# The Twelvefold Way of Non-Sequential Lossless Compression

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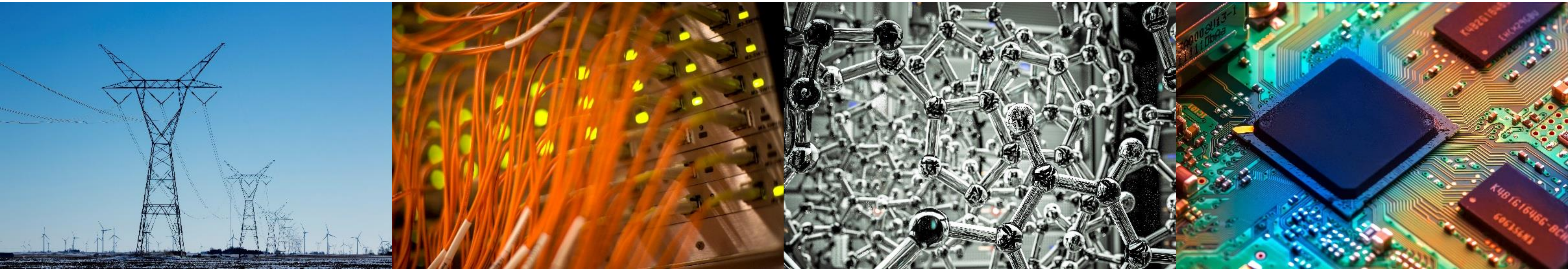

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## The polymath blog

February 20, 2021

### Polymath projects 2021

Filed under: polymath proposals — Gil Kalai @ 4:11 pm

After the success of Polymath1 and the launching of Polymath3 and Polymath4, Tim Gowers wrote a blog post "Possible future Polymath projects" for planning the next polymath project on his blog. The post mentioned 9 possible projects. (Four of them later turned  to polymath projects.) Following the post and separate posts describing some of the proposed projects, a few polls were taken and a problem – the Erdős discrepancy problem, was selected for the next project polymath5. In *Combinatorics and more* I reviewed some of the proposed projects from 2009, and in the same post I briefly and sometimes vaguely discussed the 2021 list, that I plan to present and discuss in detail in the next couple of months.

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# Order and disorder in data compression

- Several important applications produce huge amounts of data that are not simple sequences of distinguishable symbols, but have certain equivalence classes for patterns of symbols

- Often the ordering of sequences of scientific data records are irrelevant and so only the histogram or type class must be represented [Varshney and Goyal, 2006; Reznik, 2011; Gripon, et al., 2012; Steinruecken, 2015; Orlitsky, et al., 2006]
- Biological data [Ginart, et al., 2018], social graph structure [Choi and Szpankowski, 2012], neural network architecture [Basu and Varshney, 2017]
- Some non-sequential sources also arise in computer science via trace theory, which can be studied explicitly using interchange entropy [Savari, 2004]

- General approach to considering non-sequentiality in data compression is via group theory
- Alternative approach to get explicit bounds/computations for limits of lossless data compression is via enumerative combinatorics

# Rota's Twelvefold Way in enumerative combinatorics

**The twelve combinatorial objects and their enumeration formulas.**

| *f*-class | Any *f* | Injective *f* | Surjective *f* |
|---|---|---|---|
| Distinct<br>$f$ | $n$-sequence in $X$<br>$x^n$ | $n$-permutation of $X$<br>$x^{\underline{n}}$ | composition of $N$ with $x$ subsets<br>$x!\left\{ {n \atop x} \right\}$ |
| $\mathbf{S}_n$ orbits<br>$f \circ S_n$ | $n$-multisubset of $X$<br>$\binom{x+n-1}{n}$ | $n$-subset of $X$<br>$\binom{x}{n}$ | composition of $n$ with $x$ terms<br>$\binom{n-1}{n-x}$ |
| $\mathbf{S}_x$ orbits<br>$S_x \circ f$ | partition of $N$ into $\leq x$ subsets<br>$\sum_{k=0}^{x} \left\{ {n \atop k} \right\}$ | partition of $N$ into $\leq x$ elements<br>$[n \leq x]$ | partition of $N$ into $x$ subsets<br>$\left\{ {n \atop x} \right\}$ |
| $\mathbf{S}_n \times \mathbf{S}_x$ orbits<br>$S_x \circ f \circ S_n$ | partition of $n$ into $\leq x$ parts<br>$p_x(n+x)$ | partition of $n$ into $\leq x$ parts 1<br>$[n \leq x]$ | partition of $n$ into $x$ parts<br>$p_x(n)$ |

# Entropy computations for the twelvefold way

- We characterized the information-theoretic lossless compression limits for a wide range of non-sequentialities governed by the twelvefold way in combinatorics
- Explicit computations for all twelve settings were carried out for i.i.d. uniform and Bernoulli distributions
- Comparisons among settings provide quantitative insight into which equivalences reduce entropy the most
- See the paper for details [https://arxiv.org/abs/2011.04069]