# On the Choice of Sampling Rates in Multi-Rate Sampling

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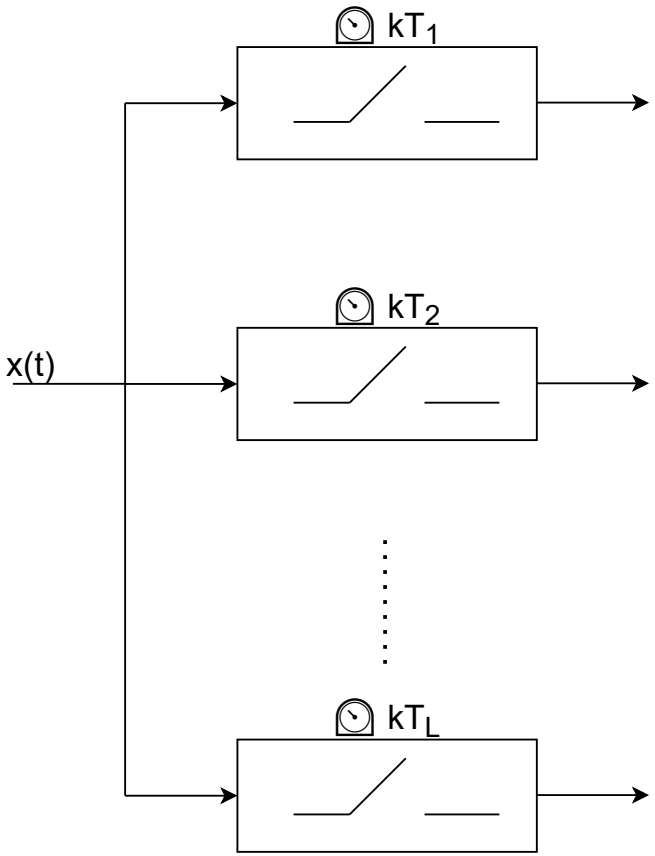
# The Multi-Rate Sampler

The Multi-Rate Sampler[1] (MRS) is a Compressed Sensing (CS) sampling scheme, based on $L$ parallel ADC's sampling at different sub-Nyquist rates $\nu_i = 1/T_i$.

For an acquisition lasting $\Delta$ seconds: $M_i = \Delta\nu_i$ samples are taken at branch $i$ ($N$ samples for an hypothetical branch sampling at the Nyquist rate $\nu_{Nyq}$).




[1] Fleyer *et al.*, Multirate synchronous sampling of sparse multiband signals. IEEE Trans. Signal Process., 2010

# Equations of the Multi-Rate Sampler

Sampling the received signal $X$ at branch $i$:

$$Z_i = F_i X. \tag{1}$$

$F_i$: folding matrix of size $M_i \times N$.

Combining samples from all $L$ branches:

$$Z = AX, \text{ where } Z = \begin{bmatrix} Z_1 \\ \vdots \\ Z_L \end{bmatrix} \text{ and } A = \begin{bmatrix} F_1 \\ \vdots \\ F_L \end{bmatrix}. \tag{2}$$

$Z$: observation vector of size $M = \sum_{i=1}^{L} M_i < N$;
$A$: measurement matrix of size $M \times N$.

# The MRS measurement matrix rank and the $\{M_i\}_i$

Our contribution: two theorems linking the rank of the MRS measurement matrix $A$ and the $\{M_i\}_i$.

**Theorem 1 (Measurement matrix rank upper-bound).**

$$\operatorname{rank} A \leq M - (L - 1). \tag{3}$$

**Theorem 2 (Equality case).**

$$\operatorname{rank} A = M - (L - 1) \Leftrightarrow \{M_i\}_i \text{ is pairwise coprime.} \tag{4}$$

# Visualizing Theorem 1 and 2 on the MRS sampling grid

Consider the sampling grid depicting sampling instants during $\Delta$ seconds for $L = 4$ and $M_i = 5$, 4, 3 and 6 respectively:

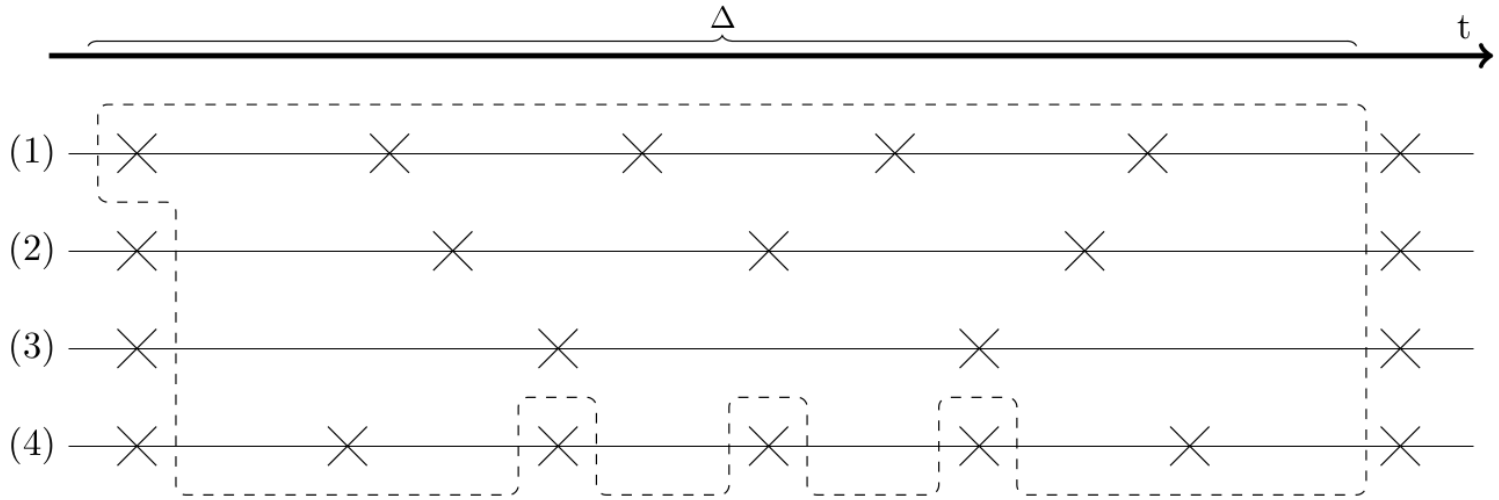



Figure: The MRS sampling grid.

Only the samples contained within the dashed area provide new information, the other ones are redundant.

# Simulation results: rank $A$ is more relevant than $M$

The $\rho - \delta$ plane: Depending on the reduction in sampling rate (from the Nyquist rate) and the sparsity (number of non-zeros) in the original signal $X$, MRS equation can be solved for $X$ (or not).

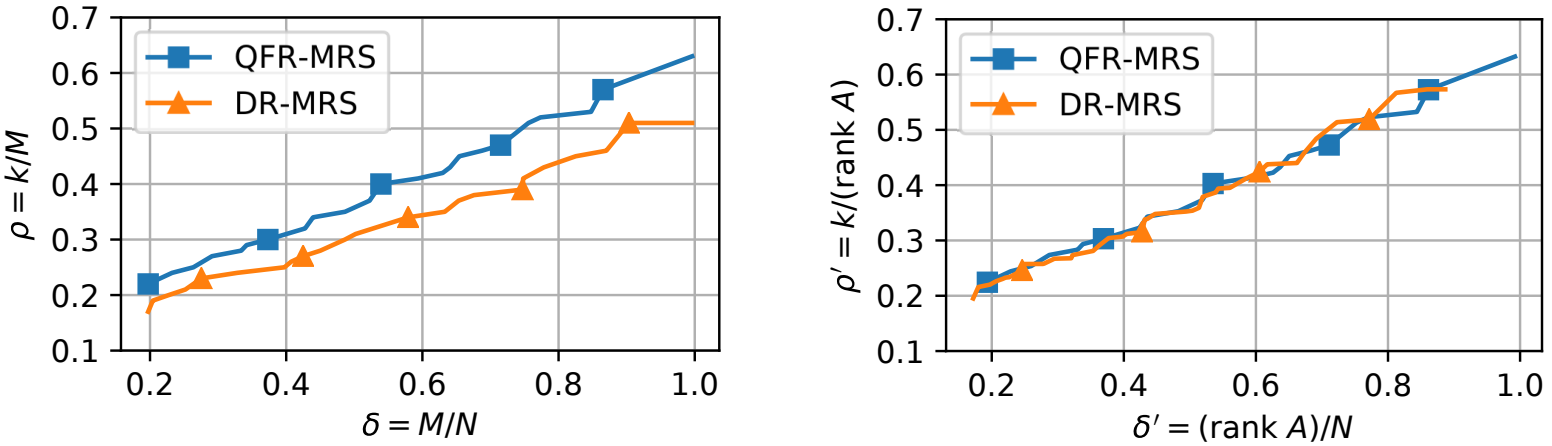



Figure: Phase transitions between successful recoveries (below lines) and failed recoveries (above lines). QFR = Quasi-Full Rank (coprime $M_i$'s), DR = Deficient Rank (non-coprime $M_i$'s).

MRS schemes with the similar matrix ranks have the same recovery performance (right), contrary to MRS schemes with similar numbers of measurements $M$ (left).