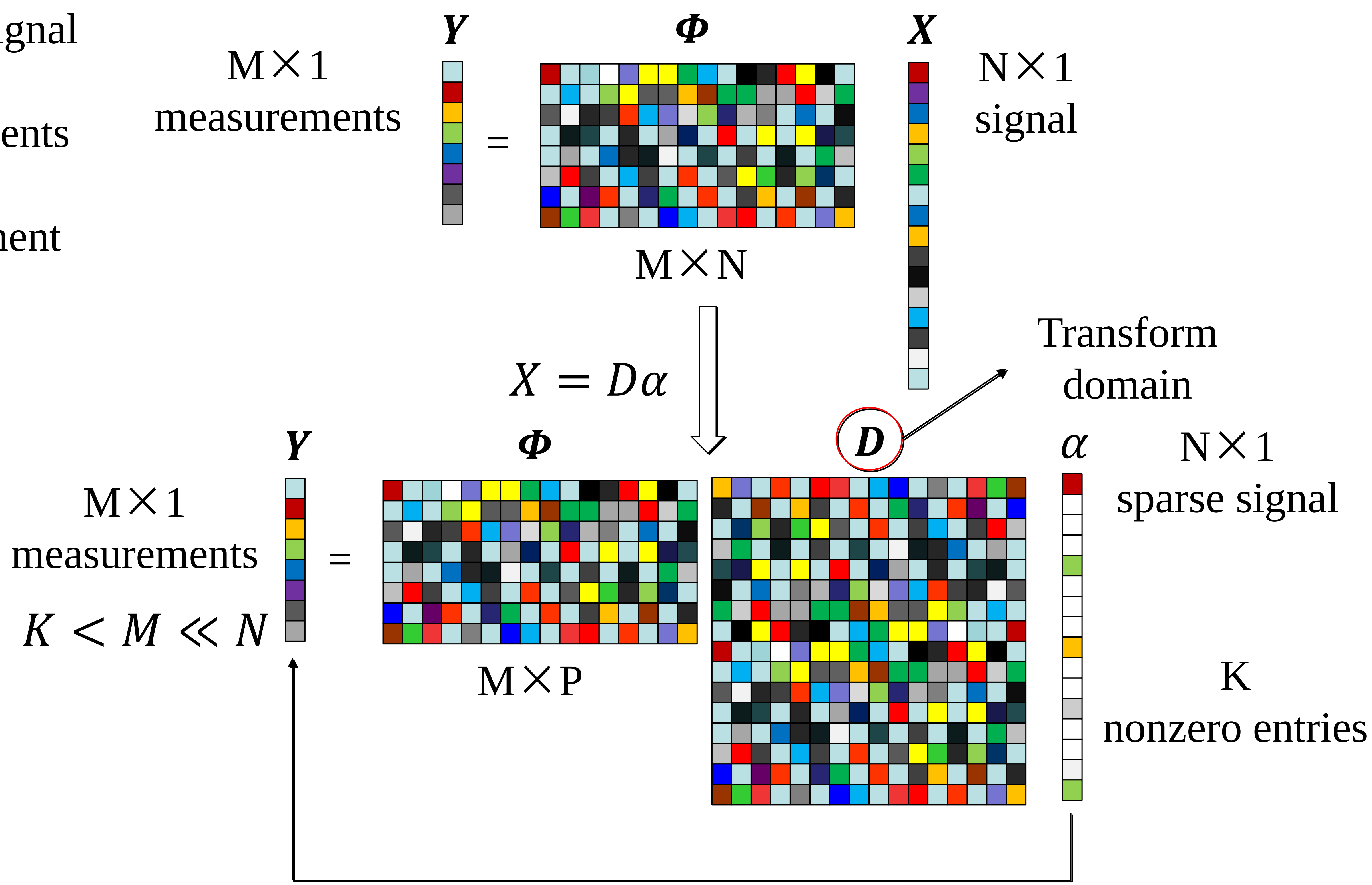
Contents

Compressive Sensing

X: Original signal

Y: Measurements

Φ: Measurement matrix



Reconstruct original signal by solving optimization problem

$$\tilde{x} = \arg\min_{x=D\alpha}\{\frac{1}{2}\|\Phi D\alpha - y\|_2^2 + g(\alpha)\}$$ → Regularization

Compressive Sensing

Objective function: $\tilde{x} = \arg\min_{x=D\alpha} \{\frac{1}{2}\|\Phi D\alpha - y\|_2^2 + g(\alpha)\}$

ill Problem

$g(\alpha) = \|\alpha\|_0$ Non-convex, lower semi-continuous, semi-algebraic

$g(\alpha) = \|\alpha\|_1$ convex, continuous

Conventional optimization methods:

Focal under determined system solver (FOCUSS)
Iteratively reweighted least square (IRLS)
Bayesian evolutionary pursuit algorithm (BEPA)

Advantages:

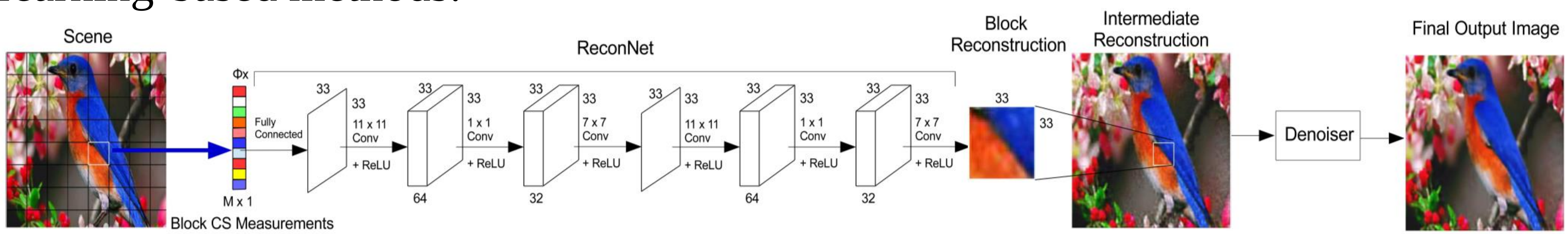
- Strong convergence
- Theoretical analysis

Disadvantages:

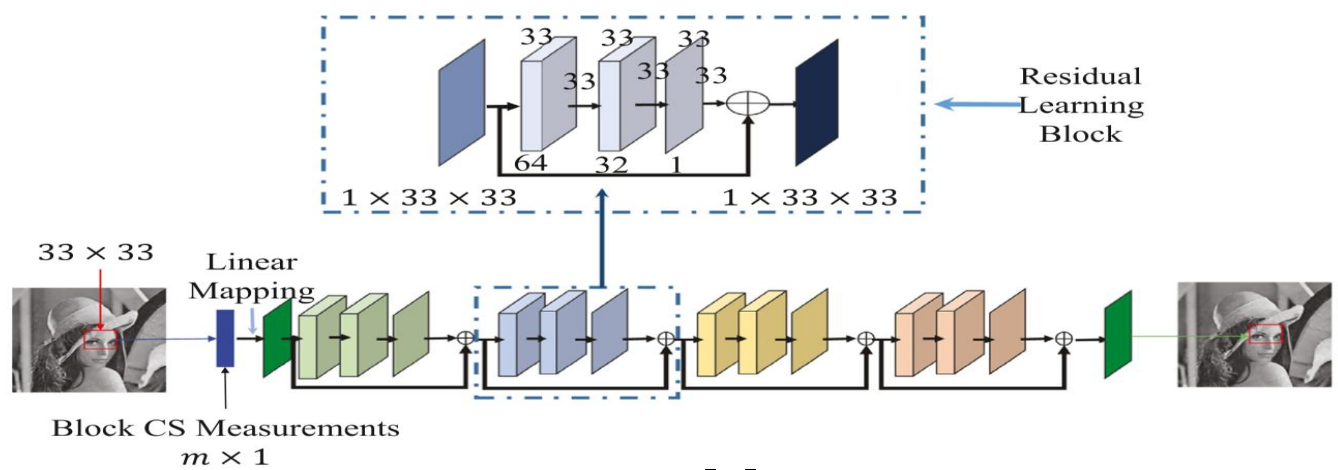
- High computational complexity
- Hard to choose optimal transforms and tune parameters

Compressive Sensing

Deep Learning-based methods:



ReconNet[1]



DR2-Net[2]

Advantages:

- Low time complexity
- Impressive reconstruction performance

Disadvantages:

- Hard to design the framework
- Lack theoretical guarantee

[1] Kulkarni K, Lohit S, Turaga P, et al. Reconnet: Non-iterative reconstruction of images from compressively sensed measurements [C]//Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. 2016: 449-458.
[2] Yao H, Dai F, Zhang S, et al. Dr2-net: Deep residual reconstruction network for image compressive sensing[J]. Neurocomputing, 2019, 359: 483-493.

Compressive Sensing

Alternating Direction Multiplier Method(ADMM)-based:

ADMM-Net: A Deep Learning Approach for Compressive Sensing MRI [1]

D-LADMM: Differentiable Linearized ADMM [2]

- Only apply for $\Phi = PF$, which means the measurements sharing the same size with original signal

Iterative Shrinkage Thresholding Algorithm(ISTA)-based:

ISTA-Net: Interpretable Optimization-Inspired Deep Network for Image Compressive Sensing [3]

- Solve a ℓ_1-norm constraint problem
- Don't strictly follow the iterative optimization of ISTA

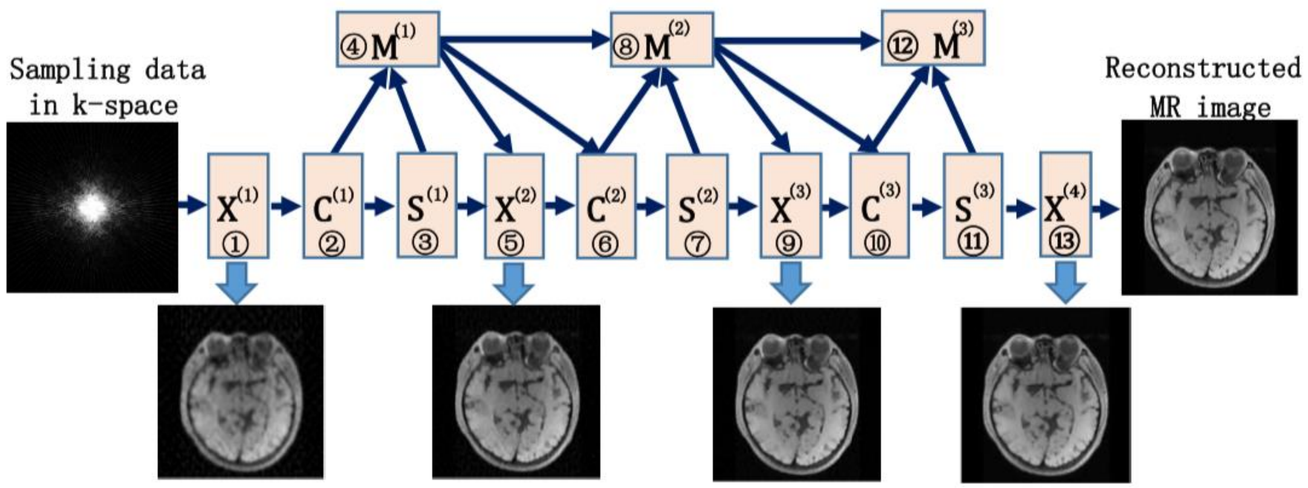
[1] Sun J, Li H, Xu Z. Deep ADMM-Net for compressive sensing MRI[J]. Advances in neural information processing systems, 2016, 29: 10-18.
[2] Xie, Xingyu, et al. "Differentiable linearized admm." International Conference on Machine Learning. PMLR, 2019.
[3] Zhang, Jian, and Bernard Ghanem. "ISTA-Net: Interpretable optimization-inspired deep network for image compressive sensing." Proceedings of the IEEE conference on computer vision and pattern recognition. 2018.

Compressive Sensing

ADMM-Net

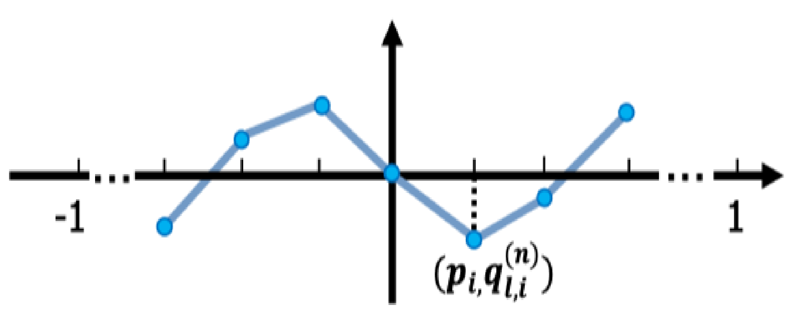
$$\arg\min_x \frac{1}{2}\|Ax-y\|_2^2 + \sum_{l=1}^{L}\lambda_l \boxed{g(z_l)},\ s.t.\ z_l = D_l x$$

$$\begin{cases} x^n = F^T[P^TP + \sum_{l=1}^{L}\rho_l F D_l^T D_l F^T]^{-1} \\ \quad [P^Ty + \sum_{l=1}^{L}\rho_l F D_l^T(z_l^{n-1} - \beta_l^{n-1})] \\ z_l^n = \mathcal{S}(D_l x^n + \beta_l^{n-1}; \lambda_l/\rho_l) \\ \beta_l^n = \beta_l^{n-1} + \eta_l(D_l x^n - z_l^n) \end{cases}$$



ADMM-Net[1]

$\mathcal{S}$



Drawbacks:

- The sampling matrix must be $\Phi = PF$
- The framework failed to derive from ADMM strictly
- the objective function failed to obey theory of sparse coding

[1] Sun J, Li H, Xu Z. Deep ADMM-Net for compressive sensing MRI[J]. Advances in neural information processing systems, 2016, 29: 10-18.

Contents

Convolutional Sparse Coding

Convolutional Sparse Coding

$$\min_{\{Z_i\}_{i=1}^m} \sum_{i=1}^{m} \|Z_i\|_0 \ \text{s.t.}\ X = \sum_{i=1}^{m} d_i * Z_i$$

X: Original Signal

Z_i: K-sparse signal

d_i: Convolutional filters

Merits of CSC:

- Learns a shift-invariant dictionary.
- Reduces dictionary redundancy.

Drawback:

- slow convergence speed
- rigid iterative structures of parameters.

Proposed Model:

$$\min_{x,\alpha,z} \frac{1}{2}\|\Phi x - y\|_2^2 + \lambda\Omega(z), \quad \text{s.t. } x = D\alpha = d * \alpha, z = \alpha$$

$$\Omega(z) = \|z\|_0$$

$$X = \sum_{i=1}^{m} d_i * Z_i \longrightarrow D \times Z$$

$$D = [D_1, D_2, D_3 \cdots D_\mathrm{m}]$$

$$D_i = Toep(d_i) = d_i * \mathrm{I}$$

[1] Zeiler M D, Krishnan D, Taylor G W, et al. Deconvolutional networks[C]//2010 IEEE Computer Society Conference on computer vision and pattern recognition. IEEE, 2010: 2528-2535.

Compressive Sensing via Unfolded ι_0-constrained Convolutional Sparse Coding

Objective function: $$\min_{x,\alpha,z} \frac{1}{2}\|\Phi x - y\|_2^2 + \lambda\Omega(z), \quad \text{s.t. } x = D\alpha = d * \alpha, z = \alpha$$

Steps:

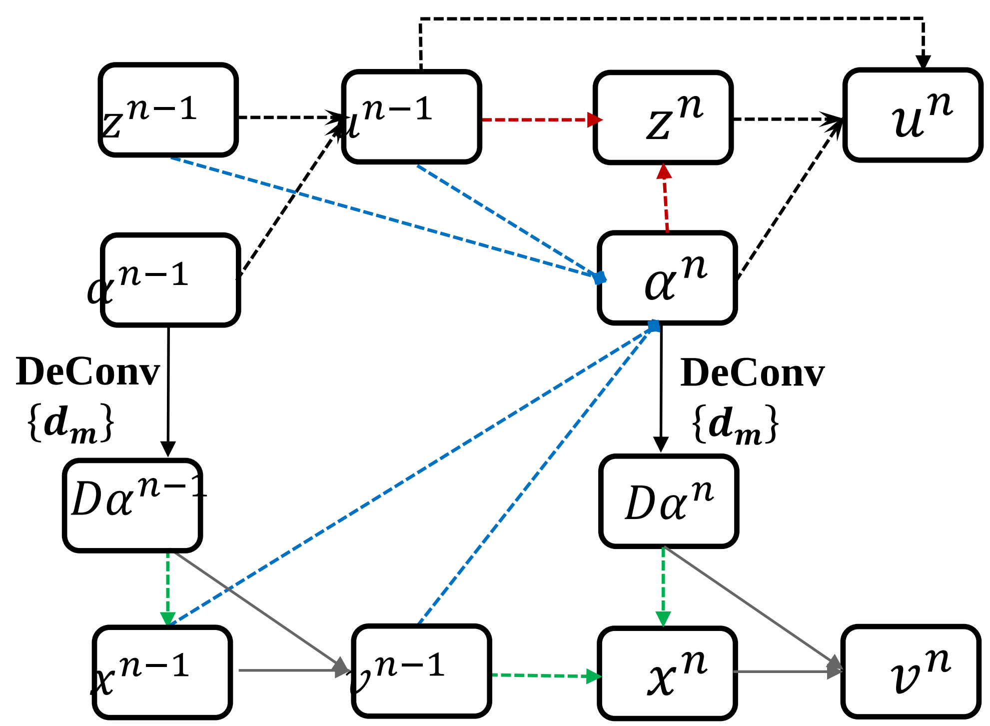
$$\alpha^{t+1} = [I + \rho D^T D]^{-1}[\rho D^T(x^t - v^t) + (z^t - u^t)]$$

$$z^{t+1} = prox_{\|\cdot\|_0 \lambda/\rho_2}(\alpha^{t+1} + u^t)$$

$$u^{t+1} = u^t + \alpha^{t+1} - z^{t+1}$$

$$x^{t+1} = [I + \frac{1}{\rho_1}\Phi^T\Phi]^{-1}[\rho_1\Phi^T y + D\alpha^{t+1} + v^{t+1}]$$

$$v^{t+1} = v^t + D\alpha^{t+1} - x^{t+1}$$



• Jiaqi Sun, Wenrui Dai, Chenglin Li, Junni Zou, Hongkai Xiong, "Compressive Sensing via Unfolded l0-constrained Convolutional Sparse Coding," **DCC 2021**.

Compressive Sensing via Unfolded ι_0-constrained Convolutional Sparse Coding

Objective function: $\min_{x,\alpha,z} \frac{1}{2}\|\Phi x - y\| + \lambda\Omega(z), \quad s.t.\, x = D\alpha = d * \alpha, z = \alpha$

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$$x^{t+1} = [I + \frac{1}{\rho_1}\Phi^T\Phi]^{-1}[\rho_1 \Phi^T y + D\alpha^{t+1} + v^{t+1}]$$

$$v^{t+1} = v^t + D\alpha^{t+1} - x^{t+1}$$

$[I + \rho D^T D]^{-1}$ → $\mathbf{I} + a_1\mathbf{M} + a_2\mathbf{M}^2 + \cdots$

$$\mathbf{M} = \mathbf{B}^{\mathbf{T}}\, \mathrm{Re}\, Lu(\mathbf{B})$$

Merits:

- derived strictly from iterative steps
- has theoretical guarantee

• Jiaqi Sun, Wenrui Dai, Chenglin Li, Junni Zou, Hongkai Xiong, “Compressive Sensing via Unfolded l0-constrained Convolutional Sparse Coding,” **DCC 2021**.

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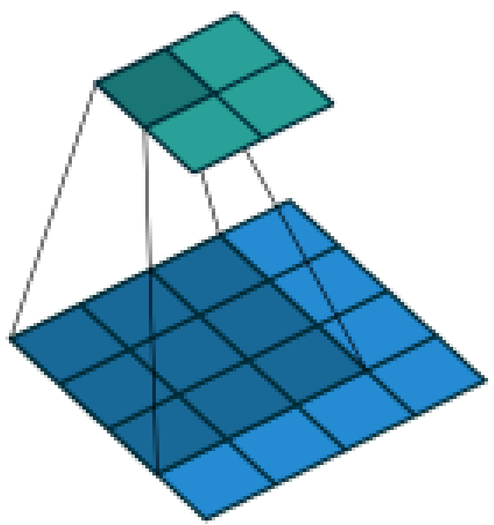
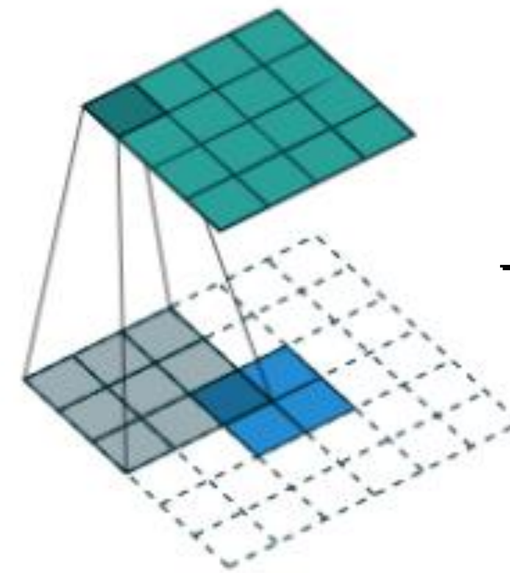


$\longrightarrow D^T\alpha$

Conv2d

Convolving a 3 × 3 kernel over a 4 × 4 input using unit strides



$\longrightarrow D\alpha$

Transposed Conv2d

Transpose of convolving a 3 × 3 kernel over a 4 × 4 input using unit strides

$$\mathbf{I} + a_1\mathbf{M} + a_2\mathbf{M}^2 + \cdots$$

$$\alpha^{t+1} = [I + \rho D^T D]^{-1}[\rho D^T(x^t - v^t) + (z^t - u^t)]$$

Theorem 1. *If $x \in \mathbb{R}^N$, and given any matrix $\mathbf{B} \in \mathbb{R}^{s\times N}(s \ll N)$, define an operator $\mathcal{F}(x) : \mathbb{R}^N \rightarrow \mathbb{R}^N$ given by $\mathcal{F}(x) = \mathbf{B}^\mathbf{T} ReLu(\mathbf{B}x)$, then $\mathcal{F} \succ 0$ means that the operator $\mathcal{F}(x)$ is a positive definite operator.*

$$\mathbf{M} = \mathbf{B}^\mathbf{T}\,\mathrm{Re}\,Lu(\mathbf{B})$$

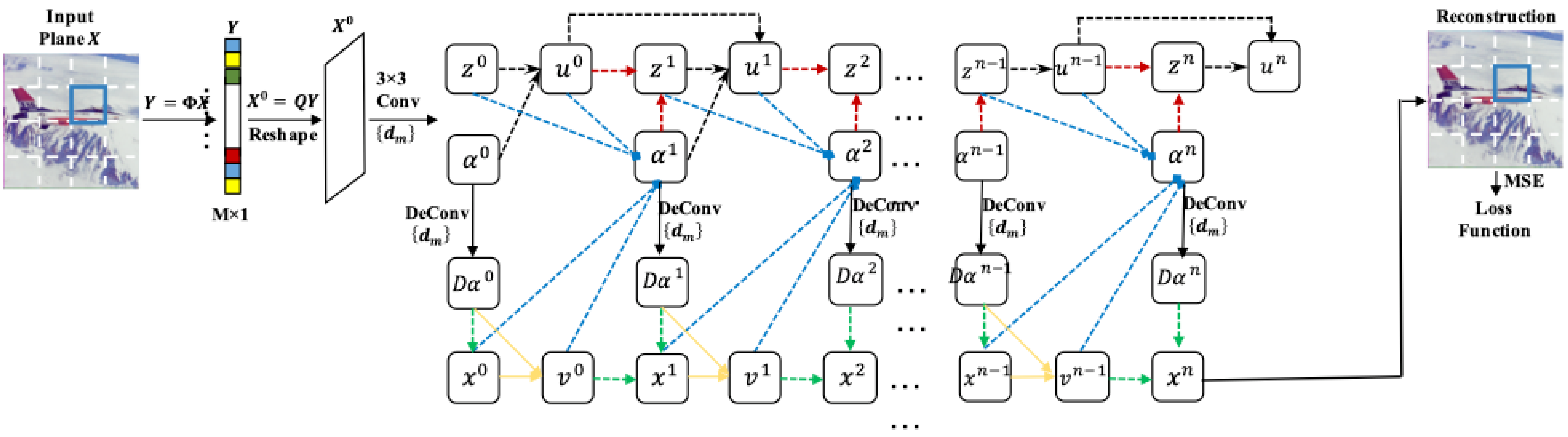
Positive definite operator

- Jiaqi Sun, Wenrui Dai, Chenglin Li, Junni Zou, Hongkai Xiong, "Compressive Sensing via Unfolded l0-constrained Convolutional Sparse Coding," **DCC 2021**.

Compressive Sensing via Unfolded ι_0-constrained Convolutional Sparse Coding

Objective function: $\min_{x,\alpha,z} \frac{1}{2}\|\Phi x - y\| + \lambda\Omega(z), \quad s.t.\, x = D\alpha = d * \alpha, z = \alpha$

Loss Function: $L_{MSE} = \frac{1}{N}\sum_{i=1}^{N}\|x_i - x_i^n\|_2^2$



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Contents

Compressive Sensing via Unfolded ι_0-constrained Convolutional Sparse Coding

Experiments:

Methods	Measurment Rates				# of Parameters
	0.25	0.1	0.04	0.01	
ReconNet	25.60	24.28	20.63	17.27	34893
ISTA-Net	31.53	25.80	21.23	17.30	336960
ISTA-Net+	32.57	26.64	21.31	17.34	336960
Proposed	27.58	25.16	21.64	17.95	37440

Tabel.1 Reconstruction performance in PSNR (dB) obtained by the proposed method, ReconNet, ISTA-Net on the Set11 dataset under the MRs of 0.01, 0.04, 0.10, and 0.25.

$$\Phi \in \mathbb{R}^{M \times N}$$

algorithm	CS rate		
	0.01	0.04	0.10
ReconNet	18.97	21.66	24.15
ISTA-Net	19.11	22.06	25.23
My	19.65	22.21	24.58

Table 2: Reconstruction performance in PSNR (dB) obtained by the proposed method, ReconNet, and ISTA-Net on the BSD68 dataset under the low MRs of 0.01, 0.04, and 0.10.

- Jiaqi Sun, Wenrui Dai, Chenglin Li, Junni Zou, Hongkai Xiong, "Compressive Sensing via Unfolded l0-constrained Convolutional Sparse Coding," **DCC 2021**.

Compressive Sensing via Unfolded ι_0-constrained Convolutional Sparse Coding

Experiments:

$$\mathbf{\Phi} \in \mathbb{R}^{\boldsymbol{N} \times \boldsymbol{N}}$$

Methods	ratio				
	10%	20%	30%	40%	50%
FFCSC	14.56	15.94	18.05	20.18	21.83
ADMM-Net	26.98	29.7	31.8	34.23	35.32
DLADMM	**27.78**	31.12	32.28	35.19	36.61
Proposed	27.70	**31.92**	**34.09**	**35.55**	**36.89**

Table 3: MRI Reconstruction performance in PSNR (dB) obtained by the proposed method, FFCSC, ADMM-Net, and DLADMM, when 10%, 20%, 30%, 40% and 50% pixels are missing.

- Jiaqi Sun, Wenrui Dai, Chenglin Li, Junni Zou, Hongkai Xiong, “Compressive Sensing via Unfolded l0-constrained Convolutional Sparse Coding,” **DCC 2021**.

Contents

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Contributions:

- The proposed method is the first attempt to develop a well-designed network architecture under the framework of ℓ_0-constrained convolutional sparse coding.

- The proposed method bridges the gap between DNN-based and conventional optimized-based CS methods.

- The proposed method incorporates DNNs to enhance the efficiency of reconstruction by strictly following the iterative alternating optimization and this method is guaranteed to converge.

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Q & A

Thanks !