Iterative Geometry Calibration from Distance Estimates for Wireless Acoustic Sensor Networks

Tobias Gburrek, Joerg Schmalenstroeer, Reinhold Haeb-Umbach

University of Paderborn, Germany

Motivation

- Task: Geometry calibration in wireless acoustic sensor networks from observed speech signals
  - Estimate relative positions of nodes
- Observations: DNN-based distance estimates from signals Coherence-to-Diffuse-Power-Ratio (CDR) (see [1])
- Advantages of the approach:
  - Solely works with distance estimates between acoustic sources and sensor nodes
  - No special calibration signals required, natural speech sufficient.
  - Requires only coarse synchronization between sensor nodes

Optimization problem

- N sensor nodes at positions $\Omega_{\boldsymbol{P}} := \{\boldsymbol{P}_1, \dots, \boldsymbol{P}_N\}$
- K spatially distributed acoustic sources at positions $\Omega_{\boldsymbol{O}} := \{\boldsymbol{O}_1, \dots, \boldsymbol{O}_K\}$
- GARDE delivers estimates of all unknown positions $\Omega = \Omega_{\boldsymbol{P}} \cup \Omega_{\boldsymbol{O}}$ by solving

$$\hat{\Omega} = \underset{\Omega}{\operatorname{argmin}} \underbrace{\sum_{k=1}^{K} \sum_{n=1}^{N} \left(\hat{d}_{n,k}^2 - \|\boldsymbol{P}_n - \boldsymbol{O}_k\|_2^2 \right)^2}_{J(\Omega)}. \quad (1)$$

- Gradient of $J(\Omega)$ w.r.t. g-th sensor position $\boldsymbol{P}_g$

$$\left.\frac{\partial J}{\partial \boldsymbol{P}_g}\right|_{\hat{\Omega}} \overset{!}{=} 0 \Leftrightarrow \sum_{k=1}^{K} \left(\hat{\boldsymbol{P}}_g - \hat{\boldsymbol{O}}_k \right) \underbrace{\left(\hat{d}_{g,k}^2 - \left\| \hat{\boldsymbol{P}}_g - \hat{\boldsymbol{O}}_k \right\|_2^2 \right)}_{e_{gk}} = \boldsymbol{0} \quad (2)$$

- Gradient of $J(\Omega)$ w.r.t. h-th acoustic source position $\boldsymbol{O}_h$

$$\left.\frac{\partial J}{\partial \boldsymbol{O}_h}\right|_{\hat{\Omega}} \overset{!}{=} 0 \Leftrightarrow \sum_{n=1}^{N} \left(\hat{\boldsymbol{P}}_n - \hat{\boldsymbol{O}}_h \right) \underbrace{\left(\hat{d}_{n,h}^2 - \left\| \hat{\boldsymbol{P}}_n - \hat{\boldsymbol{O}}_h \right\|_2^2 \right)}_{e_{nh}} = \boldsymbol{0} \quad (3)$$

- No closed form solution $\rightarrow$ Iterative algorithm required
  - Assume either $\Omega_{\boldsymbol{P}}$ or $\Omega_{\boldsymbol{O}}$ to be known
  - Similarity of (2) and (3) enables use of common functions

Weighted Least Squares

$$\underbrace{\begin{bmatrix} 2\tilde{P}_{1,x} & 2\tilde{P}_{1,y} \\ \vdots & \vdots \\ 2\tilde{P}_{N,x} & 2\tilde{P}_{N,y} \end{bmatrix}}_{\boldsymbol{R}} \begin{bmatrix} \tilde{O}_{k,x} \\ \tilde{O}_{k,y} \end{bmatrix} = \underbrace{\begin{bmatrix} \hat{d}_{\nu,k}^2 + \tilde{P}_{1,x}^2 + \tilde{P}_{1,y}^2 - \hat{d}_{1,k}^2 \\ \vdots \\ \hat{d}_{\nu,k}^2 + \tilde{P}_{N,x}^2 + \tilde{P}_{N,y}^2 - \hat{d}_{N,k}^2 \end{bmatrix}}_{\boldsymbol{b}}$$

- Estimate location $\hat{\boldsymbol{O}}_k$ with weights $W_{n,n} = 1/\tilde{d}_{n,k}^2$

$$\hat{\boldsymbol{O}}_k = \left(\boldsymbol{R}^T \boldsymbol{W} \boldsymbol{R} \right)^{-1} \boldsymbol{R}^T \boldsymbol{W} \boldsymbol{b} + \boldsymbol{P}_\nu \quad (4)$$

GARDE algorithm

Algorithm 1: GARDE algorithm

Data: Observed distances $\boldsymbol{A}_P$;

Init: $\alpha = 0.2$; $\beta = 0.2$;
for $i = 1 \rightarrow N$, $j = 1 \rightarrow N$ **do**
  $\widehat{D}_{ij} = \frac{1}{2} \max_l (|\hat{d}_{il} - \hat{d}_{jl}|) + \frac{1}{2} \min_k (\hat{d}_{iu} + \hat{d}_{ju})$
end
$\widehat{\Omega}_{\boldsymbol{P}} = \texttt{MultiDimensionalScaling}(\widehat{\boldsymbol{D}})$;
$\widehat{\Omega}_{\boldsymbol{O}} = \texttt{LSPos}\left(\widehat{\Omega}_{\boldsymbol{P}}, \boldsymbol{A}_P, \hat{d}_{n,k}\right)$;
$\widehat{\Omega} = \texttt{Iterate}(\widehat{\Omega}, \boldsymbol{A}_P)$;
$\widehat{\Omega}_o = \widehat{\Omega}$;
for $g = 1 \rightarrow \texttt{NumAnnealing}$ **do**
  $\widehat{\Omega} = \texttt{Iterate}(\widehat{\Omega}, \boldsymbol{A}_P)$;
  $\widehat{\Omega}_o = \texttt{OptSelect}(\widehat{\Omega}, \widehat{\Omega}_o, \boldsymbol{A}_P)$;
  $\hat{\boldsymbol{P}}_n = \hat{\boldsymbol{P}}_{n,o} + \mu(g) \cdot \texttt{randn}(), \forall n \in \{1, \dots, N\}$;
  $\hat{\boldsymbol{O}}_k = \hat{\boldsymbol{O}}_{k,o} + \mu(g) \cdot \texttt{randn}(), \forall k \in \{1, \dots, K\}$;
end

Result: $\widehat{\Omega}_o$;

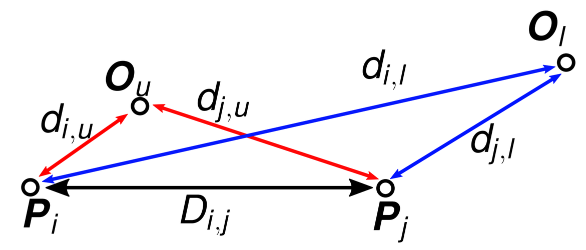
Function *Iterate* $(\widehat{\Omega}, \boldsymbol{A}_P)$:
  for $i = 1 \rightarrow \texttt{NumIterations}$ **do**
    $\widehat{\Omega}_{\boldsymbol{O},fit} = \texttt{FitSelect}(\widehat{\Omega}, \boldsymbol{A}_P)$;
    $\widehat{\Omega}_{\boldsymbol{P},fit} = \texttt{LSPos}(\widehat{\Omega}_{\boldsymbol{O},fit}, \boldsymbol{A}_P, \left\|\hat{\boldsymbol{P}}_n - \hat{\boldsymbol{O}}_k\right\|_2)$;
    $\widetilde{\Omega}_{\boldsymbol{P},fit} = \texttt{Map2Ref}(\widehat{\Omega}_{\boldsymbol{P},fit} \rightarrow \widehat{\Omega}_{\boldsymbol{P}})$;
    $\hat{\boldsymbol{P}}_n = \alpha \cdot \hat{\boldsymbol{P}}_n + (1 - \alpha) \cdot \widetilde{\boldsymbol{P}}_{n,fit}, \forall n \in \{1, \dots, N\}$;
    $\widetilde{\Omega}_{\boldsymbol{O}} = \texttt{LSPos}(\widehat{\Omega}_{\boldsymbol{P}}, \boldsymbol{A}_P, \hat{d}_{n,k})$;
    $\hat{\boldsymbol{O}}_k = \beta \cdot \hat{\boldsymbol{O}}_k + (1 - \beta) \cdot \widetilde{\boldsymbol{O}}_k, \forall k \in \{1, \dots, K\}$;
  end
  return $\widehat{\Omega}$;

Multidimensional Scaling Initialization

- Approximation by triangular inequality

$$\max_l (|\hat{d}_{i,l} - \hat{d}_{j,l}|) \leq D_{i,j} \leq \min_u (\hat{d}_{i,u} + \hat{d}_{j,u})$$
$$\widehat{D}_{i,j} = [\max_l (|\hat{d}_{i,l} - \hat{d}_{j,l}|) + \min_u (\hat{d}_{i,u} + \hat{d}_{j,u})]/2,$$

where i, j are sensor node indices and u, l are acoustic source indices.



Experiments

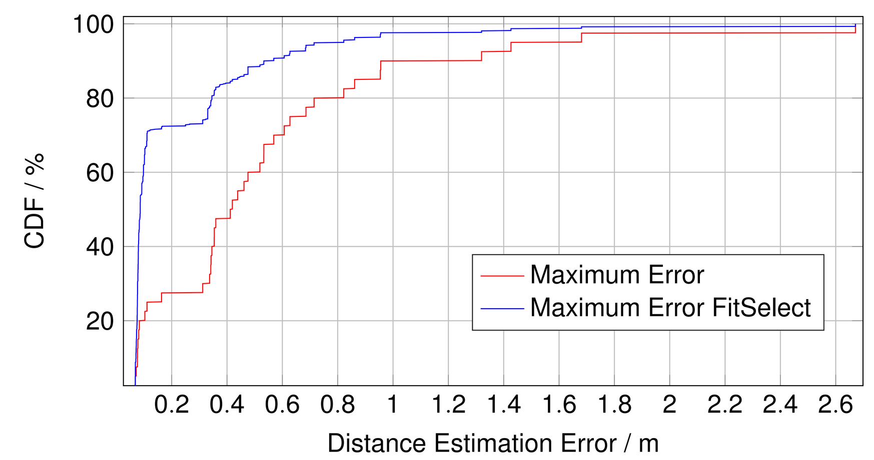


Figure 1: Cumulative density function (CDF) of maximum distance error in $\boldsymbol{A}_p$ and selected subset of $\boldsymbol{A}_p$ by FitSelect function.

Method	$T_{60} = 200\,\text{ms}$	$T_{60} = 400\,\text{ms}$
DoA + Scaling	0.043 m	0.103 m
GARDE	0.017 m	0.032 m

Table 1: RMSE of the sensor positions for different geometry calibration methods

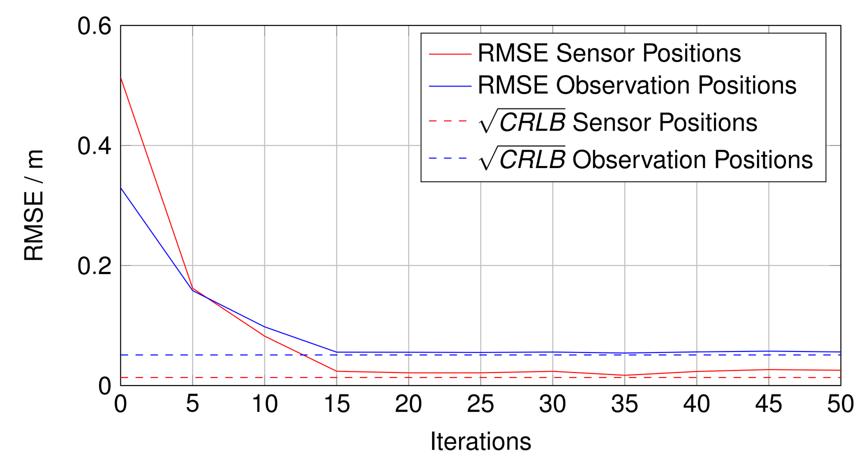


Figure 2: Comparison between CRLB of estimator and RMSE of observation and sensor positions. Number of iterations and annealing rounds were equally chosen.

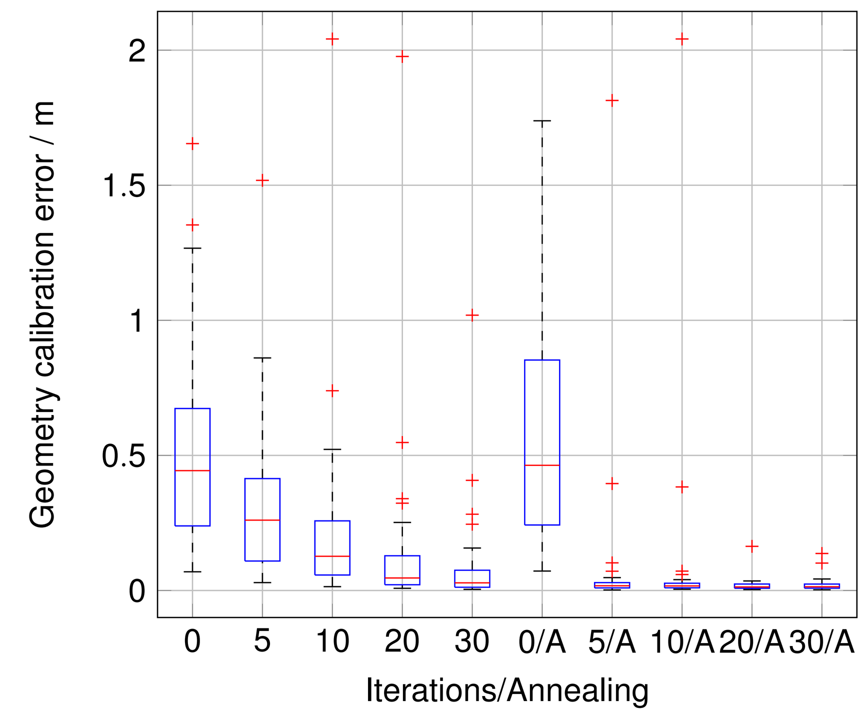


Figure 3: Geometry calibration error for different number of iterations and optionally 30 annealing rounds (results marked with "/A").

Conclusions

- GARDE[1] algorithm: Iterative WLS-based algorithm for geometry calibration of WASNs
  - Positions of sensor nodes and acoustic sources
- Derived CRLB for the geometry calibration approach

[1]Python implementation of GARDE is available in the paderwasn repository: `https://github.com/fgnt/paderwasn`

[1] T. Gburrek, J. Schmalenstroeer, A. Brendel, W. Kellermann, and R. Haeb-Umbach, "Deep neural network based distance estimation for geometry calibration in acoustic sensor networks," in EUSIPCO 2021

Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) - Project 282835863