ON OVERFITTING IN DISCRETE SUPER-RESOLUTION RECOVERY

Paper #2213

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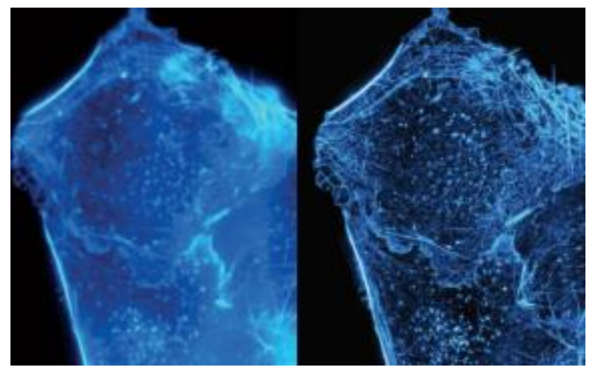
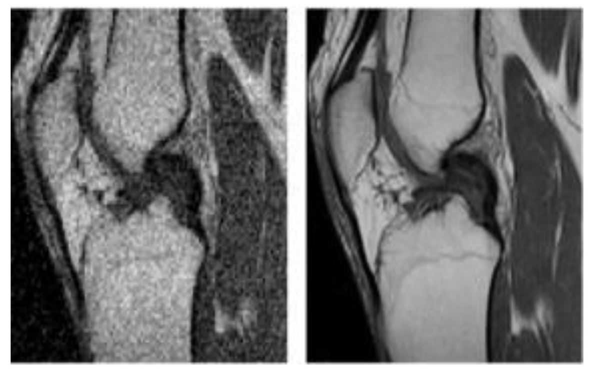
Application: Super-resolution



Microscopy imaging [EKB]

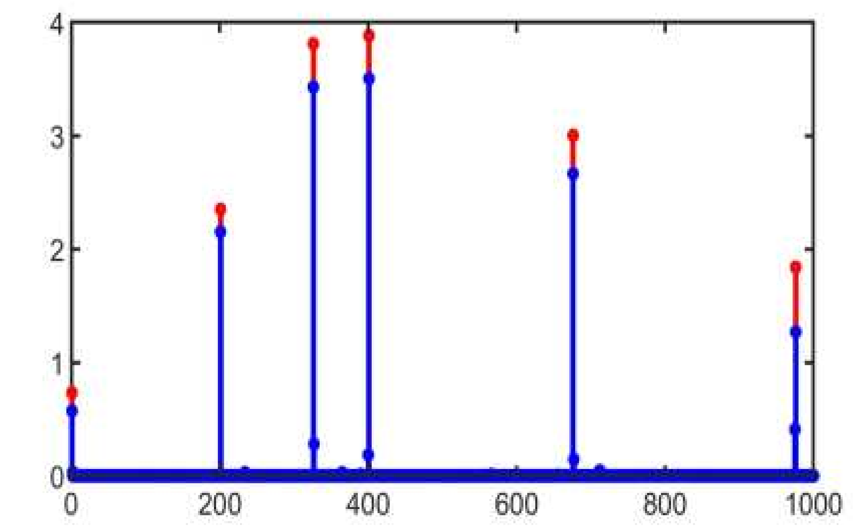


Medical imaging [Medium]

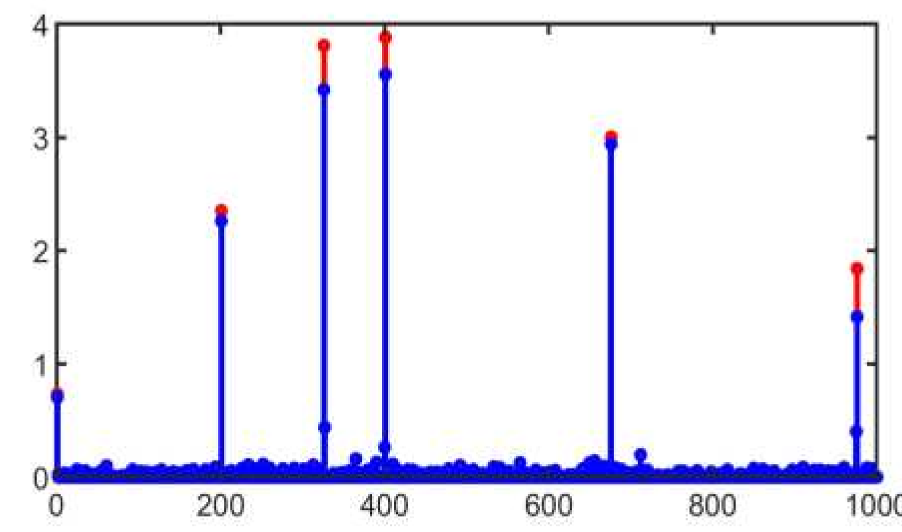
Astronomy [Nap]

Goal: Reconstruct the sparse target signal from its low-frequency measurements

Motivation: Comparison between Non-overfitting and Overfitting Recoveries



Non-overfitting BPDN-2 Recovery



Overfitting BPDN Recovery

Key observation: Estimates of non-overfitting and overfitting recoveries are similar.

Question: Is the prior knowledge of the noise necessary for a stable estimate?

Proof Sketch

Quotient property

Lemma 1: $Au = w, \|u\|_q \le c(M,N)\|w\|$

Null space property

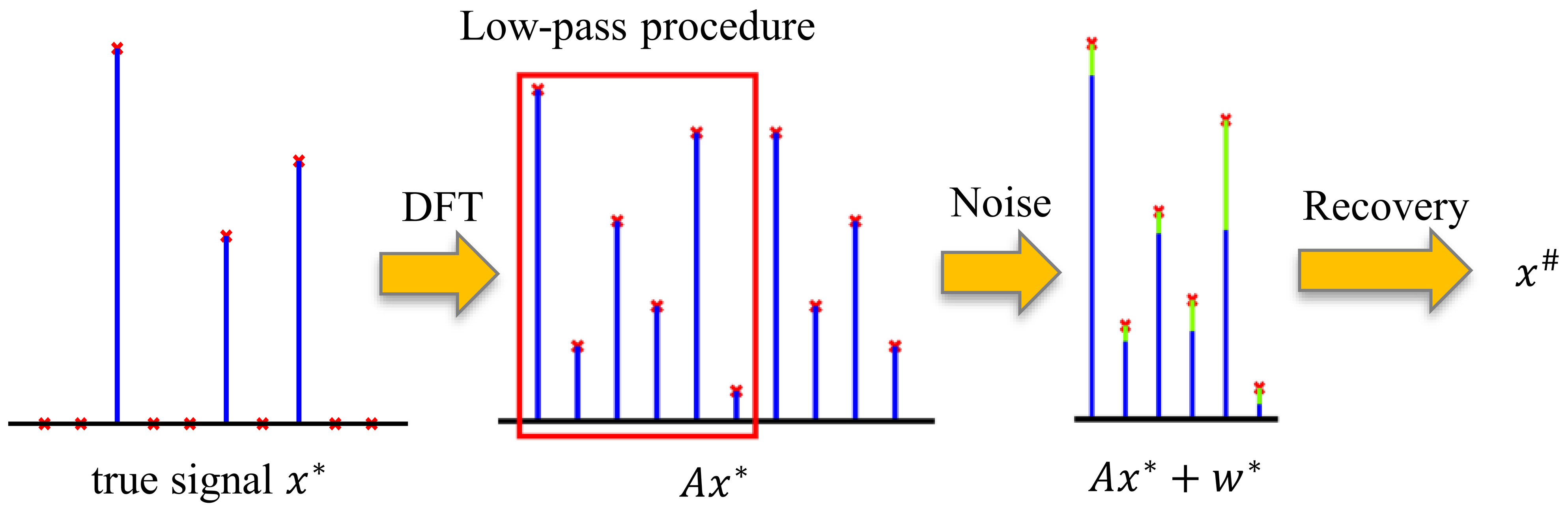
Lemma 2: $\|V_S\|_q \le \frac{\gamma}{s^{1-1/q}}\|V_{S^c}\|_1 + \tau\|Av\|$

→ **Overfitting stability guarantee**

Challenge: RIP/Coherence based techniques don't apply to deterministic A

New insight: An interpolation-based technique under the separation condition replaces NSP. $\|V_S\|_1 \le (1-\rho)\|V_{S^c}\|_1 + \|q\|_2\|Av\|_2$

Super-resolution Recovery Framework



Studied overfitting recovery:

$$x^\# = \min_x \|x\|_1,$$
$$s.t.\ Y = Ax$$

No prior knowledge of noise needed

Main Result

Theorem: consider the overfitting recovery algorithm with partial DFT matrix A and the true sparse source signal x^*. Suppose the true support S satisfies the separation condition,

$$\min_{m,n\in S, m<n}\left\{\frac{|m-n|}{N}, \frac{|m-n+N|}{N}\right\} \ge \frac{2.5}{f_c}\ with\ f_c \ge 128,$$

then for any additive noise w^*, we have

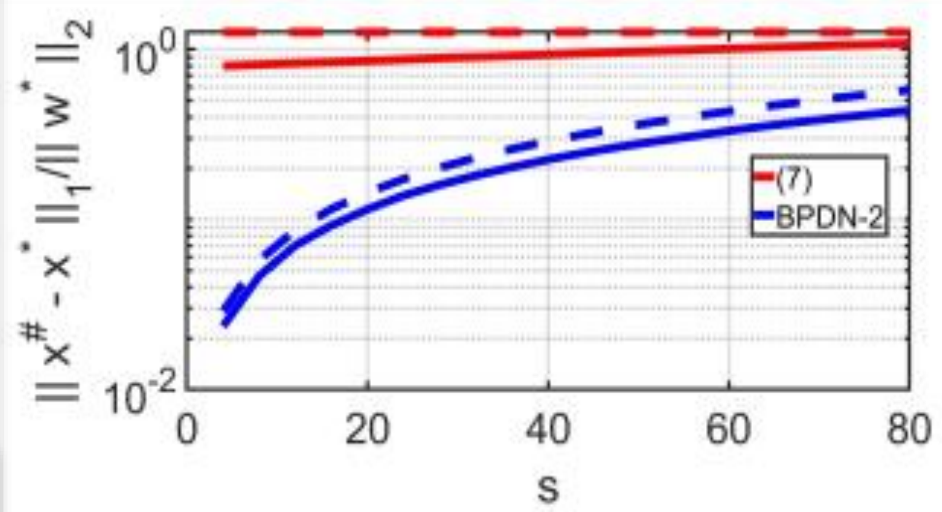
$$\left\|x_{BP}^\# - x^*\right\|_1 \le \frac{4}{\rho}\|w^*\|_2 \le 12\|w^*\|_2\left(\frac{N}{f_c}\right)^2.$$

Simulations

Comparison of the estimation error $\left\|x_{BP}^\# - x^*\right\|_1$.

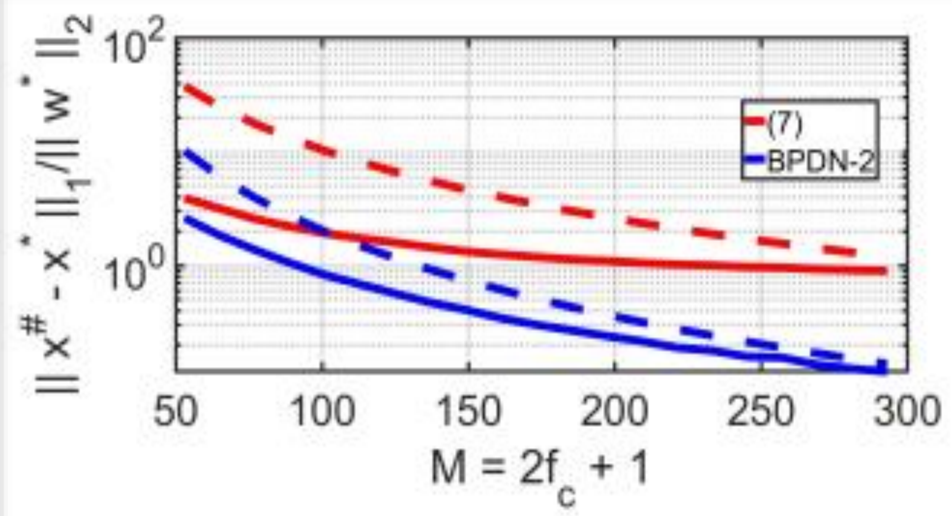
Setting: N = 1600, $\sigma_x^2 = 10$, $\sigma_w^2 = 1$ and $\epsilon = 1.02\ \|w^*\|_2$.

Experiment A



Fix M = 521, change s

Experiment B



Fix s = 8, change M